Robust Quaternion-based Cooperative Manipulation without Force/Torque Information [*]

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Abstract: This paper proposes a task-space control protocol for the collaborative manipulation of a single object by N robotic agents. The proposed methodology is decentralized in the sense that each agent utilizes information associated with its own and the object's dynamic/kinematic parameters and no on-line communication takes place. Moreover, no feedback of the contact forces/torques is required, therefore employment of corresponding sensors is avoided. An adaptive version of the control scheme is also introduced, where the agents' and object's dynamic parameters are considered unknown. We also use unit quaternions to represent the object's orientation. In addition, load sharing coefficients between the agents are employed and internal force regulation is guaranteed. Finally, experimental studies with two robotic arms verify the validity and effectiveness of the proposed control protocol.

Keywords: Robotic manipulators, Multi-agent systems, Cooperative control, Adaptive control, Robust control.

1. INTRODUCTION

Multi-agent manipulation has gained a notable amount of attention lately. Difficult tasks including manipulation of heavy loads that cannot be handled by a single robotic arm necessitate the employment of multiple agents. Early works develop control architectures where the robotic agents communicate and share information with each other (Schneider and Cannon, 1992), and completely decentralized schemes (Liu et al., 1996; Liu and Arimoto, 1998; Zribi and Ahmad, 1992; Khatib et al., 1996; Caccavale et al., 2000) where each agent uses only local information or observers (Gudiño-Lau et al., 2004), avoiding potential communication delays.

Impedance and force/motion control is the most common methodology utilized in the related literature (Schneider and Cannon, 1992; Caccavale et al., 2008; Heck et al., 2013; Erhart and Hirche, 2013; Kume et al., 2007; Szewczyk et al., 2002; Tsiamis et al., 2015; Ficuciello et al., 2014; Ponce-Hinestroza et al., 2016; Gueaieb et al., 2007). Most of the aforementioned works employ force/torque sensors to acquire knowledge of the manipulator-object contact forces/torques which however may result to performance decline due to sensor noise or mounting difficulties. Force/Torque sensor-free methodologies can be found in (Wen and Kreutz-Delgado, 1992; Yoshikawa and Zheng, 1993; Liu et al., 1996; Kume et al., 2007; Zribi and Ahmad,

1992), which have inspired the dynamic modeling in this work.

Another important characteristic is the representation of the agent and object orientation. The most commonly used tools for orientation representation consist of rotation matrices, Euler angles and the angle/axis convention. Rotation matrices, however, are rarely used in robotic manipulation tasks due to the difficulty of extracting an error vector from them. Moreover, the mapping from Euler angles and angle/axis values to angular velocities exhibits singularities at certain points, rendering thus these representations incompetent. On the other hand, the representation using unit quaternions, which is employed in this work, constitutes a singularity-free orientation representation, without complicating the control design. Unit quaternions are employed in (Campa et al., 2006; Caccavale et al., 2000, 2008; Aghili, 2011) for manipulation tasks and in (Erhart and Hirche, 2016) for the analysis of the interaction dynamics in cooperative manipulation.

In addition, most of the works in the related literature consider known dynamic parameters regarding the object and the robotic agents. However, the accurate knowledge of such parameters, such as masses or moments of inertia, can be a challenging issue; (Liu and Arimoto, 1998) proposes an adaptive control scheme through gain tuning and (Caccavale et al., 2000) considers the robust pose regulation problem. The adaptive control of single manipulation tasks with uncertain kinematic and dynamic parameters is tackled in (Cheah et al., 2006; Wang, 2017).

In (Erhart and Hirche, 2013) and (Erhart et al., 2013) kinematic uncertainties are considered whereas (Erhart and Hirche, 2015) performs an internal force and load

distribution analysis. In (Tsiamis et al., 2015) a leader-follower scheme is employed, and in (Wang and Schwager, 2015) a decentralized force consensus algorithm is developed; (Murphey and Horowitz, 2008) and (Chaimowicz et al., 2003) address the problem employing hybrid control schemes. A formation-control approach is considered in (Bai and Wen, 2010) and a fuzzy control methodology is presented in (Li et al., 2015). In (Petitti et al., 2016) the agent dynamics are not taken into account and (Wang and Schwager, 2016) considers a kinematic decentralized approach using force feedback. Finally, mobile manipulator approaches are treated in (Sugar and Kumar, 2002; Tanner et al., 2003; Ponce-Hinestroza et al., 2016).

In this paper, we propose a novel nonlinear control scheme for trajectory tracking of an object rigidly grasped by N robotic agents. The main novelty of our approach is the *combination* of i) coupled object-agents dynamic formulation which does not require contact forces/torques measurements from corresponding sensors, ii) an extension to an adaptive version, where the dynamic parameters of the object and the agents are considered unknown and iii) the employment of unit quaternions for the object orientation, avoiding thus potential representation singularities. Moreover, the overall scheme is decentralized in the sense that each agent utilizes information regarding only its own state, and internal force regulation can be also guaranteed. Furthermore, in contrast to the majority of the related literature, we utilize coefficients for load sharing among the robotic arms, which may exhibit different power capabilities. To the best of the authors' knowledge, the integration of the aforementioned attributes for cooperative manipulation has not been addressed before, and turns out to be a challenging problem, due to the high complexity of the coupled object-agents dynamics. Finally, experimental studies verify the validity and effectiveness of the proposed framework.

The rest of the paper is organized as follows: Section 2 introduces notation and preliminary background. Section 3 describes the problem formulation and the overall system's model. The control scheme is presented in Section 4 and Section 5 verifies our approach with an experimental setup. Finally, Section 6 concludes the paper.

2. NOTATION AND PRELIMINARIES

2.1 Notation

The set of positive integers is denoted as $\mathbb{N}$ and, given $n \in \mathbb{N}$, $\mathbb{R}^n$ is the real n-coordinate space, $\mathbb{R}^n_{\geq 0}$ and $\mathbb{R}^n_{>0}$ are the sets of real n-vectors with all elements nonnegative and positive, respectively, and S^n is the n-D sphere; $I_n \in \mathbb{R}^{n \times n}_{\geq 0}$ and $0_{n \times m} \in \mathbb{R}^{n \times m}, n, m \in \mathbb{N}$, denote the unit matrix and the matrix with all entries zero, respectively. The vector connecting the origins of coordinate frames $\{A\}$ and $\{B\}$ expressed in frame $\{C\}$ coordinates in 3D space is denoted as $p^C_{B/A} \in \mathbb{R}^3$. Given $a \in \mathbb{R}^3$, $S(a) \in \mathbb{R}^{3 \times 3}$ is the skew-symmetric matrix defined according to $S(a)b = a \times b$. The rotation matrix from $\{A\}$ to $\{B\}$ is denoted as $R_{B/A} \in SO(3)$, where $SO(3)$ is the 3D rotation group. The angular velocity of frame $\{B\}$ with respect to $\{A\}$, expressed in $\{C\}$, is denoted as $\omega^C_{B/A} \in \mathbb{R}^3$ and it holds that (Siciliano et al., 2010) $\dot{R}_{B/A} = S(\omega^C_{B/A})R_{B/A}$. We further
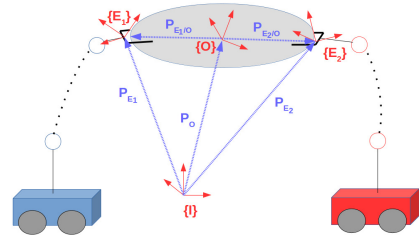

Fig. 1. Two robotic agents rigidly grasping an object.

denote as $\phi_{A/B} \in \mathbb{T}^3$ the Euler angles representing the orientation of $\{B\}$ with respect to $\{A\}$, where $\mathbb{T}^3$ is the 3D torus. We also define the set $\mathbb{M} = \mathbb{R}^3 \times \mathbb{T}^3$. For notational brevity, when a coordinate frame corresponds to an inertial frame of reference $\{I\}$, we will omit its explicit notation (e.g., $p_B = p^I_{B/I}, \omega_B = \omega^I_{B/I}, R_B = R_{B/I}$ etc.). Finally, all vector and matrix differentiations will be with respect to an inertial frame $\{I\}$, unless otherwise stated.

