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Continuous Optimization

A generalised approach for efficient computation of look ahead security constrained optimal power flow

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ABSTRACT

We consider a generalised comprehensive Look-ahead Security-constrained Optimal Power Flow (LASCOPF) formulation under the $N-1$ contingency criterion over multiple dispatch intervals. We observe that the number of decision variables varies quadratically with the number of intervals. To improve scalability, we propose a reduced LASCOPF formulation for which the number of decision variables varies only linearly. We extend these formulations to the $N-k$ contingency criterion. For reduced LASCOPF we observe that the number of decision variables varies with the number of k-permutations of contingencies. To improve scalability, we propose a formulation that is further reduced to vary only with the number of k-combinations. Also, we show that our formulations can be extended simply to model recovery from the corresponding outages. Furthermore, we present LASCOPF under the $N-1$ contingency criterion using DC and AC power flow under generator contingencies. We prove that, barring borderline cases, solving the reduced formulation is equivalent to solving the comprehensive formulation. We extend these results to the $N-k$ contingency criterion. Finally, we present numerical results on the IEEE 14 bus, IEEE 30 bus and IEEE 300 bus test cases, and the 1354 bus part of the European power system using AC power flow to demonstrate the computational advantage of the reduced formulations under the $N-1$ and $N-2$ contingency criteria.

1. Introduction

Power system operation relies on variants of the optimal power flow (OPF) problem (Skolfield & Escobedo, 2022). OPF entails minimising generation costs given a set of physical constraints for individual dispatch intervals. However, in recent years, the increase in the amounts of renewable energy sources has increased intermittency in the available generation capacity (Bjørndal, Bjørndal, Cai, & Panos, 2018; WWEA, 2018) and traditional energy sources are often called upon to accommodate for this. Therefore, it is important to take energy sources' ramping limits into account, which couples consecutive dispatch intervals (Han, Gooi, & Kirschen, 2001).

The Look-ahead OPF (LAOPF) offers an extension to OPF that takes ramping limits and coupling into account (Choi & Xie, 2017; Schiro, Zheng, Zhao, & Litvinov, 2016; Xia & Elaiw, 2010). LAOPF is intended to be robust to unanticipated changes in net demand. Accordingly, LAOPF considers generation cost minimisation over multiple consecutive dispatch intervals, viz., the planning horizon based on short term weather forecasts, which in turn, affect the net demand, i.e., the demand less the renewable generation (Lorca, Sun, Litvinov, & Zheng, 2016). The solution to LAOPF is used to determine the locational marginal prices in the next dispatch interval and serves as a prediction for subsequent dispatch intervals (Hua, Schiro, Zheng, Baldick, & Litvinov, 2019). Then, after the dispatch interval is realised the problem is solved again based on an updated forecast to determine the locational marginal prices for the dispatch interval after. This follows the principle of receding horizon control (Kwon & Han, 2005).

Over the years, an increasing number of Independent System Operators (ISOs) have implemented LAOPF for the real time operation of power systems (viz., multi-interval real-time markets) such as the ISO (2019b), the ISO (2019a), the Midcontinent ISO (Ma et al., 2009) Ontario's Independent Electricity System Operator (IESO) (Yu, Cohen, & Danai, 2005) and the Interconnection (2022). ERCOT,ERCOT uses LAOPF over five minute intervals with an hour look-ahead to obtain indicative prices. Its use is also being considered in Australia (Hesamzadeh, Galland, & Biggar, 2014).

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Apart from the coupling of consecutive dispatch intervals, ISOs consider security against contingencies as well. Security against a contingency entails that immediately after the contingency and as automatic actions occur to respond to the contingency, all generation levels and line flows will stay within emergency ratings. Thereafter, all generation levels and line flows will be returned to non-emergency levels within a predetermined amount of time. if generation levels and line flows exceed these limits, there is a risk that of a cascading effect that would result in a system-wide blackout. Since the operation follows the principle of receding horizon control, eventually the system will be restored to being secure with respect to another contingency; however, this may not be explicitly represented. Security often involves ex ante preparedness against the contingency by the dispatch in the base-case, i.e, in the case where there is no contingency, e.g., ensuring that the system has ramping capabilities to attain non-emergency levels. This takes the form of constraints on the base-case that ensure that the demand can be satisfied even despite the corresponding outage of a component such as transmission line or a generator.

For power systems, the $N-k$ contingency criterion is commonly used as a measure of security, which requires that a system be secure against k simultaneous contingencies. Typically, power systems are operated under the $N-1$ contingency criterion. However, several markets have adopted the $N-k$ contingency criterion for $k>1$ in order to improve security. The government of the state of New South Wales in Australia has imposed an $N-2$ planning standard in the Sydney region transmission network (Commission, 2013) and TenneT (2017) in Netherlands considers the $N-2$ contingency criterion as a benchmark to test its transmission systems.

Stott, Alsac, & Monticelli (1987) proposed including security constraints in the OPF over a single dispatch interval, viz., Security-constrained OPF (SCOPF). Single interval SCOPF formulations, by definition, cannot explicitly accommodate a change in dispatch and therefore, can only explicitly model those contingencies that allow the dispatch to remain the same following the outage, such as transmission line contingencies (Madani, Lavaei, & Baldick, 2017). On the contrary, SCOPF formulations with multiple intervals are able to model changes in dispatch following a contingency and after the automated short-term responses to the contingency, thus allowing for more flexibility in the operation for contingencies such as transmission line contingencies. In addition, contingencies which require a change in dispatch such as generator contingencies may also be modelled, thus improving security. Accordingly, later works by Arroyo & Galiana (2005); Zaoui & Fliscounakis (2006) consider two dispatch intervals, where if there is an outage in the first interval, the dispatch in the second interval is changed. Attarha & Amjady (2016); Karangelos & Wehenkel (2019); Zaoui & Fliscounakis (2006) modelled SCOPF with AC power flow. Li & McCalley (2009) proposed a decomposition method to solve SCOPF efficiently. Huang, Pan, & Guan (2021); Laur, Nieto-Martin, Bunn, & Vicente-Pastor (2020); Ordoudis, Pinson, & Morales (2019) consider generation reserves as a means to recover from outages but do not explicitly model the corresponding contingencies. Ramping limits become relevant while considering generation reserves since the limits constrain reserves deployment. This motivated the consideration of LAOPF with generation reserves without explicitly modelling generator contingencies (Han & Gooi, 2007; Han et al., 2001).

