FOURIERSERIE:

$$f(t) \sim \sum_{n=-\infty}^{\infty} c_n \cdot e^{int} \qquad \left(= \sum \hat{f}(n)\, e^{int} \right)$$

$$c_n := \frac{1}{2\pi} \int_{\mathbb{T}} f(t) \cdot e^{-int}\, dt \qquad \left(= \hat{f}(n) \right)$$

"REELL FORM":

$$f(t) \sim \frac{a_0}{2} + \sum_{n \geq 1} a_n \cdot \cos nt + b_n \cdot \sin nt$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cdot \cos nt\, dt$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cdot \sin nt\, dt$$

HÄRLEDNING:

$$\sum c_n e^{int} = c_0 + \sum_{n \geq 1} \left(c_n e^{int} + c_{-n} \cdot e^{-int} \right)$$

$$= c_0 + \sum_{n \geq 1} \left(c_n \cdot (\cos nt + i \sin nt) + c_{-n} \cdot (\cos nt - i \sin nt) \right)$$

$$= c_0 + \sum_{n \geq 1} \underbrace{(c_n + c_{-n})}_{a_n} \cdot \cos nt + \underbrace{i \cdot (c_n - c_{-n})}_{b_n} \cdot \sin nt$$

$$a_n = \frac{1}{2\pi} \int f(t) \cdot e^{-int} dt + \frac{1}{2\pi} \int f(t)\, e^{int} dt = \frac{1}{2\pi} \int f(t) \cdot \left(e^{-int} + e^{int} \right) dt$$

$$\left(e^{int} + e^{-int} = \cos nt + i \sin nt + \cos nt - i \sin nt = 2 \cdot \cos nt \right)$$

$$= \frac{1}{2\pi} \int f(t) \cdot 2 \cdot \cos nt\, dt = \frac{1}{\pi} \int f(t) \cdot \cos nt\, dt$$

$$b_n = \frac{i}{2\pi} \int f(t) \cdot \left(e^{-int} - e^{int} \right) dt$$

$$\left(e^{-int} - e^{int} = \cos nt - i \sin nt - \cos nt - i \sin nt = -2i \sin nt \right)$$

$$= \frac{i}{2\pi} \int f(t) \cdot (-2i \sin nt)\, dt = \frac{1}{\pi} \int f(t) \cdot \sin nt\, dt$$

$$\left. \begin{array}{l} c_0 = \frac{1}{2\pi} \int f(t)\, dt \\ a_0 = \frac{1}{\pi} \int f(t)\, dt \end{array} \right\} \Rightarrow a_0 = 2 \cdot c_0 \iff c_0 = \frac{a_0}{2}$$

$\boxed{2.10}$ $\underbrace{1+0-1}+\underbrace{1+0-1}+\underbrace{1+0-1}+\dots$

$$s_k = \begin{cases} 1, & k \neq n\cdot 3 \\ 0, & k = n\cdot 3 \end{cases}$$

$$\sigma_n = \frac{1}{n}(s_1+s_2+\dots+s_n) = \frac{1}{n}\cdot\left(\overbrace{1+1+0}^{=2}+\overbrace{1+1+0}^{=2}+\dots\right)$$

$$\sigma_{3n} = \frac{1}{3n}\cdot n\cdot 2 = \frac{2}{3}$$

$$\sigma_{3n+1} = \frac{1}{3n+1}\cdot(n\cdot 2+1) = \frac{2n}{3n+1}+\frac{1}{3n+1} \longrightarrow \frac{2}{3}$$

$$\sigma_{3n+2} = \frac{1}{3n+2}\cdot(n\cdot 2+2) = \frac{2n}{3n+2}+\frac{2}{3n+2} \longrightarrow \frac{2}{3}$$

$$\Longrightarrow \sigma_n \to \frac{2}{3}$$

2012-12-11

$\boxed{A1}$

$$K(x) = \begin{cases} \frac{3}{4}(1-x^2), & |x| < 1 \\ 0, & |x| \geq 1 \end{cases}$$

a) VISA ATT $K_n(x) := n \cdot K(nx)$ ÄR EN POSITIV SUMMATIONSKÄRNA.