2.2 Unit Quaternions

Given two frames $\{A\}$ and $\{B\}$, we define a unit quaternion $\xi_{B/A} = [\eta_{B/A}, \varepsilon^T_{B/A}]^T \in S^3$ describing the orientation of $\{B\}$ with respect to $\{A\}$, with $\eta_{B/A} \in \mathbb{R}, \varepsilon_{B/A} \in S^2$, subject to the constraint $\eta^2_{B/A} + \varepsilon^T_{B/A}\varepsilon_{B/A} = 1$. The relation between $\xi_{B/A}$ and the corresponding rotation matrix $R_{B/A}$ as well as the axis/angle representation can be found in (Siciliano et al., 2010). For a given quaternion $\xi_{B/A} = [\eta_{B/A}, \varepsilon^T_{B/A}]^T \in S^3$, its conjugate, that corresponds to the orientation of $\{A\}$ with respect to $\{B\}$, is (Siciliano et al., 2010) $\xi^*_{B/A} = [\eta_{B/A}, -\varepsilon^T_{B/A}]^T \in S^3$. Moreover, given two quaternions $\xi_i = [\eta_i, \varepsilon^T_i]^T, i \in \{1, 2\}$, the quaternion product is defined as (Siciliano et al., 2010)

$$\xi_1 \otimes \xi_2 = \begin{bmatrix} \eta_1\eta_2 - \varepsilon^T_1\varepsilon_2 \\ \eta_1\varepsilon_2 + \eta_2\varepsilon_1 + S(\varepsilon_1)\varepsilon_2 \end{bmatrix} \in S^3. \tag{1}$$

The time derivative of a quaternion $\xi_{B/A} = [\eta_{B/A}, \varepsilon^T_{B/A}]^T \in S^3$ is given by (Siciliano et al., 2010):

$$\dot{\xi}_{B/A} = \frac{1}{2}E(\xi_{B/A})\omega^A_{B/A}, \tag{2a}$$

where $E : S^3 \to \mathbb{R}^{4 \times 3}$ is defined as:

$$E(\xi) = \begin{bmatrix} -\varepsilon^T \\ \eta I_3 - S(\varepsilon) \end{bmatrix}.$$

Finally, it can be shown that $E^T(\xi)E(\xi) = I_3$ and hence

$$\omega^A_{B/A} = 2E^T(\xi_{B/A})\dot{\xi}_{B/A}. \tag{2b}$$

3. PROBLEM FORMULATION

Consider N fully actuated robotic agents rigidly grasping an object (see Fig. 1). We denote as $q_i \in \mathbb{R}^{n_i}$ the generalized joint-space variables of the ith agent and as $\{E_i\}$, $\{O\}$ the end-effector and object's center of mass frames, respectively; $\{I\}$ corresponds to an inertial frame of reference, as mentioned in Section 2.1. The rigidity assumption implies that the agents can exert both forces and torques along all directions to the object. We consider that each agent has access to the position and velocity of its own joint variables and that no interaction force/torque measurements or on-line information exchange between the agents is required. Moreover, it is assumed that the

desired object profile as well as relevant geometric features (e.g., center of mass location) are transmitted off-line to the agents. Finally, we consider that the agents operate away from kinematic singularity poses (Siciliano et al., 2010). In the following, we present the modeling of the coupled kinematics and dynamics of the object and the agents.

3.1 Kinematics

In view of Fig. 1, we have that:
$$p_{E_i}(t) = p_O(t) + p_{E_i/O}(q_i) = p_O(t) + R_{E_i}(q_i)p_{E_i/O}^{E_i}, \quad (3a)$$
$$\phi_{E_i}(t) = \phi_O(t) + \phi_{E_i/O}, \quad (3b)$$
$\forall i \in \mathcal{N}$, where $p_{E_i}, \phi_{E_i}, p_O, \phi_O$ are the ith end-effector's and object's pose, respectively, and $p_{E_i/O}^{E_i}$ and $\phi_{E_i/O}$ are the constant distance and orientation offset between $\{O\}$ and $\{E_i\}$, which are considered known. Differentiation of (3a) along with the fact that, due to the grasping rigidity, it holds that $\omega_{E_i} = \omega_O$, leads to
$$v_i(t) = J_{O_i}(q_i)v_O(t), \quad (4)$$
and, by differentiation, to
$$\dot{v}_i(t) = \dot{J}_{O_i}(q_i, \dot{q}_i)v_O(t) + J_{O_i}(q_i)\dot{v}_O(t), \quad (5)$$
where $v_O, v_i : \mathbb{R}_{\geq 0} \to \mathbb{R}^6$ with $v_O(t) = [\dot{p}_O^T(t), \omega_O^T(t)]^T, v_i(t) = [\dot{p}_{E_i}^T(t), \omega_{E_i}^T(t)]^T$ are the object's center of mass' and end-effectors' velocities respectively. Also, $J_{O_i} : \mathbb{R}^{n_i} \to \mathbb{R}^{6 \times 6}$ is the object-to-agent Jacobian matrix, with
$$J_{O_i}(q_i) = \begin{bmatrix} I_3 & S(p_{O/E_i}(q_i)) \\ 0_{3 \times 3} & I_3 \end{bmatrix}, \quad (6)$$
which is always full-rank due to the grasp rigidity.

Remark 1. Each agent i can compute p_{E_i}, ϕ_{E_i} and v_i via its forward and differential kinematics (Siciliano et al., 2010) $p_{E_i}(t) = k_{p_i}(q_i), \phi_{E_i}(t) = k_{\eta_i}(q_i)$ and $v_i(t) = J_i(q_i)\dot{q}_i$, respectively, where $k_{p_i} : \mathbb{R}^{n_i} \to \mathbb{R}^3, k_{\eta_i} : \mathbb{R}^{n_i} \to \mathbb{T}^3$ are the forward kinematics and $J_i : \mathbb{R}^{n_i} \to \mathbb{R}^{6 \times 6}$ is the geometric Jacobian of agent $i \in \mathcal{N}$. In addition, since $p_{E_i/O}^{E_i}$ and $\phi_{E_i/O}$ are known, p_O, ϕ_O and v_O can be computed by inverting (3) and (4), respectively, without employing any sensory data for the object's configuration. Moreover, from ϕ_O, we can compute the unit quaternion ξ_O (Siciliano et al., 2010) to represent the object's orientation, since the desired pose for the object's center of mass will be given in terms of a desired position trajectory $p_{O,\mathrm{d}}(t)$ and a desired quaternion trajectory $\xi_{O,\mathrm{d}}(t)$.