The inclusion of security constraints into LAOPF results in the Look-ahead security-constrained OPF (LASCOPF) problem (Javadi, Amraee, & Capitanescu, 2019). Chakrabarti & Baldick (2020) considered LASCOPF under the $N-1$ contingency criterion using the DC power flow but with a voltage-phase angle representation. They propose a message passing based decomposition algorithm to handle the vast computational complexity of the problem. Their formulation considers security against transmission line contingencies in every dispatch interval. However, ramping constraints are imposed only on the base-case (non-contingency) dispatches, thus ignoring the effect of an outage in one dispatch interval on subsequent ones. A similar (but simpler) LASCOPF formulation is also used by ISOs. Murillo-Sánchez, Zimmerman, Anderson, & Thomas (2013) developed a stochastic LASCOPF formulation under the $N-1$ contingency criterion using AC power flow for use in the event of unreliable forecasts. Alizadeh, Usman, & Capitanescu (2022) developed a similar formulation while also considering flexible resources. However, these formulations require a computationally tractable solution to LASCOPF. Varawala, Hesamzadeh, Dán, & Baldick (2022) proposed a tractable formulation for the DC power flow while considering a comprehensive description of security against generator contingencies under the $N-k$ contingency criterion and discussed computationally tractable approaches such as Benders decomposition and contingency filtering (Capitanescu, Glavic, Ernst, & Wehenkel, 2007; Papavasiliou & Oren, 2013). There are, however, no tractable solutions for LASCOPF using non-convex AC power flow, which, in addition to its large size, is NP hard (Bienstock & Verma, 2019).

The layout of the rest of the article and our contributions are as follows. In Section 2, we propose a LASCOPF formulation with a generalised objective function and generalised constraints, LASCOPF$_1$. We consider security against a set of contingencies under the $N-1$ contingency criterion for a planning horizon of multiple dispatch intervals. We consider that outages corresponding to any contingencies may take place in any dispatch interval and model their effect during the remainder of the planning horizon. Since we consider the ex post dispatch in the remainder of the planning horizon, the number of decision variables in the problem is quadratic in the length of the planning horizon. In order to overcome this, we propose a reduced formulation, LASCOPF-r$_1$, for which the decision variables are defined to be independent of the interval of the contingency, such that there is only one set of decision variables per interval and hence, the number of decision variables are linear in the length of the planning horizon. In Section 3, we extend the LASCOPF$_1$ and LASCOPF-r$_1$ formulations to the $N-k$ contingency criterion where $k \in \mathbb{N}, k \geq 1$, where $\mathbb{N}$ is the set of positive integers, viz., LASCOPF$_k$ and LASCOPF-r$_k$ respectively. Since we consider multiple contingencies with outages that could occur in any order the number of decision variables varies with the number of possible k permutations of contingencies. To overcome this, we propose LASCOPF-ru$_k$ for which the number of decision variables varies with the number of possible k permutations of contingencies.

In Section 4, we present DC-LASCOPF$_1$ where we explicitly model a cost minimisation objective and constraints under DC power flow and consider security against generator contingencies. The objective function for both the DC-LASCOPF$_1$ and DC-LASCOPF-r$_1$ formulations are identical and depend only upon the base-case. Therefore, the contingency scenario decision variables only serve to add constraints on the base-case. We prove that these constraints are identical for LASCOPF$_1$ and LASCOPF-r$_1$ under certain realistically fulfilled conditions and therefore, the optimal objective value is equal for both formulations. We extend our results and show that DC-LASCOPF$_k$, DC-LASCOPF-r$_k$ and DC-LASCOPF-ru$_k$ have the same optimal objective value.

We present DC-LASCOPF$_1$ in order to set the stage for AC-LASCOPF$_1$ under AC power flow which we present in Section 5. We conjecture that certain observations made for DC-LASCOPF$_1$ also apply to AC-LASCOPF$_1$ and therefore the same results would apply to both. In Section 6, we demonstrate the usefulness of the proposed AC-LASCOPF-r$_1$ and AC-LASCOPF-ru$_k$ formulations by comparing their computational time to AC-LASCOPF$_1$ and AC-LASCOPF-r$_k$ respectively. Finally, we conclude in Section 7.

2. LASCOPF under $N-1$ contingency criterion

2.1. General formulation: LASCOPF$_1$

In what follows, we develop a formulation for the look-ahead security-constrained optimal power flow problem under the $N-1$ contingency criterion (LASCOPF$_1$). We do so for a power system with a set of buses[1] $\mathcal{N}$ and generators $\mathcal{G}$. We compute LASCOPF$_1$ for a planning horizon of $T \in \mathbb{N}$ dispatch intervals, assumed to be of equal duration, for simplicity, given the *present* dispatch[2] of the system. The present, for which the dispatch has already been chosen, is represented as dispatch interval (interval, in short) 0. Our objective is to choose the dispatch for intervals 1 to T. Under a base-case dispatch, i.e., when no outage has taken place, the voltage[3] at bus $n \in \mathcal{N}$ in interval $t \in \mathbb{N}_0; t \le T$, where $\mathbb{N}_0$ refers to the set of non-negative integers, is denoted by $v_{n,t}^{\{0\}} \in \mathbb{C}$. Note that the values of variables, such as voltage and parameters, such as demand at a bus may vary continuously with time and therefore, also during a single interval. For brevity, we refer by 'in interval t' to a value at the end of interval t. The active power generation by generator $g \in \mathcal{G}$ is $p_{g,t}^{\{0\}} \in \mathbb{R}_{\ge 0}$ and the reactive power generation is $q_{g,t}^{\{0\}} \in \mathbb{R}$. In the LASCOPF$_1$ formulation, the base-case dispatch of the system in interval t is fully determined by $(v_{n,t}^{\{0\}}, p_{g,t}^{\{0\}}, q_{g,t}^{\{0\}} | n \in \mathcal{N}; g \in \mathcal{G})$.

The LASCOPF$_1$ formulation considers security under the $N-1$ contingency criterion against a given set of contingencies $\mathcal{C}$. Consider that an outage corresponding to contingency $c \in \mathcal{C}$ may take place in any interval $u \in \mathbb{N}; u < T$. Accordingly, we need to explicitly re-dispatch other generators (including the deployment of capacity that would be designated as spinning reserves in a conventional dispatch formulation) over the subsequent intervals, i.e., beginning with interval[4] are implicitly incorporated, e.g., for a generator contingency $c \in \mathcal{G}$ which entails the complete outage of an operational generator, we are implicitly assuming that the stored kinetic energy in the inertia of other generators will make up for the shortfall in the seconds after the outage with other automated actions operating subsequently until new dispatch instructions can be set to generators and implemented. Accordingly, our formulation obtains the dispatch across several consecutive dispatch intervals that is cognisant of such automatic responses, but with the understanding that spinning reserves would be dispatched explicitly in our formulation over several intervals to relieve the capacity providing the automatic actions. $u+1$ until the end of the planning horizon to make up for the shortfall. In order to model the contingency scenario for contingency c, we must consider the modified physical constraints that would apply to the corresponding modified dispatch. Accordingly, a contingency scenario is denoted by the tuple (c,u): in interval $t > u$, the voltage at bus n, and the active and reactive power at generator g are $v_{n,t}^{(c,u)} \in \mathbb{C}$, $p_{g,t}^{(c,u)} \in \mathbb{R}_{\ge 0}$ and $q_{g,t}^{(c,u)} \in \mathbb{R}$, respectively. The dispatch of the system under scenario (c,u) in interval $t > u$ is determined by $(v_{n,t}^{(c,u)}, p_{g,t}^{(c,u)}, q_{g,t}^{(c,u)} | n \in \mathcal{N}; g \in \mathcal{G})$.

