

ALGEBRA AND GEOMETRY SEMINAR

Wednesday March 9, 2011, 13.15-15.00

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Explicit structure of some infinite graded Lie algebras
in algebraic topology and in the theory of numerical semigroups.

Abstract

Infinite graded Lie algebras occur in several cases in practice. Here are some typical examples.

1) Let G be a group. The descending central series of G is defined by first forming the iterated commutators;

$$G_1 \supseteq G_2 \supseteq G_3 \dots \supseteq G_n \dots$$

where $G_1 = G$ and $G_n = [G, G_{n-1}]$ for $n \geq 2$, where $[,]$ is the group of commutators. The graded associated abelian group $\oplus_{i \geq 1} G_i/G_{i+1}$ is in a natural way a graded Lie ring and tensoring it with \mathbf{Q} we obtain a graded Lie algebra, whose structure is largely unknown, but there are favourable cases where lots of things can be said.

2) Another example is found by taking a topological space X and taking the homotopy groups $\pi_i(X)$ under Whitehead product.

3) Some time ago Sturmfels and myself found a toric variety defined by some monomials $s^a t^b$ inside $k[s,t]$ whose ring R had quadratic relations but R was certainly not Koszul since the generating series of the dimensions of the $\text{Tor}_i^R(k, k)$ was a transcendental function. As noted by Fröberg and myself this example also gave a numerical semigroup ring inside $k[t]$ whose Tor series had similar properties.

In the talk I will define most of the things mentioned above. I will also describe simplified methods for obtaining the previous results using a Mathematica programme Liedim by Clas Löfwall (with C++ -variants by Stefan Pettersson and Samuel Lundqvist), a programme BERGMAN by Backelin and others and the programme M2 (Macaulay 2). As a bonus I will deduce an example of a Gorenstein numerical semigroup ring with transcendental series which seems to be the first one of its kind (note that the examples treated by Zariski and others in connection with plane curve singularities gave complete intersections which have rational series.)

The seminar will be given in room 306, Kräftriket, Roslagsvägen 101, hus 6.

Stockholm, February 28, 2011

Torsten EKEDAHL, Carel FABER, Ralf FRÖBERG, Sergei MERKULOV, Sandra di ROCCO, Jan-Erik ROOS, Roy SKJELNES