3.2 Dynamics

Next, we consider the following second order dynamics for the object, which can be derived based on the Newton-Euler formulation:
$$M_O(x_O)\dot{v}_O + C_O(x_O, \dot{x}_O)v_O + g_O(x_O) = f_O, \quad (7)$$
where $x_O : \mathbb{R}_{\geq 0} \to \mathbb{M}$, with $x_O(t) = [p_O^T(t), \phi_O^T(t)]^T$, $M_O : \mathbb{M} \to \mathbb{R}^{6 \times 6}$ is the positive definite inertia matrix, $C_O : \mathbb{M} \times \mathbb{R}^6 \to \mathbb{R}^{6 \times 6}$ is the Coriolis matrix, $g_O : \mathbb{M} \to \mathbb{R}^6$ is the gravity vector, and $f_O \in \mathbb{R}^6$ is the vector of generalized forces acting on the object's center of mass.

The task space agent dynamics are given by (Siciliano et al., 2010):
$$M_i(q_i)\dot{v}_i + C_i(q_i, \dot{q}_i)v_i + g_i(q_i) = u_i - f_i, \quad (8)$$

where $M_i : \mathbb{R}^{n_i} \to \mathbb{R}^{6 \times 6}$ is the positive definite inertia matrix, $C_i : \mathbb{R}^{n_i} \times \mathbb{R}^{n_i} \to \mathbb{R}^{6 \times 6}$ is the Coriolis matrix, $g_i : \mathbb{R}^{n_i} \to \mathbb{R}^6$ is the task-space gravity term, $f_i \in \mathbb{R}^6$ is the vector of generalized forces that agent i exerts on the grasping point with the object and u_i is the task space wrench acting as the control input, $\forall i \in \mathcal{N}$.

The agent dynamics (8) can be written in vector form as:
$$M(q)\dot{v} + C(q, \dot{q})v + g(q) = u - f, \quad (9)$$
where $q = [[q_i^T]_{i \in \mathcal{N}}]^T \in \mathbb{R}^n$, with $n = \sum_{i \in \mathcal{N}} n_i$, $v = [[v_i^T]_{i \in \mathcal{N}}] \in \mathbb{R}^{6N}, M = \mathrm{diag}\{[M_i]_{i \in \mathcal{N}}\} \in \mathbb{R}^{6N \times 6N}, C = \mathrm{diag}\{[C_i]_{i \in \mathcal{N}}\} \in \mathbb{R}^{6N \times 6N}, f = [[f_i^T]_{i \in \mathcal{N}}]^T \in \mathbb{R}^{6N}, u = [[u_i^T]_{i \in \mathcal{N}}]^T \in \mathbb{R}^{6N}, g = [[g_i^T]_{i \in \mathcal{N}}]^T \in \mathbb{R}^{6N}$.

Remark 2. The task space wrench u_i can be translated to joint space inputs $\tau_i \in \mathbb{R}^{n_i}$ via $\tau_i = J_i^T(q_i)u_i + (I_{n_i} - J_i^T(q_i)\bar{J}_i^T(q_i))\tau_{i0}$, where $\bar{J}_i$ is a generalized inverse of J_i (Siciliano et al., 2010); τ_{i0} concerns redundant agents ($n_i > 6$) and does not contribute to end-effector forces.

Moreover, the following property holds:

Lemma 1. (Siciliano et al., 2010) The matrices $\dot{M}_O - 2C_O$ and $\dot{M}_i - 2C_i$ are skew-symmetric.

The kineto-statics duality (Siciliano et al., 2010) along with the grasp rigidity suggest that the force f_O acting on the object's center of mass and the generalized forces $f_i, i \in \mathcal{N}$, exerted by the agents at the grasping points, are related through:
$$f_O = G^T(q)f, \quad (10)$$
where $G : \mathbb{R}^n \to \mathbb{R}^{6N \times 6}$ is the full column-rank grasp matrix, with
$$G(q) = [J_{O_1}^T(q_1), \ldots, J_{O_N}^T(q_N)]^T.$$

By substituting (9) into (10), we obtain:
$$f_O = G^T(q)(u - M(q)\dot{v} - C(q, \dot{q})v - g(q)), \quad (11)$$
which, after substituting (4), (5), (7), and rearranging terms, yields the overall system coupled dynamics:
$$\tilde{M}(q, x_O)\dot{v}_O + \tilde{C}(q, \dot{q}, x_O, \dot{x}_O)v_O + \tilde{g}(q, x_O) = G^T(q)u, \quad (12)$$
where
$$\tilde{M} = M_O + G^T M G \quad (13a)$$
$$\tilde{C} = C_O + G^T C G + G^T M \dot{G} \quad (13b)$$
$$\tilde{g} = g_O + G^T g. \quad (13c)$$
Moreover, the following Lemma holds.

Lemma 2. The matrix $\tilde{M}$ is symmetric and positive definite and the matrix $\dot{\tilde{M}} - 2\tilde{C}$ is skew-symmetric.

Proof. By employing the definition of M and the positive definiteness of $M_i, \forall i \in \mathcal{N}$, it is straightforward to prove the positive definiteness of M. Then, in view of (13a) and by invoking the positive definiteness of M_O and the fact that G is full column-rank, we deduce the positive definiteness of $\tilde{M}$.

Regarding the skew symmetry of $\dot{\tilde{M}} - 2\tilde{C}$, notice first that the definitions of M, C as well as Lemma 1 imply the skew-symmetry of $\dot{M} - 2C$. Moreover, by defining $A = \dot{G}^T M \dot{G}$, we have from (13a), (13b):
$$\dot{\tilde{M}} - 2\tilde{C} = \dot{M}_O - 2C_O + G^T(\dot{M} - 2C)G + A - A^T,$$

from which, by employing Lemma 1, we obtain: $(\dot{\tilde{M}} - 2\tilde{C})^T = -(\dot{\tilde{M}} + 2\tilde{C})$, which completes the proof.