Fig. 1 illustrates our model of operation under contingency scenarios.

The LASCOPF$_1$ formulation over the set of variables $\mathcal{S} = (v_{n,t}^{\{0\}}, p_{g,t}^{\{0\}}, q_{g,t}^{\{0\}}, v_{n,t}^{(c,u)}, p_{g,t}^{(c,u)}, q_{g,t}^{(c,u)} | c \in \mathcal{C}; n \in \mathcal{N}; g \in \mathcal{G}; t, u \in \mathbb{N}; u < t \le T)$ is

$$\min_{\mathcal{S}} f\left(v_{n,t}^{\{0\}}, p_{g,t}^{\{0\}}, q_{g,t}^{\{0\}} | n \in \mathcal{N}; g \in \mathcal{G}; t \in \mathbb{N}; t \le T\right), \tag{1a}$$

subject to
(Base-case dispatch constraints:)

$$h_t^{\{0\}}\left(v_{n,t}^{\{0\}}, p_{g,t}^{\{0\}}, q_{g,t}^{\{0\}} | n \in \mathcal{N}; g \in \mathcal{G}\right) \le 0, \tag{1b}$$

$$\tilde{h}_t^{\{0\}}\left(v_{n,t}^{\{0\}}, p_{g,t}^{\{0\}}, q_{g,t}^{\{0\}} | n \in \mathcal{N}; g \in \mathcal{G}\right) = 0, \tag{1c}$$

$$-\overline{R}_g \le p_{g,t}^{\{0\}} - p_{g,t-1}^{\{0\}} \le \overline{R}_g, \tag{1d}$$

(Contingency scenario constraints on the corresponding dispatch:)

$$h_t^{\{c\}}\left(v_{n,t}^{(c,u)}, p_{g,t}^{(c,u)}, q_{g,t}^{(c,u)} | n \in \mathcal{N}; g \in \mathcal{G}\right) \le 0, \tag{1e}$$

$$\tilde{h}_t^{\{c\}}\left(v_{n,t}^{(c,u)}, p_{g,t}^{(c,u)}, q_{g,t}^{(c,u)} | n \in \mathcal{N}; g \in \mathcal{G}\right) = 0, \tag{1f}$$

$$-\overline{R}_g \le p_{g,t}^{(c,u)} - p_{g,t-1}^{(c,u)} \le \overline{R}_g \text{ if } g \ne c; t > u+1, \tag{1g}$$

$$-\overline{R}_g \le p_{g,t}^{(c,u)} - p_{g,t-1}^{\{0\}} \le \overline{R}_g \text{ if } g \ne c; t = u+1, \tag{1h}$$

$\forall c \in \mathcal{C}; \forall g \in \mathcal{G}; \forall t, u \in \mathbb{N}; u < t \le T$.

Here, (1a) is a generalised objective function that represents minimisation over any desired base-case quantity[5] the objective to be a function of the base-case dispatch only (Capitanescu, Glavic, Ernst, & Wehenkel, 2006). This is because (1) the probability of individual outages is in practice difficult to determine, (2) the probability of individual outages is low and therefore, considering the quantity under contingency scenarios would not affect the outcome much and the accompanied increase in computational costs would not be justified and (3) if a contingency actually occurs, then LASCOPF will be re-solved for the system in the next dispatch interval with the lost component(s) removed, treating what was the post-contingency state of the system as the new base-case, so that any sub-optimality will begin to be addressed within a dispatch interval, typically 5 to 15 minutes. Although the contingency scenario quantities are not considered ex ante, based on the principle of receding horizon control, they would be optimised ex post where what was previously the contingency scenario would be the base-case., e.g., total generation cost and transmission line losses. This will turn out to be crucial in enabling a simplification of the problem. The power system is subject to certain static physical constraints, i.e., physical constraints that apply independently to each instant of time, such as active power generation limits and power balance. For the base-case dispatch, (1b) and (1c) represent an aggregate of these inequality and equality constraints respectively. In addition to static physical constraints, every generator $g \in \mathcal{G}$ has ramping limits $\overline{R}_g \in \mathbb{R}_{\ge 0}$, respectively, which constrain the difference in its active power generation between consecutive intervals. For the base-case dispatch, the ramping constraints are expressed in (1d). In Fig. 1, (1d) are represented by solid arrows (which represent no change in scenario) connecting the filled circles (which represent the base-case). It is the existence of ramping constraints that requires consideration of multiple intervals while operating the power system.

The static physical constraints that would apply to the contingency scenario dispatch, including any constraints that characterise the outage corresponding to the contingency, are represented in

[1] See Appendix A for a comprehensive list of notation used throughout this article.

[2] In practice, this *dispatch* would be a re-dispatch in the real-time market compared to the day-ahead market schedules.

[3] For brevity, we use *voltage* to refer to the *voltage phasor*.

[4] We assume that automatic responses carried out immediately after the contingency

[5] A theoretically optimal approach would consider the desired quantity under all scenarios, i.e., the base-case and the contingency scenarios and discount it with their probabilities. However, it is customary in the literature to approximate.