(i) $K_n(x) \geq 0$?

$$K_n(x) = \begin{cases} n \cdot \frac{3}{4} \cdot (1-(nx)^2), & |nx| < 1 \\ 0, & |nx| \geq 1 \end{cases}$$

SÅ OM $|nx| < 1$ SÅ $K_n(x) = n \cdot \frac{3}{4} \cdot (1-(nx)^2) > 0$

OM $|nx| \geq 1$ SÅ $K_n(x) = 0$

D.V.S. $K_n(x) \geq 0$ $\underline{\underline{\text{OK}}}$

(ii) $\int_{-\infty}^{\infty} K_n(x)\,dx = 1$?

$$\int_{-\infty}^{\infty} K_n(x)\,dx = \int_{-\frac{1}{n}}^{\frac{1}{n}} n \cdot \frac{3}{4}\left(1-(nx)^2\right) = \frac{3n}{4}\left[x - n^2 \cdot \frac{x^3}{3}\right]_{-\frac{1}{n}}^{\frac{1}{n}}$$

$$= \frac{3n}{4}\left(\frac{2}{n} - \left(n^2 \cdot \frac{1}{3n^3} + n^2 \frac{1}{3n^3}\right)\right) = \frac{3n}{4}\left(\frac{2}{n} - \frac{2}{3n}\right)$$

$$= \frac{3}{2} - \frac{1}{2} = 1 \qquad \underline{\underline{\text{OK}}}$$

(iii) $\lim_{n\to\infty} \int_{|x|>\delta} K_n(x)\,dx = 0$?

$\delta > 0$ ÄR GIVET. VÄLJ $n \geq N$ DÄR $\frac{1}{N} < \delta$. D.V.S. $\frac{1}{n} < \delta$, $\forall n \geq N$.

DÅ OM $n \geq N$ SÅ

$$\int_{|x|>\delta} K_n(x)\,dx \leq \int_{|x|>\frac{1}{n}} K_n(x)\,dx = \int_{|x|>\frac{1}{n}} 0\,dx = 0 \qquad \underline{\underline{\text{OK}}}$$

A1

b) f KONTINUERLIGT DERIVERBAR. BESTÄM $\lim_{n\to\infty} \int_{-\frac{1}{n}}^{\frac{1}{n}} K_n'(x) f(x)\, dx$.

$$\int_{-\frac{1}{n}}^{\frac{1}{n}} K_n'(x) f(x)\, dx = \Big[K_n(x) f(x)\Big]_{-\frac{1}{n}}^{\frac{1}{n}} - \int_{-\frac{1}{n}}^{\frac{1}{n}} K_n(x) \cdot f'(x)\, dx$$

$$= 0 - \int_{-\frac{1}{n}}^{\frac{1}{n}} K_n(x) f'(x)\, dx \longrightarrow -f'(0) \quad \text{DÅ } n \to \infty$$

EFTERSOM $K_n(x)$ ÄR POSITIV SUMMATIONS-KÄRNA.

2014-03-12

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$$K_n(t) = \begin{cases} n, & |t| < \frac{1}{2n} \\ 0, & |t| \geq \frac{1}{2n} \end{cases} \quad , \quad f \text{ KONTINUERLIG}$$

BEVISA ATT

$$\lim_{n \to \infty} \int_{-1}^{1} K_n(t) f(t) dt = f(0)$$

ENL. DEF. AV lim SKA VISAS ATT: $\forall \varepsilon > 0 \exists N$ S.A. $\forall n \geq N$ SÅ GÄLLER

$$\left| \int_{-1}^{1} K_n(t) f(t) dt - f(0) \right| < \varepsilon .$$

LÅT $\varepsilon > 0$ VARA GIVET.

$$\left| \int_{-1}^{1} K_n(t) f(t) \, dt - f(0) \right| = \left| \int_{-\frac{1}{2n}}^{\frac{1}{2n}} n \cdot f(t) dt - f(0) \cdot n \cdot \int_{-\frac{1}{2n}}^{\frac{1}{2n}} dt \right|$$

$$= \left| n \cdot \int_{-\frac{1}{2n}}^{\frac{1}{2n}} (f(t) - f(0)) dt \right| \leq n \cdot \int_{-\frac{1}{2n}}^{\frac{1}{2n}} |f(t) - f(0)| \, dt$$