Formally, the problem treated in this paper is the following:

Problem 1. Given a desired bounded object pose specified by $p_{O,\mathrm{d}}(t) \in \mathbb{R}^3$, $\xi_{O,\mathrm{d}}(t) = [\eta_{O,\mathrm{d}}, \varepsilon_{O,\mathrm{d}}^T]^T \in S^3$, with bounded first and second derivatives, find u in (12) that achieves

$$\lim_{t \to \infty} \begin{bmatrix} p_O(t) \\ \xi_O(t) \end{bmatrix} = \begin{bmatrix} p_{O,\mathrm{d}}(t) \\ \xi_{O,\mathrm{d}}(t) \end{bmatrix}.$$

4. MAIN RESULTS

We need first to define the errors associated with the object pose and the desired pose trajectory. We first define the position error $e_p : \mathbb{R}_{\geq 0} \to \mathbb{R}^3$:

$$e_p(t) = p_O(t) - p_{O,\mathrm{d}}(t). \tag{14}$$

Since unit quaternions do not form a vector space, they cannot be subtracted to form an orientation error; instead we should use the properties of the quaternion group algebra. Let $e_\xi = [e_\eta, e_\varepsilon^T]^T : \mathbb{R}_{\geq 0} \to S^3$ be the unit quaternion describing the orientation error. Then, it holds that (Siciliano et al., 2010),

$$e_\xi(t) = \xi_{O,\mathrm{d}}(t) \otimes \xi_O^*(t) = \begin{bmatrix} \eta_{O,\mathrm{d}}(t) \\ \varepsilon_{O,\mathrm{d}}(t) \end{bmatrix} \otimes \begin{bmatrix} \eta_O(t) \\ -\varepsilon_O(t) \end{bmatrix},$$

which, by using (1), becomes:

$$e_\xi(t) = \begin{bmatrix} e_\eta(t) \\ e_\varepsilon(t) \end{bmatrix} =$$
$$= \begin{bmatrix} \eta_O(t)\eta_{O,\mathrm{d}}(t) + \varepsilon_O^T(t)\varepsilon_{O,\mathrm{d}}(t) \\ \eta_O(t)\varepsilon_{O,\mathrm{d}}(t) - \eta_{O,\mathrm{d}}(t)\varepsilon_O(t) + S(\varepsilon_O)(t)\varepsilon_{O,\mathrm{d}}(t) \end{bmatrix}. \tag{15}$$

By taking the time derivative of (14) and (15), employing (2) and certain properties of skew-symmetric matrices (Campa et al., 2006), it can be shown that (Siciliano et al., 2010)

$$\dot{e}_p(t) = \dot{p}_O(t) - \dot{p}_{O,\mathrm{d}}(t) \tag{16a}$$
$$\dot{e}_\eta(t) = \tfrac{1}{2} e_\varepsilon^T(t) e_\omega(t) \tag{16b}$$
$$\dot{e}_\varepsilon(t) = -\tfrac{1}{2} \left(e_\eta(t) I_3 + S(e_\varepsilon(t)) \right) e_\omega - S(e_\varepsilon(t))\omega_{O,\mathrm{d}}(t), \tag{16c}$$

where $e_\omega : \mathbb{R}_{\geq 0} \to \mathbb{R}^3$, with $e_\omega(t) = \omega_O(t) - \omega_{O,\mathrm{d}}(t)$ and $\omega_{O,\mathrm{d}}(t) = 2E^T(\xi_{O,\mathrm{d}})\dot{\xi}_{O,\mathrm{d}}(t)$, as indicated by (2b).

Notice that, considering the properties of unit quaternions, when $\xi_O = \xi_{O,\mathrm{d}}$, then $e_\xi(t) = [1, 0_{1\times 3}]^T \in S^3$. If $\xi_O = -\xi_{O,\mathrm{d}}$, then $e_\xi(t) = [-1, 0_{1\times 3}]^T \in S^3$, which, however, represents the same orientation. Therefore, the control objective established in Problem 1 is equivalent to

$$\lim_{t \to \infty} \begin{bmatrix} e_p(t) \\ |e_\eta(t)| \\ e_\varepsilon(t) \end{bmatrix} = \begin{bmatrix} 0_{3\times 1} \\ 1 \\ 0_{3\times 1} \end{bmatrix}. \tag{17}$$

Next, we design control protocols such that the specification (17) is met. Firstly, we consider that the dynamics parameters of the object and the agents are known. Then, we extend the proposed scheme to also compensate for unknown dynamic parameters, using adaptive control techniques (Slotine and Li, 1987; Siciliano et al., 2010).

4.1 Non-Adaptive Control Scheme

Define the velocity reference signals $v_O^r : \mathbb{R}_{\geq 0} \to \mathbb{R}^6$:

$$v_O^r(t) = \begin{bmatrix} \dot{p}_O^r(t) \\ \omega_O^r(t) \end{bmatrix}$$
$$= \begin{bmatrix} \dot{p}_{O,\mathrm{d}}(t) - k_p e_p(t) \\ \omega_{O,\mathrm{d}}(t) - k_\varepsilon e_\eta(t) e_\varepsilon(t) \end{bmatrix} = v_{O,\mathrm{d}}(t) - Ke(t), \tag{18}$$

where $v_{O,\mathrm{d}}(t) = [\dot{p}_{O,\mathrm{d}}^T(t), \omega_{O,\mathrm{d}}^T(t)]^T \in \mathbb{R}^6$, $k_p, k_\varepsilon \in \mathbb{R}_{>0}$, $K = \mathrm{diag}\{k_p I_3, k_\varepsilon I_3\} \in \mathbb{R}_{\geq 0}^{6\times 6}$ and $e(t) = [e_p^T(t), -e_\eta(t)e_\varepsilon^T(t)]^T \in \mathbb{R}^6$.

Furthermore, define the reference velocity error $e_v : \mathbb{R}_{\geq 0} \to \mathbb{R}^6$ as:

$$e_v(t) = v_O(t) - v_O^r(t), \tag{19}$$

and design the decentralized control law for $u_i : \mathbb{R}_{\geq 0} \to \mathbb{R}^6$ in (12), $i \in \mathcal{N}$, as:

$$u_i(t) = \mu_i(t) + f_{i,\mathrm{d}}(t) \tag{20}$$

where

$$\mu_i(t) = g_i + \left(C_i J_{O_i} + M_i \dot{J}_{O_i} \right) v_O^r(t) + M_i J_{O_i} \dot{v}_O^r(t)$$
$$- J_{O_i}^{-T} \left(k_{v_i} e_v(t) + c_i e(t) \right)$$
$$f_{i,\mathrm{d}}(t) = c_i J_{O_i}^{-T} \left(M_O \dot{v}_O^r(t) + C_O v_O^r(t) + g_O \right),$$

$k_{v_i} \in \mathbb{R}_{>0}$ is a positive gain, $c_i \in \mathbb{R}_{\geq 0}$ are load sharing coefficients with $0 \leq c_i \leq 1, \forall i \in \mathcal{N}, \sum_{i \in \mathcal{N}} c_i = 1$, and we have also exploited the dependence of $q_i, \dot{q}_i, x_O, \dot{x}_O$ on time.

The control law (20) can be also written in vector form:

$$u = \mu + f_\mathrm{d}, \tag{21}$$

where

$$\mu = g + \left(CG + M\dot{G} \right) v_O^r(t) + MG\dot{v}_O^r(t)$$
$$- \tilde{G}^T \left(K_v e_v(t) + C_f e(t) \right)$$
$$f_\mathrm{d} = \tilde{G}^T C_f \left(M_O \dot{v}_O^r(t) + C_O v_O^r(t) + g_O \right),$$

$K_v = [k_{v_1} I_6, \ldots, k_{v_N} I_6]^T \in \mathbb{R}^{6N\times 6}$, $C_f = [c_1 I_6, \ldots, c_N I_6]^T \in \mathbb{R}^{6N\times 6}$, $f_\mathrm{d} = [[f_{i,\mathrm{d}}^T]_{i \in \mathcal{N}}]^T \in \mathbb{R}^{6N}$, and finally, $\tilde{G} = \mathrm{diag}\{[J_{O_i}^{-1}]_{i \in \mathcal{N}}\} \in \mathbb{R}^{6N\times 6N}$.