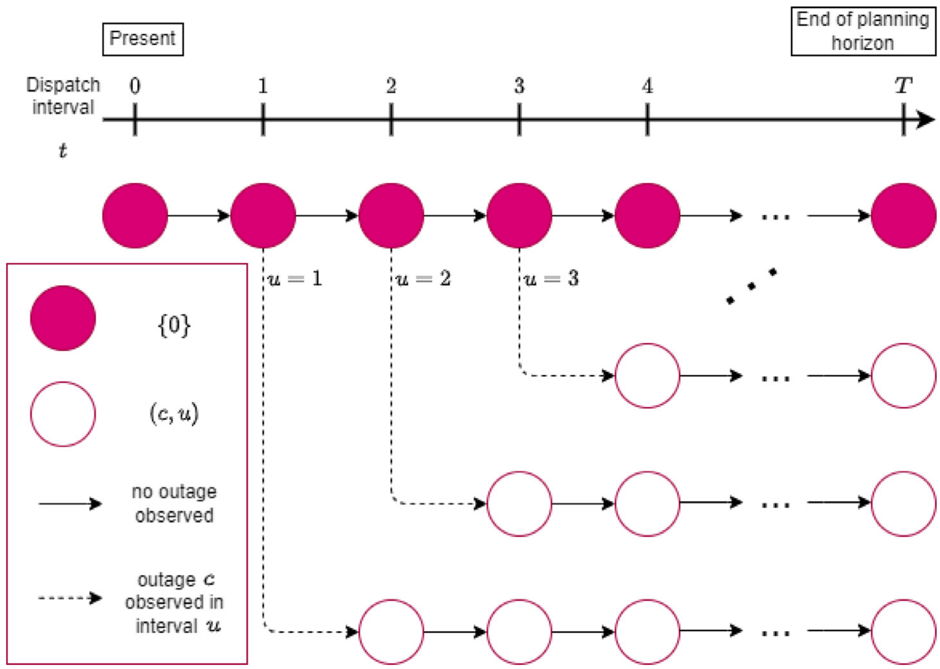

Fig. 1. Illustration of the dispatch under base-case $\{0\}$ and contingency scenario (c, u) under contingency $c \in \mathcal{C}$ in interval $u \in \mathbb{N}; u < T$, and the change from base-case to contingency scenario (c, u) in interval $t = u + 1$ for LASCOPF$_1$.

(1e) and (1f). For example, for a generator contingency $g \in \mathcal{C}$, the characteristic constraint would describe the outage of generator g. Other static physical constraints on other generators and the system would replace the base-case dispatch constraints. Observe that while the arguments of the functions represented by $h_t^{\{c\}}$ and $\tilde{h}_t^{\{c\}}$ depend upon u, the parameters of the functions themselves do not. In addition, consecutive contingency scenario dispatches from interval $u+1$ to interval T must satisfy ramping constraints expressed in (1g). In Fig. 1, constraints (1g) are represented by solid arrows connecting empty circles (which represent a contingency scenario). Finally, since an outage corresponding to a contingency in interval u would require a change from the base-case to the contingency scenario, the ramping constraints (1h) between the base-case dispatch in interval u and the contingency scenario dispatch in interval $u+1$ would apply. In Fig. 1, constraints (1h) are represented by dashed arrows (which represent a change of scenario when an outage corresponding to a contingency takes place). The condition $g \neq c$ ensures that if the contingency is a generator contingency, i.e., $c \in \mathcal{G}$, the ramping constraints would not apply to generator c and if the contingency is not a generator contingency, i.e., $c \notin \mathcal{G}$, it holds for all generators. The distinction between (1g) and (1h) is the reason why u is required to specify a contingency scenario. Our optimisation objective does not include the costs in contingency scenarios, i.e., we do not consider the costs under contingency scenarios ex ante. However, due to the receding horizon control, the dispatch will have to be recomputed ex post and this will move the post-contingency dispatch towards optimality in a receding horizon control fashion given the contingency. In this article, we restrict our attention to the dispatch ex ante.

To represent the contingency reserve limits (Huang et al., 2021), a *conventional* SCOPF formulation would need the surrogate constraint

$$-\underline{S}_{g,t} \leq p_{g,t}^{(c,u)} - p_{g,t}^{\{0\}} \leq \overline{S}_{g,t} \ \forall c \in \mathcal{C}; \quad \forall g \in \mathcal{G}; g \neq c; \forall t, u \in \mathbb{N}; u < t \leq T. \tag{1i}$$

where $\underline{S}_{g,t} \in \mathbb{R}_{\geq 0}$ and $\overline{S}_{g,t} \in \mathbb{R}_{\geq 0}$ represent the lower and upper contingency reserve limits respectively. On the other hand, our formulation explicitly considers contingencies and ensures that generators can be re-dispatched to make up for the shortfall due to a generator contingency. That is, the reservation of capacity and ramping capability is done implicitly by enforcing supply-demand balance in the post-contingency system, rather than by explicitly defining a spinning reserve requirement. This eliminates the need for the surrogate constraint. In fact, we can obtain the parameters $\underline{S}_{g,t}$ and $\overline{S}_{g,t}$ from our formulation as

$$\underline{S}_{g,t} = \min\left\{0, p_{g,t}^{(c,u)} - p_{g,t}^{\{0\}} | c \in \mathcal{C}, c \neq g, u \in \mathbb{N}, u < t\right\} \forall t \in \mathbb{N}; t \leq T, \tag{1j}$$

$$\overline{S}_{g,t} = \max\left\{0, p_{g,t}^{(c,u)} - p_{g,t}^{\{0\}} | c \in \mathcal{C}, c \neq g, u \in \mathbb{N}, u < t\right\} \forall t \in \mathbb{N}; t \leq T. \tag{1k}$$

Recall that base-case dispatches are defined in every interval t, thus the number of base-case dispatches is T. On the contrary, contingency scenario dispatches are defined for every contingency scenario (c, u) in every remaining interval t where $c \in \mathcal{C}$ and $t, u \in \mathbb{N}, u < t \leq T$, thus the number of contingency scenario dispatches is $|\mathcal{C}| \times \sum_{u \in \mathbb{T}, u < T}(T-u) = |\mathcal{C}| \times T(T-1)/2$. This can be inferred from Fig. 1. Accordingly, the total number of decision variables and constraints of LASCOPF$_1$ increases quadratically in T. The number of constraints scales similarly, as discussed in Appendix B. This renders the problem computationally intractable for large values of T.

Finally, observe that our formulation differs from existing formulations of LASCOPF implemented at certain ISOs. Those formulations only model outages that would take place in the upcoming interval, i.e., $u = 1$. In addition, they may or may not consider the effect of the outage on the remainder of the planning horizon but rather only for a single interval following the outage, i.e., $t = u + 1 = 2$. In other words, they do not consider the entire set of constraints in (1h) and may or may not consider the constraint set (1g). On the contrary, our model enforces both (1h) and (1g) and is thus, a more comprehensive formulation of LASCOPF.