EFTERSOM ATT f ÄR KONT. I 0 SÅ $\exists \delta$ S.A. $|f(t) - f(0)| < \varepsilon$ FÖR ALLA $|t| < \delta$. VÄLJ NU N SÅ ATT $\frac{1}{2N} < \delta$, D.V.S. $N > \frac{1}{2\delta}$. ~~[illegible]~~ ~~[illegible]~~ OM $n \geq N$ SÅ $\frac{1}{2n} \leq \frac{1}{2N} < \delta$, VILKET GER

$$n \int_{-\frac{1}{2n}}^{\frac{1}{2n}} |f(t) - f(0)| \, dt < n \cdot \int_{-\frac{1}{2n}}^{\frac{1}{2n}} \varepsilon \, dt = \varepsilon, \quad \forall n \geq N.$$

VILKET SKULLE VISAS.

OM f HAR PRIMITIV FUNKTION F SÅ

$$\int_{-1}^{1} K_n(t) f(t) \, dt = n \cdot \int_{-\frac{1}{2n}}^{\frac{1}{2n}} f(t) dt = n \cdot \left(F\left(\tfrac{1}{2n}\right) - F\left(-\tfrac{1}{2n}\right) \right)$$

$$= \frac{1}{2} \left(\frac{F\left(\frac{1}{2n}\right) - F(0)}{\frac{1}{2n}} + \frac{F\left(-\frac{1}{2n}\right) - F(0)}{-\frac{1}{2n}} \right) \longrightarrow \frac{1}{2} \cdot \left(F'(0) + F'(0) \right) = f(0)$$

$\boxed{4.4}$ BERÄKNA FOURIERSERIE FÖR $f(t) = t+1, \quad |t| < \pi$.

FOURIERSERIE KAN SKRIVAS

$$f(t) \sim \frac{a_0}{2} + \sum_{n \geq 1} a_n \cdot \cos nt + b_n \cdot \sin nt$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cdot \cos nt \, dt$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cdot \sin nt \, dt$$

BERÄKNA:

$$(n>0) \quad a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (t+1) \cdot \cos nt \, dt = \frac{1}{\pi} \int_{-\pi}^{\pi} \underbrace{t \cdot \cos nt}_{\text{UDDA}} \, dt + \frac{1}{\pi} \int_{-\pi}^{\pi} \underbrace{\cos nt}_{\text{JÄMN}} \, dt$$

$$= \frac{2}{\pi} \int_{0}^{\pi} \cos nt \, dt = \frac{2}{\pi} \left[\frac{1}{n} \sin nt \right]_0^{\pi} = \frac{2}{\pi \cdot n} \Big(\underbrace{\sin n\pi}_{=0} - \underbrace{\sin 0}_{=0} \Big) = 0$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (t+1) \cdot \sin nt \, dt = \frac{1}{\pi} \int_{-\pi}^{\pi} \underbrace{t \cdot \sin nt}_{\text{JÄMN}} \, dt + \frac{1}{\pi} \int_{-\pi}^{\pi} \underbrace{\sin nt}_{\text{UDDA}} \, dt$$

$$= \frac{2}{\pi} \int_{0}^{\pi} t \cdot \sin nt \, dt = \frac{2}{\pi} \cdot \left(\left[-\frac{t}{n} \cdot \cos nt \right]_0^{\pi} + \frac{1}{n} \underbrace{\int_0^{\pi} \cos nt \, dt}_{=0 \text{ ENL. OVAN}} \right)$$

$$= \frac{2}{\pi \cdot n} \Big(-\pi \cdot \cos n\pi + 0 \cdot \cos 0 \Big) = -\frac{2 \cdot \cos n\pi}{n}$$

$$= \frac{2}{n} \cdot (-1)^{n+1}$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} (t+1) \, dt = \frac{1}{\pi} \cdot \int_{-\pi}^{\pi} dt = 2$$

$$\underline{\text{SVAR: } f(t) \sim 1 + \sum_{n \geq 1} \frac{2 \cdot (-1)^{n+1}}{n} \cdot \sin nt}$$