By employing the fact that $G^T \tilde{G}^T = [I_6, \ldots, I_6] \in \mathbb{R}^{6\times 6N}$ as well as $\sum_{i \in \mathcal{N}} c_i = 1$, we multiply (21) by G^T to obtain:

$$G^T u = \tilde{M}\dot{v}_O^r(t) + \tilde{C}v_O^r(t) + \tilde{g} - \sum_{i \in \mathcal{N}} k_{v_i} e_v(t) - e(t), \tag{22}$$

that will be used in the sequel.

The following theorem summarizes the main results of this subsection.

Theorem 1. Consider N robotic agents rigidly grasping an object with coupled dynamics described by (12) under the control protocol (21). Then, under the assumption $e_\eta(0) \neq 0$, the object pose converges asymptotically to the desired one with all closed loop signals being bounded, i.e., Problem 1 is solved.

Proof. Consider the positive definite and radially unbounded Lyapunov function :

$$V(e_p, e_\eta, e_v, t) = \frac{1}{2} e_p^T e_p + e_\eta^2 - 1 + e_\varepsilon^T e_\varepsilon$$
$$+ \frac{1}{2} e_v^T \tilde{M}(q(t), x_O(t)) e_v.$$

By differentiating V with respect to time and substituting the error dynamics (16), we obtain:

$$\dot{V} = e_p^T \dot{e}_p + e_\eta e_\varepsilon^T e_\omega + e_v^T \tilde{M} \dot{e}_v + \frac{1}{2} e_v^T \dot{\tilde{M}} e_v$$
$$= e_p^T (\dot{p}_O - \dot{p}_{O,\mathrm{d}}) + e_\eta e_\varepsilon^T (\omega_O - \omega_{O,\mathrm{d}}) +$$
$$e_v^T \tilde{M} (\dot{v}_O - \dot{v}_O^r) + \frac{1}{2} e_v^T \dot{\tilde{M}} e_v,$$

from which, in view of (18), (19) and (12), we derive:

$$\dot{V} = -e^T K e + e^T e_v + e_v^T \left(G^T u - \tilde{C} v_O - \tilde{g} \right) -$$
$$e_v^T \tilde{M} \dot{v}_O^r + \frac{1}{2} e_v^T \dot{\tilde{M}} e_v$$
$$= -e^T K e + e^T e_v + e_v^T \left(G^T u - \tilde{C} v_O^r - \tilde{g} \right) -$$
$$e_v^T \tilde{M} \dot{v}_O^r + e_v^T \left(\frac{1}{2} \dot{\tilde{M}} - \tilde{C} \right) e_v.$$

Then, by employing Lemma 2, $\dot{V}$ becomes:

$$\dot{V} = -e^T K e - e_v^T (-e + \tilde{M} \dot{v}_O^r + \tilde{C} v_O^r + \tilde{g} - G^T u),$$

and after substituting (22):

$$\dot{V} = -k_p e_p^T e_p - k_\varepsilon e_\eta^2 e_\varepsilon^T e_\varepsilon - \sum_{i \in \mathcal{N}} k_{v_i} e_v^T e_v, \quad (23)$$

which is non-positive. We conclude therefore that the system is stable and V is a non-increasing function, deducing the boundedness of $e_p, e_\eta, e_\varepsilon, e_v$. Hence, invoking also the boundedness of $p_{O,\mathrm{d}}$ and $\omega_{O,\mathrm{d}}$ and of their derivatives, we employ (18) to prove the boundedness of v_O^r and (19) to prove the boundedness of v_O and therefore of v_i, since the boundedness of J_{O_i} and G is straightforward. From the aforementioned conclusions, invoking also the fact that $M_i(\cdot), C_i(\cdot), M_O(\cdot), g_i(\cdot), C_O(\cdot), g_O(\cdot)$ are continuous functions, we can deduce the boundedness of q_i and $\dot{q}_i, \forall i \in \mathcal{N}$ and of $\tilde{M}, \tilde{C}, \tilde{g}$. Moreover, the error derivatives (16a)-(16c) are all bounded and thus, in view of (18), $\dot{v}_O^r$ is bounded as well. Hence, we also deduce the boundedness of u_i. Finally, by differentiating (19) and substituting (12) and (21) we also deduce the boundedness of $\dot{e}_v$ and therefore of $\dot{v}_O$.

Combining the aforementioned statements we can conclude the boundedness of $\ddot{V}$ and hence the uniform continuity of $\dot{V}$. Invoking Barbalat's lemma (Slotine et al., 1991), we deduce that $\dot{V} \to 0$ and therefore through (23) that $(e_p, e_\eta e_\varepsilon, e_v) \to (0_{3\times 1}, 0_{3\times 1}, 0_{6\times 1})$. The equilibrium $e_\eta = 0$ can be proven to be unstable (Mayhew et al., 2011) and hence, since $e_\eta(0) \neq 0$, we conclude that $(e_p, e_\varepsilon, e_v) \to (0_{3\times 1}, 0_{3\times 1}, 0_{6\times 1})$. Furthermore, we also conclude that $e_\eta^2 \to 1$ since $e_\xi \in S^3$ is a unit quaternion, which leads to the completion of the proof.

Remark 3. The assumption $e_\eta(0) \neq 0$ is a necessary assumption to guarantee asymptotic stability of the orientation error e_ξ. In terms of Euler angles, it states that the initial orientation errors (in the x, y, z directions) should not be 180 degrees.

4.2 Adaptive Control Scheme

Consider now that the dynamic parameters of the object and the agents (e.g., masses and inertia moments), are unknown. We propose an adaptive version of (21) that does not incorporate the aforementioned parameters and still guarantees the solution of Problem 1.

It can be shown (Siciliano et al., 2010) that the object and agent dynamics can be written in the form:

$$M_i(q_i)\dot{v}_i + C_i(q_i, \dot{q}_i)v_i + g_i(q_i) = H_i(q_i, \dot{q}_i, v_i, \dot{v}_i)\theta_i \quad (24a)$$
$$M_O(x_O)\dot{v}_O + C_O(x_O, \dot{x}_O)v_O + g_O(x_O) =$$
$$Y_O(x_O, \dot{x}_O, v_O, \dot{v}_O)\theta_O, \quad (24b)$$