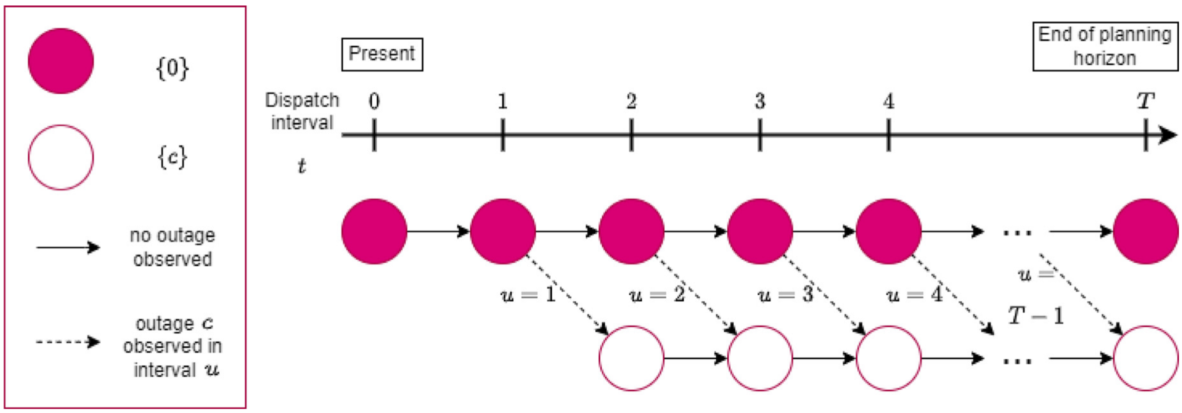

Fig. 2. Illustration of the dispatch under base-case $\{0\}$ and contingency scenario (c, u) under contingency $c \in \mathcal{C}$ in interval $u \in \mathbb{N}; u < T$, and the change from base-case to contingency scenario (c, u) in interval $t = u + 1$ for LASCOPF-r$_1$.

2.2. Reduced formulation: LASCOPF-r$_1$

In LASCOPF$_1$, the quadratic dependence on the number of contingency scenario dispatches on T is due to their dependence on u. In what follows, we propose LASCOPF-r$_1$, a formulation in which every contingency scenario dispatch is *chosen* to be independent of the interval u in which the outage may take place. Accordingly, under scenario (c, u) the contingency scenario dispatch in interval t is $(\upsilon^{(c)}_{n,t}, p^{(c)}_{g,t}, q^{(c)}_{g,t} | n \in \mathcal{N}; g \in \mathcal{G})$. LASCOPF-r$_1$ is essentially LASCOPF$_1$ with the additional constraint

$$\left(\upsilon^{(c,u)}_{n,t}, p^{(c,u)}_{g,t}, q^{(c,u)}_{g,t} | n \in \mathcal{N}; g \in \mathcal{G}\right) = \left(\upsilon^{(c)}_{n,t}, p^{(c)}_{g,t}, q^{(c)}_{g,t} | n \in \mathcal{N}; g \in \mathcal{G}\right)$$
$$\forall c \in \mathcal{C}; \forall t, u \in \mathbb{N}; u < t \le T, \tag{1l}$$

and with $\mathcal{S} = (\upsilon^{\{0\}}_{n,t}, p^{\{0\}}_{g,t}, q^{\{0\}}_{g,t}, \upsilon^{(c)}_{n,t}, p^{(c)}_{g,t}, q^{(c)}_{g,t} | c \in \mathcal{C}; n \in \mathcal{N}; g \in \mathcal{G}; t \in \mathbb{N}; t \le T)$.

Constraint (1l) essentially means that the contingency scenario dispatch $(\upsilon^{(c)}_{n,t}, p^{(c)}_{g,t}, q^{(c)}_{g,t} | n \in \mathcal{N}; g \in \mathcal{G})$ must simultaneously satisfy all the constraints that were satisfied in LASCOPF$_1$ separately by the contingency scenario dispatches $(\upsilon^{(c,u)}_{n,t}, p^{(c,u)}_{g,t}, q^{(c,u)}_{g,t} | n \in \mathcal{N}; g \in \mathcal{G})$ over individual values of u as illustrated in Fig. 2. Observe that constraints (1e) and (1f) depend only on contingency c and not on u and are thus identical for $(\upsilon^{(c,u)}_{n,t}, p^{(c,u)}_{g,t}, q^{(c,u)}_{g,t} | n \in \mathcal{N}; g \in \mathcal{G})$ over all values of u. Consequently, each of them represents only a single set of constraints on $(\upsilon^{(c)}_{n,t}, p^{(c)}_{g,t}, q^{(c)}_{g,t} | n \in \mathcal{N}; g \in \mathcal{G})$. This only leaves (1g) and (1h) as distinct constraints to be obeyed simultaneously by $(\upsilon^{(c)}_{n,t}, p^{(c)}_{g,t}, q^{(c)}_{g,t} | n \in \mathcal{N}; g \in \mathcal{G})$.

In the reduced formulation, there is one contingency scenario dispatch for every contingency c in every interval $t > 1$ and so the number of contingency scenario dispatches is $|\mathcal{C}| \times (T - 1)$. This can be inferred from Fig. 2. Accordingly, the number of decision variables and constraints of LASCOPF-r$_1$ increases only linearly in T rendering the problem more computationally feasible than LASCOPF$_1$ for large values of T.

Recall that certain ISO LASCOPF formulations only model outages that would take place in the upcoming interval, i.e., $u = 1$ but consider their effect throughout the planning horizon, i.e., $t \in \mathbb{N}, 1 < t \le T$. Our LASCOPF-r$_1$ formulation, owing to the independence of u, LASCOPF-r$_1$, would have the same number of decision variables as existing ISO formulations. The only difference between the two is the additional consideration of (1h) in LASCOPF-r$_1$ since LASCOPF-r$_1$ considers that the outage may take place in any interval of the planning horizon and therefore, the corresponding ramping constraints must be obeyed. We expect that this addition does not increase the computational complexity much compared to existing ISO formulations while allowing for a more comprehensive consideration of contingencies.

It follows from the definition of LASCOPF-r$_1$ that its feasible region is a subset of that of LASCOPF$_1$.

Observation 1. If LASCOPF-r$_1$ is feasible, then LASCOPF$_1$ is feasible.

Lemma 1. *For every feasible solution of* LASCOPF$_1$, *the feasible regions defined by (1g) and (1h) for* $(\upsilon^{(c)}_{n,t}, p^{(c)}_{g,t}, q^{(c)}_{g,t} | n \in \mathcal{N}; g \in \mathcal{G}; \forall t \in \mathbb{N}; t \le T)$ *intersect* $\forall c \in \mathcal{C}$.

Proof. Consider a feasible instance of LASCOPF$_1$ and a feasible solution $(\upsilon^{\{0\}}_{n,t}, p^{\{0\}}_{g,t}, q^{\{0\}}_{g,t}, \upsilon^{(c,u)}_{n,t}, p^{(c,u)}_{g,t}, q^{(c,u)}_{g,t} | c \in \mathcal{C}; n \in \mathcal{N}; g \in \mathcal{G}; t, u \in \mathbb{N}; u < t \le T)$. Given the base-case dispatch, $(\upsilon^{\{0\}}_{n,t}, p^{\{0\}}_{g,t}, q^{\{0\}}_{g,t} | n \in \mathcal{N}; g \in \mathcal{G}; t \in \mathbb{N}; t \le T)$, observe that (1g) and (1h) together with (1l) only place constraints on $(p^{(c)}_{g,t} | g \in \mathcal{G}; \forall t \in \mathbb{N}; t \le T)$ $\forall c \in \mathcal{C}$. First, consider a contingency $c = c'$ and interval $t = 2$. Let $(p^{(c',1)}_{g,2} | g \in \mathcal{G})$ be the feasible set of dispatch during interval $t = 2$ for contingency scenario (c') occurring during interval $u = 1$. Now consider the corresponding LASCOPF-r$_1$ formulation and the set of dispatch $(p^{(c')}_{g,2} | g \in \mathcal{G})$. For $t = 2$, the only possible value that u can take on is $u = 1$, since $u \in \mathbb{N}, u < t = 2$, so from (1l) $(p^{(c')}_{g,2} | g \in \mathcal{G}) = (p^{(c',1)}_{g,2} | g \in \mathcal{G})$. As a result, $(p^{(c')}_{g,2} | g \in \mathcal{G})$ satisfies (1h) for $u = 1$. Observe that (1g) does not involve $(p^{(c')}_{g,2} | g \in \mathcal{G})$.

Let us now consider interval $t = 3$ and let $(p^{(c',1)}_{g,3} | g \in \mathcal{G})$ and $(p^{(c',2)}_{g,3} | g \in \mathcal{G})$ be the feasible sets of dispatch for LASCOPF$_1$ for the outage occurring during intervals $u = 1$ and $u = 2$, and consequently satisfying (1g) and (1h) respectively. For LASCOPF-r$_1$, let $(p^{(c')}_{g,3} | g \in \mathcal{G})$ be the corresponding set of dispatch. From (1l), $(p^{(c')}_{g,3} | g \in \mathcal{G})$ is constrained by (1g) for $u = 1$ and by (1h) for $u = 2$. Let $\mathcal{Y}$ be the feasible region defined by (1g) for $u = 1$. We can decompose $\mathcal{Y} = \prod_{g \in \mathcal{G}} \mathcal{Y}_g$, where

$$\mathcal{Y}_g = \begin{cases} \left[-\bar{R}_g + p^{(c',1)}_{g,2}, \bar{R}_g + p^{(c',1)}_{g,2}\right] & \text{if } g \ne c', \\ \mathbb{R} & \text{otherwise.} \end{cases} \tag{2}$$

Similarly, let $\mathcal{Z}$ be the feasible region defined by (1h) for $u = 2$, which can also be decomposed as $\mathcal{Z} = \prod_{g \in \mathcal{G}} \mathcal{Z}_g$, where

$$\mathcal{Z}_g = \begin{cases} \left[-\bar{R}_g + p^{\{0\}}_{g,2}, \bar{R}_g + p^{\{0\}}_{g,2}\right] & \text{if } g \ne c', \\ \mathbb{R} & \text{otherwise.} \end{cases} \tag{3}$$

In the next step, we show that for generator $g = g'$, we have $\mathcal{Y}_{g'} \cap \mathcal{Z}_{g'} = \emptyset$. If $g' = c'$, then $\mathcal{Y}_{g'} \cap \mathcal{Z}_{g'} = \mathbb{R} \ne \emptyset$. If $g' \ne c'$, $\mathcal{Y}_{g'} \cap \mathcal{Z}_{g'} = [-\bar{R}_{g'} + \max\{p^{(c',1)}_{g',2}, p^{\{0\}}_{g',2}\}, \bar{R}_{g'} + \min\{p^{(c',1)}_{g',2}, p^{\{0\}}_{g',2}\}]$. To show that the intersection above is non-empty, let us first consider (1h) for

$g = g'$, $u = 1$ and $t = 2$, which is satisfied by $p_{g',2}^{(c',1)}$. After rearrangement we obtain

$$-\overline{R}_{g'} + p_{g',2}^{(c',1)} \leq p_{g',1}^{\{0\}} \leq \overline{R}_{g'} + p_{g',2}^{(c',1)}. \quad (4)$$

Now consider (1d) for $g = g'$ and $t = 2$, which is satisfied by $p_{g',2}^{\{0\}}$. After rearrangement we obtain

$$-\overline{R}_{g'} + p_{g',2}^{\{0\}} \leq p_{g',1}^{\{0\}} \leq \overline{R}_{g'} + p_{g',2}^{\{0\}}. \quad (5)$$

Since LASCOPF$_1$ is feasible, we know that $\exists\, p_{g',1}^{\{0\}}$, $\exists\, p_{g',2}^{(c',1)}$ and $\exists\, p_{g',2}^{\{0\}}$ satisfying the above. Therefore, after combining the above we obtain

$$-\overline{R}_{g'} + \max\left\{p_{g',2}^{(c',1)}, p_{g',2}^{\{0\}}\right\} \leq \overline{R}_{g'} + \min\left\{p_{g',2}^{(c',1)}, p_{g',2}^{\{0\}}\right\}, \quad (6)$$

which implies $\mathcal{Y}_g \cap \mathcal{Z}_g \neq \emptyset$ for $g' \neq c'$. Therefore, the feasible regions defined by (1g) and (1h) for $(p_{g,3}^{(c')}|g \in \mathcal{G})$ intersect.

Let us now consider interval $t = 4$. The dispatch $(p_{g,4}^{(c')}|g \in \mathcal{G})$ is constrained by (1g) for $u \in \{1, 2\}$ and by (1h) for $u = 3$. First, recall for $t = 3$ that constraints (1g) for $u = 1$ and (1h) for $u = 2$ allow $(p_{g,3}^{(c')}|g \in \mathcal{G}) = (p_{g,3}^{(c',1)}|g \in \mathcal{G}) = (p_{g,3}^{(c',2)}|g \in \mathcal{G})$. If we require this to be the case, we can see that the feasible regions defined by (1g) for $u \in \{1, 2\}$ when $t = 4$ are identical. We can show for $t = 4$, similarly to the approach for $t = 3$, that the feasible region defined by (1h) for $u = 3$ intersects with the others. So far we have shown that feasible regions intersect up to $t = 4$. We can repeat the above analysis for interval $t = t'$ starting with $t' = 5$ up to $t' = T$ in increasing order and then for all contingencies $c \in \mathcal{C}$. This concludes the proof. □