$\forall i \in \mathcal{N}$, where $\theta_i \in \mathbb{R}^\ell, \theta_O \in \mathbb{R}^{\ell_O}$ are vectors of unknown but constant dynamic parameters of the agents and the object, appearing in the terms M_i, C_i, g_i and M_O, C_O, g_O, respectively, and $H_i \in \mathbb{R}^{6\times \ell}, i \in \mathcal{N}, Y_O \in \mathbb{R}^{6\times \ell_O}$ are known regressor matrices, independent of θ_i, θ_O. It is worth noting that the choice for ℓ and ℓ_O is not unique and depends on the factorization method used (Siciliano et al., 2010). In the same vein, since J_{O_i}, as given in (6), depends only on q_i and not on $\theta_i, \theta_O, \forall i \in \mathcal{N}$, we can write:

$$J_{O_i}^T M_i J_{O_i} \dot{v}_i + (J_{O_i}^T M_i \dot{J}_{O_i} + J_{O_i}^T C_i J_{O_i})v_i + J_{O_i}^T g_i =$$
$$Y_i(q_i, \dot{q}_i, v_i, \dot{v}_i)\theta_i, \quad (25)$$

where $Y_i \in \mathbb{R}^{6\times \ell}$ is another regressor matrix independent of θ_i, θ_O. Hence, in view of (13), (24) and (25), the left-hand side of (12) can be written as:

$$\tilde{M}(q, x_O)\dot{v}_O + \tilde{C}(q, \dot{q}, x_O, \dot{x}_O)v_O + \tilde{g}(q, x_O) =$$
$$Y_O(x_O, \dot{x}_O, v_O, \dot{v}_O)\theta_O + Y^T(q, \dot{q}, v_O, \dot{v}_O)\theta \quad (26)$$

where $Y(q, \dot{q}, v_O, \dot{v}_O) = [Y_1(q_1, \dot{q}_1, v_O, \dot{v}_O), \ldots, Y_N(q_N, \dot{q}_N, v_O, \dot{v}_O)]^T \in \mathbb{R}^{N\ell \times 6}$ and $\theta = [[\theta_i^T]_{i\in\mathcal{N}}]^T \in \mathbb{R}^{N\ell}$.

Let us now denote as $\hat{\theta}_O^i : \mathbb{R}_{\geq 0} \to \mathbb{R}^{\ell_O}$ and $\hat{\theta}_i : \mathbb{R}_{\geq 0} \to \mathbb{R}^\ell$ the estimates of θ_O and θ_i, respectively, by agent $i \in \mathcal{N}$, and the corresponding stack vectors $\hat{\theta}_O(t) = [[(\hat{\theta}_O^i(t))^T]_{i\in\mathcal{N}}]^T \in \mathbb{R}^{N\ell_O}, \hat{\theta}(t) = [[\hat{\theta}_i^T(t)]_{i\in\mathcal{N}}]^T \in \mathbb{R}^{N\ell}$, for which we formulate the associated errors $e_{\theta_O} : \mathbb{R}_{\geq 0} \to \mathbb{R}^{N\ell_O}, e_\theta : \mathbb{R}_{\geq 0} \to \mathbb{R}^{N\ell}$ as

$$e_{\theta_O}(t) = \begin{bmatrix} e_{\theta_O}^1(t) \\ \vdots \\ e_{\theta_O}^N(t) \end{bmatrix} = \begin{bmatrix} \theta_O - \hat{\theta}_O^1(t) \\ \vdots \\ \theta_O - \hat{\theta}_O^N(t) \end{bmatrix} = \bar{\theta}_O - \hat{\theta}_O(t) \quad (27a)$$

$$e_\theta(t) = \begin{bmatrix} e_{\theta_1}(t) \\ \vdots \\ e_{\theta_N}(t) \end{bmatrix} = \begin{bmatrix} \theta_1 - \hat{\theta}_1(t) \\ \vdots \\ \theta_N - \hat{\theta}_N(t) \end{bmatrix} = \theta - \hat{\theta}(t), \quad (27b)$$

$\forall i \in \mathcal{N}$, where $\bar{\theta}_O = [\underbrace{\theta_O^T, \ldots, \theta_O^T}_{N \text{times}}]^T \in \mathbb{R}^{N\ell_O}$.

Then, with the reference velocity signal v_O^r defined as in (18) and the corresponding error e_v as in (19), we design the adaptive control law $u_i : \mathbb{R}_{\geq 0} \to \mathbb{R}^6$ in (12), for each agent $i \in \mathcal{N}$, as:

$$u_i(t) = J_{O_i}^{-T} \left(Y_i(q_i, \dot{q}_i, v_O, \dot{v}_O)\hat{\theta}_i(t) - c_i e(t) - k_{v_i} e_v(t) \right.$$
$$\left. + c_i Y_O(x_O, \dot{x}_O, v_O, \dot{v}_O)\hat{\theta}_O^i(t) \right),$$

which can be written in vector form as

$$u(t) = \tilde{G}^T \left(\tilde{Y}(\cdot)\hat{\theta}(t) + \tilde{Y}_O(\cdot)\hat{\theta}_O(t) - C_f e(t) - K_v e_v(t) \right), \quad (28)$$

where $\tilde{Y}(\cdot) = \mathrm{diag}\{[Y_i(q_i, \dot{q}_i, v_O, \dot{v}_O)]_{i\in\mathcal{N}}\} \in \mathbb{R}^{6N\times N\ell}$, $\tilde{Y}_O(\cdot) = \mathrm{diag}\{[c_i Y_O(x_O, \dot{x}_O, v_O, \dot{v}_O)]_{i\in\mathcal{N}}\} \in \mathbb{R}^{6N\times N\ell_O}, \tilde{G}, C_f, K_v$ as defined in (21), and e as defined in (18).

Moreover, we design the adaptation laws $\dot{\hat{\theta}}_O^i : \mathbb{R}_{\geq 0} \to \mathbb{R}^{\ell_O}, \dot{\hat{\theta}}_i : \mathbb{R}_{\geq 0} \to \mathbb{R}^\ell, i \in \mathcal{N}$, for each agent, as:

$$\dot{\hat{\theta}}_O^i(t) = -c_i Y_O^T(x_O, \dot{x}_O, v_O, \dot{v}_O)e_v(t)$$

$$\dot{\hat{\theta}}_i(t) = -\gamma_i Y_i^T(q_i, \dot{q}_i, v_O, \dot{v}_O)e_v(t),$$

which is written in vector form as

$$\dot{\hat{\theta}}_O(t) = \begin{bmatrix} \dot{\hat{\theta}}_O^1(t) \\ \vdots \\ \dot{\hat{\theta}}_O^N(t) \end{bmatrix} = -\tilde{C}_f Y_O^T(x_O, \dot{x}_O, v_O, \dot{v}_O)e_v(t) \quad (29a)$$

$$\dot{\hat{\theta}}(t) = \begin{bmatrix} \dot{\hat{\theta}}_1(t) \\ \vdots \\ \dot{\hat{\theta}}_N(t) \end{bmatrix} = -\Gamma Y(q, \dot{q}, x_O, \dot{x}_O)e_v(t), \quad (29b)$$

where $\Gamma = \text{diag}\{[\gamma_i I_\ell]_{i \in \mathcal{N}}\} \in \mathbb{R}_{\geq 0}^{N\ell \times N\ell}, \gamma_i \in \mathbb{R}_{>0}$, and $\tilde{C}_f = [c_1 I_{\ell_O}, \dots, c_N I_{\ell_O}]^T \in \mathbb{R}^{N\ell_O \times \overline{\ell_O}}$.

The following theorem summarizes the main results of this subsection.