3. LASCOPF under $N - k$ contingency criterion

3.1. Comprehensive formulation: LASCOPF$_k$

In the following section, we propose LASCOPF$_k$, a generalised formulation for LASCOPF under the $N - k$ contingency criterion. LASCOPF$_k$ differs from LASCOPF$_1$ in that we require the system to be secure against k contingencies over the planning horizon. We consider temporally ordered sequences of $s \in \mathbb{N}; s \leq k$ contingencies $(c_1, \ldots, c_s | c_1, \ldots, c_s \in \mathcal{C}; c_1 \neq \ldots \neq c_s)$. For contingencies, it is also necessary to specify the sequence of intervals $(u_1, \ldots, u_s | u_1, \ldots, u_s \in \mathbb{N}; u_1 \leq \ldots \leq u_s < T)$ in which the corresponding outages may take place. Observe that we have allowed multiple outages to take place in the same interval.[6] A contingency scenario is denoted by the tuple $(c_1, u_1, \ldots, c_s, u_s)$: in interval $t > u_s$ the dispatch of the system is $(v_{n,t}^{(c_1,u_1,\ldots,c_s,u_s)}, p_{g,t}^{(c_1,u_1,\ldots,c_s,u_s)}, q_{g,t}^{(c_1,u_1,\ldots,c_s,u_s)}|n \in \mathcal{N}; g \in \mathcal{G})$. Accordingly, the LASCOPF$_k$ formulation over the set of variables $\mathcal{S} = (v_{n,t}^{\{0\}}, p_{g,t}^{\{0\}}, q_{g,t}^{\{0\}}, v_{n,t}^{(c_1,u_1,\ldots,c_s,u_s)}, p_{g,t}^{(c_1,u_1,\ldots,c_s,u_s)}, q_{g,t}^{(c_1,u_1,\ldots,c_s,u_s)}|s \in \mathbb{N}; s \leq k; c_1, \ldots, c_s \in \mathcal{C}; c_1 \neq \ldots \neq c_s; n \in \mathcal{N}; g \in \mathcal{G}; t, u_1, \ldots, u_s \in \mathbb{N}; u_1 \leq \ldots \leq u_s < t \leq T)$ is

$$\min_{\mathcal{S}} f\left(v_{n,t}^{\{0\}}, p_{g,t}^{\{0\}}, q_{g,t}^{\{0\}}|n \in \mathcal{N}; g \in \mathcal{G}; t \in \mathbb{N}; t \leq T\right), \quad (7a)$$

subject to
(Base-case dispatch constraints, (7b) to (7d):) (1b) to (1d),
(Contingency scenario constraints on the corresponding dispatch:)

$$h_t^{\{c_1,\ldots,c_s\}}\left(v_{n,t}^{(c_1,u_1,\ldots,c_s,u_s)}, p_{g,t}^{(c_1,u_1,\ldots,c_s,u_s)}, q_{g,t}^{(c_1,u_1,\ldots,c_s,u_s)}|n \in \mathcal{N}; g \in \mathcal{G}\right) \leq 0, \quad (7b)$$

$$\tilde{h}_t^{\{c_1,\ldots,c_s\}}\left(v_{n,t}^{(c_1,u_1,\ldots,c_s,u_s)}, p_{g,t}^{(c_1,u_1,\ldots,c_s,u_s)}, q_{g,t}^{(c_1,u_1,\ldots,c_s,u_s)}|n \in \mathcal{N}; g \in \mathcal{G}\right) = 0, \quad (7c)$$

$$-\overline{R}_g \leq p_{g,t}^{(c_1,u_1,\ldots,c_s,u_s)} - p_{g,t-1}^{(c_1,u_1,\ldots,c_s,u_s)} \leq \overline{R}_g \quad \text{if } g \notin \{c_1, \ldots, c_s\}; t > u_s + 1, \quad (7d)$$

$$-\overline{R}_g \leq p_{g,t}^{(c_1,u_1,\ldots,c_s,u_s)} - p_{g,t-1}^{\{0\}} \leq \overline{R}_g \quad \text{if } g \notin \{c_1, \ldots, c_s\}; u_1 = u_s; t = u_s + 1, \quad (7e)$$

$$-\overline{R}_g \leq p_{g,t}^{(c_1,u_1,\ldots,c_s,u_s)} - p_{g,t-1}^{(c_1,u_1,\ldots,c_r,u_r)} \leq \overline{R}_g \text{ if } g \notin \{c_1, \ldots, c_s\}; u_r < u_{r+1} = u_s; t = u_s + 1, \quad (7f)$$

$\forall r, s \in \mathbb{N}; s \leq k; \forall c_1, \ldots, c_s \in \mathcal{C}; c_1 \neq \ldots \neq c_s; \forall g \in \mathcal{G}; \forall t, u_1, \ldots, u_s \in \mathbb{N}; u_1 \leq \ldots \leq u_s < t \leq T.$

Constraints (7b) and (7c) are analogous to (1e) and (1f) in LASCOPF$_1$, respectively. Since these are static constraints, they only depend on the sets of contingencies $\{c_1, \ldots, c_s\}$ and not on their order. In addition, consecutive contingency scenario dispatches for dispatches under the same contingency scenario $(c_1, u_1, \ldots, c_s, u_s)$ must satisfy ramping constraints expressed in (7d). At the point of transition from the base-case to $(c_1, u_1, \ldots, c_s, u_s)$, ramping constraints expressed in (7e) would apply. Observe that a transition from the base-case would instead take place if all corresponding outages would take place in the same interval $u_1 = \ldots = u_s$. Similarly, at the point of transition from the contingency scenario $(c_1, u_1, \ldots, c_r, u_r)$ to scenario $(c_1, u_1, \ldots, c_s, u_s)$, ramping constraints expressed in (7f) would apply. In this case, all outages corresponding to contingencies c_{r+1} to c_s would take place in the same interval $u_{r+1} = \ldots = u_s$. The example in Appendix D illustrates the $N - k$ contingency criterion.

Owing to the dependence of contingency scenario dispatches on the intervals $(u_1, \ldots, u_s)$ in which the outages would take place, the numbers of contingency scenario dispatches and accordingly, the decision variables and constraints and follow $\mathcal{O}(T^{k+1})$ rendering LASCOPF$_k$ computationally intractable for large values of T.