Theorem 2. Consider N robotic agents rigidly grasping an object with coupled dynamics described by (12) and unknown dynamic parameters. Then, by applying the control protocol (28) with the adaptation laws (29), and under the assumption $e_\eta(0) \neq 0$, the object pose converges asymptotically to the desired pose with all closed loop signals being bounded, i.e, Problem 1 is solved.

Proof. Consider the positive definite, decrescent and radially unbounded Lyapunov function

$$V(e_p, e_\eta, e_v, e_\theta, e_{\theta_O}, t) = \frac{1}{2}e_p^T e_p + e_\eta^2 - 1 +$$
$$\frac{1}{2}e_v^T \tilde{M}(q(t), x_O(t))e_v + \frac{1}{2}e_\theta^T \Gamma^{-1} e_\theta + \frac{1}{2}e_{\theta_O}^T e_{\theta_O},$$

which, by time differentiation and by employing the error dynamics (16), relations (18), (19) as well as (26) and (27), becomes:

$$\dot{V} = -e^T K e + e_v^T(e - Y_O(x_O, \dot{x}_O, v_O, \dot{v}_O)\theta_O -$$
$$Y^T(q, \dot{q}, v_O, \dot{v}_O)\theta + G^T u) - e_{\theta_O}^T \dot{\hat{\theta}}_O - e_\theta^T \Gamma^{-1}\dot{\hat{\theta}}.$$

By substituting the adaptive control protocol (28), and noticing that, due to the fact that $\sum_{i \in \mathcal{N}} c_i = 1$, it holds that $Y_O(\cdot)\theta_O = Y_O(\cdot)\tilde{C}_f^T \bar{\theta}_O$, we obtain:

$$\dot{V} = -e^T K e - \sum_{i \in \mathcal{N}} k_{v_i} e_v^T e_v - e_v^T Y_O(\cdot)\tilde{C}_f^T e_{\theta_O} - e_v Y^T(\cdot)e_\theta$$
$$- e_{\theta_O}^T \dot{\hat{\theta}}_O - e_\theta^T \Gamma^{-1} e_\theta,$$

which, after substituting the adaptive laws (29), becomes

$$\dot{V} = -k_p e_p^T e_p - k_\varepsilon e_\eta e_\varepsilon^T e_\varepsilon - \sum_{i \in \mathcal{N}} k_{v_i} e_v^T e_v,$$

which is non-positive. We conclude therefore the boundedness of V and of $e_j, j \in \{p, \eta, \varepsilon, v, \theta_O, \theta\}$ and hence the boundedness of $v_O^r, v_O, v_i, \hat{\theta}_O$ and $\hat{\theta}$. By proceeding in a similar manner as in the non-adaptive scenario, we prove the boundedness of $\dot{v}_O^r, \dot{v}_O$ and of $\dot{e}_p, \dot{e}_\varepsilon, \dot{e}_v$. We deduce, therefore, the boundedness of $\ddot{V}$ and consequently the uniform continuity of $\dot{V}$ and that $(e_p, e_\varepsilon, e_v) \to (0_{3\times1}, 0_{3\times1}, 0_{6\times1})$ and hence, $e_\eta^2 \to 1$. Finally, it can be proved that the control and adaptation signals $u, \dot{\hat{\theta}}_O, \dot{\hat{\theta}}$ are also bounded, which leads to the completion of the proof.

Remark 4. Note that the dynamic parameter errors e_{θ_O}, e_θ are only guaranteed to stay bounded, not to be asymptotically driven to zero. However, that does not affect the re-
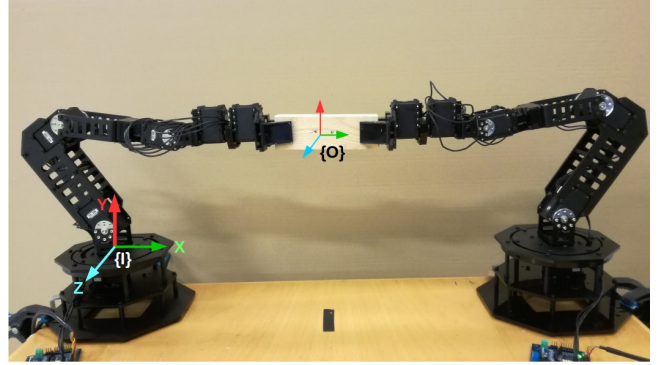

Fig. 2. Two WidowX Robot Arms rigidly grasping an object; $\{I\}$ and $\{O\}$ denote the inertial and the object's frame, respectively.

sult of the aforementioned analysis that $(e_p, e_\varepsilon, |e_\eta|, e_v) \to (0_{3\times1}, 0_{3\times1}, 1, 0_{6\times1})$. Moreover, the gains k_p, k_ε in (18) must be known by all agents $i \in \mathcal{N}$, and the load-sharing coefficients $c_i, i \in \mathcal{N}$ cannot be arbitrarily chosen by each agent, due to the constraint that $\sum_{i \in \mathcal{N}} c_i = 1$. Nevertheless, these values are constant and can be transmitted off-line to the agents.

Remark 5. In both control methodologies (21),(28), we can guarantee internal force regulation by including a vector of desired internal forces $f_{\text{int,d}} : \mathbb{R}_{\geq 0} \to \mathbb{R}^{6N}$ that belong to the nullspace of G^T, i.e., $f_{\text{int,d}}(t) = (I_{6N} - G^*G^T)\hat{f}_{\text{int,d}}$, where $G^* : \mathbb{R}^n \to \mathbb{R}^{6N\times6}$, with $G^*(q) = \frac{1}{N}[J_{O_1}^{-1}(q_1), \dots, J_{O_N}^{-1}(q_N)]^T$, and $\hat{f}_{\text{int,d}}$ a constant vector that can be transmitted off-line to the agents. It can be proved then, in view of the aforementioned analysis, that when $t \to \infty$, the generalized force vector acting on the object's center of mass consists of a term that results in its motion (for the trajectory tracking), and the term associated with the internal forces. Note though, that the computation of G^*G^T requires knowledge of all grasping points p_{E_i}, which reduces to knowledge of the constant offsets $p_{E_i/O}^O$, since, from (3a), we have that $p_{E_i}(t) = p_O(t) + R_O p_{E_i/O}^O$ and therefore, each agent can compute all $p_{E_i}(t), \forall i \in \mathcal{N}$, since it can compute the pose of the object's center of mass. Therefore, by off-line transmission of all $p_{E_i/O}^O$ to all agents, internal force regulation can be achieved.

5. EXPERIMENTAL EVALUATION

To demonstrate the efficiency of the proposed algorithm, en experimental study was carried out using two WidowX Robot Arms, as shown in Fig. 2, and the non-adaptive version of the proposed control scheme. The desired profile to be tracked by the object was determined by the planar motion $p_{O,\text{d}}(t) = [0.3 + 0.05\sin(\frac{2\pi}{15}t), 0.12, 0]^T$m and $\xi_{O,\text{d}}(t) = [\eta_{O,\text{d}}(t), \varepsilon_{O,\text{d}}^T(t)]^T = [\cos(\frac{\pi}{60}\sin(\frac{2\pi}{15}t)), 0, 0, -\sin(\frac{\pi}{60}\sin(\frac{2\pi}{15}t))]^T$, that is associated to the angle trajectory $\phi_{O,\text{d}}(t) = [0, 0, -\frac{\pi}{30}\sin(\frac{2\pi}{15}t)]^T$rad with respect to the z axis. For the execution of the task,
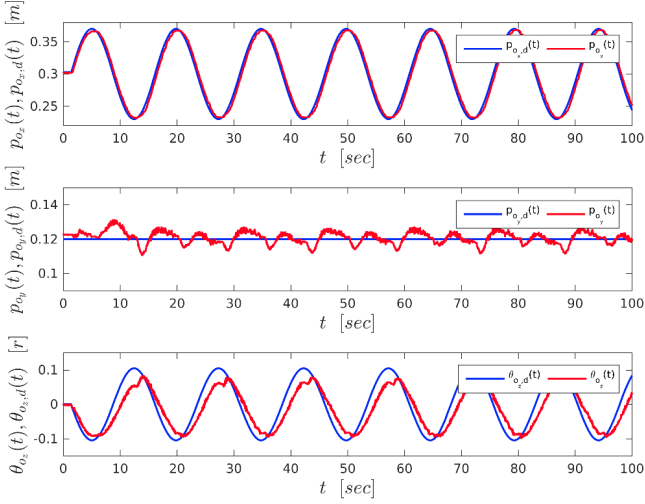
Fig. 3. The desired (with blue) and actual (with red) pose trajectory of the object's center of mass $p_{O,d}(t)$ and $p_O(t)$, respectively, for $t \in [0, 100]$s. Top: x (horizontal) direction. Middle: y (vertical) direction. Bottom: Angle $\phi_{O_z}(t)$ with respect to z axis (direction perpendicular to plane x-y).

we employed the three rotational joints with respect to the z-axis (see Fig. 2) of the arms. The object's initial pose was $p_O(0) = [0.301, 0.123, 0]^T$m, $\phi_O(0) = [0, 0, 0]^T$rad. The load sharing coefficients and the control gains were chosen as $c_1 = c_2 = 0.5$ and $k_p = 150, k_\varepsilon = 100, k_{v_1} = k_{v_2} = 2.5$, respectively.

The experimental results for $t = 10^2$s are depicted in Fig. 3-6. In particular, the tracking of the desired object pose by the actual one is illustrated in Fig. 3. Moreover, the evolution of the errors $e_p(t)$ and $e_\xi(t)$ is depicted in Fig. 4. It can be concluded from the figures that the tracking of the desired pose is achieved with some negligible oscillatory behavior that can be attributed to the deviation of the dynamics (12) from the actual coupled dynamics due to sensor noise, unmodelled friction, external disturbances and small sliding in the contact points which affects the rigidity assumption. The torque signals $\tau_i, i \in \{1, 2\}$ are pictured in Fig 5 and the task-space wrenches $u_i, i \in \{1, 2\}$ in Fig. 6. It can be seen that the proposed algorithm does not output large values for the resulting input torques and forces. A short video demonstrating the experimental setup can be found at https://youtu.be/PCnZ6C8ECFg.

6. CONCLUSION

We have proposed a novel control protocol for the cooperative manipulation of an object by N robotic agents using unit quaternions and without employing any force/torque measurements. Future efforts will be devoted towards incorporating kinematic uncertainties associated with the location of the object's center of mass, external disturbances, non-rigid grasps as well as singularity avoidance.

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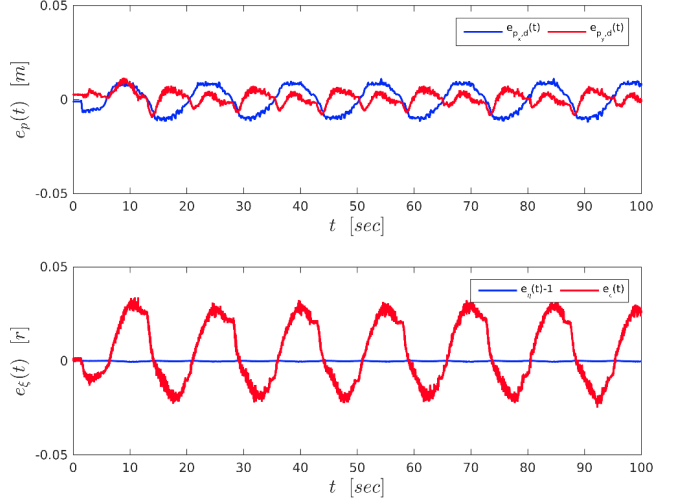
Fig. 4. The evolution of the errors $e_p(t), e_\xi(t), t \in [0, 100]$s. Top: $e_p(t)$ in x (with blue) and y (with red) direction. Bottom: $e_\eta(t) - 1$ (with blue) and $e_\varepsilon(t)$ (with red).

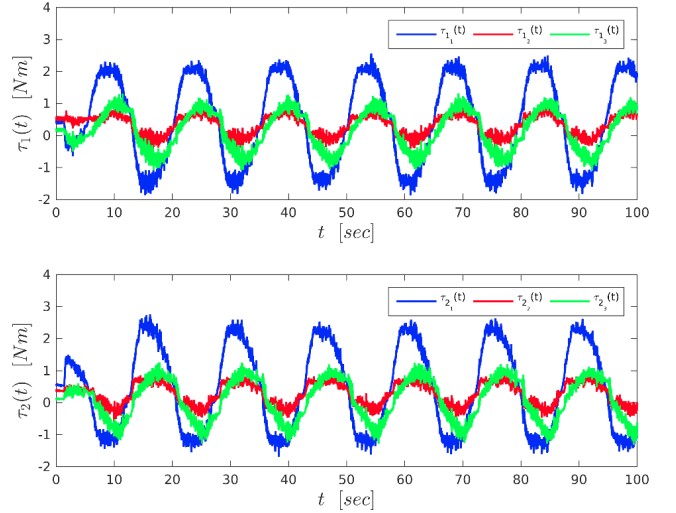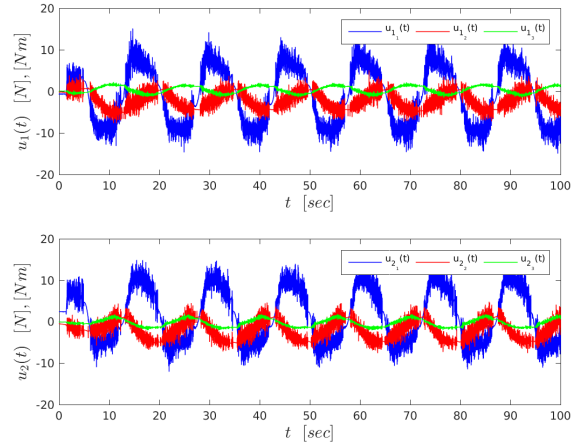Fig. 5. The resulting input torques $\tau_1(t)$ (top) and $\tau_2(t)$ (bottom), $t \in [0, 100]$s.

Fig. 6. The resulting 3D task-space wrenches $u_1(t)$ (top) and $u_2(t)$ (bottom), $t \in [0, 100]$s.

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