3.2. Reduced formulation: LASCOPF-r$_k$

To overcome the $\mathcal{O}(T^{k+1})$ dependence of the number of dispatches on T, similar to LASCOPF-r$_1$, we propose LASCOPF-r$_k$ under the $N - k$ contingency criterion. Here, under contingency scenario $(c_1, u_1, \ldots, c_s, u_s)$ the dispatch in interval t is $(v_{n,t}^{(c_1,\ldots,c_s)}, p_{g,t}^{(c_1,\ldots,c_s)}, q_{g,t}^{(c_1,\ldots,c_s)}|n \in \mathcal{N}; g \in \mathcal{G})$. Note that the indexing in the superscript of the decision variables is on the basis of unordered sets $\{c_1, \ldots, c_s\}$ in LASCOPF-ru$_k$, whereas the indexing is on the basis of ordered sets $(c_1, \ldots, c_s)$ in LASCOPF-r$_k$. Accordingly, LASCOPF-r$_k$ is essentially LASCOPF$_k$ with the additional constraint

$$\left(v_{n,t}^{(c_1,u_1,\ldots,c_s,u_s)}, p_{g,t}^{(c_1,u_1,\ldots,c_s,u_s)}, q_{g,t}^{(c_1,u_1,\ldots,c_s,u_s)}|n \in \mathcal{N}; g \in \mathcal{G}\right) = \left(v_{n,t}^{(c_1,\ldots,c_s)}, p_{g,t}^{(c_1,\ldots,c_s)}, q_{g,t}^{(c_1,\ldots,c_s)}|n \in \mathcal{N}; g \in \mathcal{G}\right)$$
$$\forall s \in \mathbb{N}; s \leq k; \forall c_1, \ldots, c_s \in \mathcal{C}; c_1 \neq \ldots \neq c_s; \forall t, u_1, \ldots, u_s \in \mathbb{N}; u_1 \leq \ldots \leq u_s < t \leq T, \quad (7g)$$

and with $\mathcal{S} = (v_{n,t}^{\{0\}}, p_{g,t}^{\{0\}}, q_{g,t}^{\{0\}}, v_{n,t}^{(c_1,\ldots,c_s)}, p_{g,t}^{(c_1,\ldots,c_s)}, q_{g,t}^{(c_1,\ldots,c_s)}|s \in \mathbb{N}; s \leq k; c_1, \ldots, c_s \in \mathcal{C}; c_1 \neq \ldots \neq c_s; n \in \mathcal{N}; g \in \mathcal{G}; t \in \mathbb{N}; t \leq T)$.

Observation 2. If LASCOPF-r$_k$ is feasible, then LASCOPF$_k$ is feasible.

[6] The contingency criterion securing against k contingencies sequentially is referred to, in the literature, as $N - 1 - 1 \ldots k$ times and simultaneously as $N - k$. The adjustment requirement is in general greater for the latter case. Since our formulation secures against simultaneous contingencies, we have chosen to refer to our contingency criterion as the stricter $N - k$ criterion.

Lemma 2. *For every feasible solution of* LASCOPF-r$_k$, *the feasible regions defined by (7d), (7e) and (7f) for* $(v_{n,t}^{(c_1,\ldots,c_s)}, p_{g,t}^{(c_1,\ldots,c_s)}, q_{g,t}^{(c_1,\ldots,c_s)} | n \in \mathcal{N}; g \in \mathcal{G}; \forall t \in \mathbb{N}; t \le T)$ *intersect* $\forall s \in \mathbb{N}; s \le k; \forall c_1, \ldots, c_s \in \mathcal{C}; c_1 \ne \ldots \ne c_s$.

Proof. The proof of Lemma 1 for the $N-1$ contingency criterion can be generalised to the $N-k$ contingency criterion. We begin by proving that the feasible regions intersect for contingency scenario dispatches with a single contingency and proceed by considering contingency scenario dispatches with increasing number of contingencies. □

Incorporating the additional constraint, we obtain the following formulation for LASCOPF-r$_k$ over the set of variables $\mathcal{S} = (v_{n,t}^{\{0\}}, p_{g,t}^{\{0\}}, q_{g,t}^{\{0\}}, v_{n,t}^{(c_1,\ldots,c_s)}, p_{g,t}^{(c_1,\ldots,c_s)}, q_{g,t}^{(c_1,\ldots,c_s)} | s \in \mathbb{N}; s \le k; c_1, \ldots, c_s \in \mathcal{C}; c_1 \ne \ldots \ne c_s; n \in \mathcal{N}; g \in \mathcal{G}; t \in \mathbb{N}; t \le T)$.

$$\min_{\mathcal{S}} f\left(v_{n,t}^{\{0\}}, p_{g,t}^{\{0\}}, q_{g,t}^{\{0\}} | n \in \mathcal{N}; g \in \mathcal{G}; t \in \mathbb{N}; t \le T\right), \tag{8a}$$

subject to
(Base-case dispatch constraints, (8b) to (8d):) (1b) to (1d),
(Contingency scenario constraints on the corresponding dispatch if $t > 1$:)

$$h_t^{\{c_1,\ldots,c_s\}}\left(v_{n,t}^{(c_1,\ldots,c_s)}, p_{g,t}^{(c_1,\ldots,c_s)}, q_{g,t}^{(c_1,\ldots,c_s)} | n \in \mathcal{N}; g \in \mathcal{G}\right) \le 0, \tag{8b}$$

$$\tilde{h}_t^{\{c_1,\ldots,c_s\}}\left(v_{n,t}^{(c_1,\ldots,c_s)}, p_{g,t}^{(c_1,\ldots,c_s)}, q_{g,t}^{(c_1,\ldots,c_s)} | n \in \mathcal{N}; g \in \mathcal{G}\right) = 0, \tag{8c}$$

$$-\overline{R}_g \le p_{g,t}^{(c_1,\ldots,c_s)} - p_{g,t-1}^{(c_1,\ldots,c_r)} \le \overline{R}_g \text{ if } g \notin \{c_1, \ldots, c_s\} \text{ if } t > 2, \tag{8d}$$

$$-\overline{R}_g \le p_{g,t}^{(c_1,\ldots,c_s)} - p_{g,t-1}^{\{0\}} \le \overline{R}_g \text{ if } g \notin \{c_1, \ldots, c_s\}, \tag{8e}$$

$\forall r, s \in \mathbb{N}; r \le s \le k; \forall c_1, \ldots, c_s \in \mathcal{C}; c_1 \ne \ldots \ne c_s; \forall g \in \mathcal{G}; \forall t, u_1, \ldots, u_s \in \mathbb{N}; u_1 \le \ldots \le u_s < t \le T$. Here, (7d) and (7f) have been represented jointly as (8d).

Since separate contingency scenario dispatches are defined only for every contingency sequence $(c_1, \ldots, c_s)$ in every interval t, the number of contingency scenario dispatches are $\sum_{k'=1}^{k} {}^{|\mathcal{C}|}P_{k'} \times T$ where ${}^{n}P_k$ represents the number of k permutations of n. Accordingly, LASCOPF-r$_k$ is more computationally feasible than LASCOPF$_k$ for large values of T. However, the dependence of the number on ${}^{|\mathcal{C}|}P_k$ could still render LASCOPF-r$_k$ computationally intractable for large values of k, despite it being easier to solve than LASCOPF$_k$.

3.3. Unordered contingencies: LASCOPF-ru$_k$

To overcome the $\sum_{k'=1}^{k} {}^{|\mathcal{C}|}P_{k'}$ dependence of the number of dispatches on k, we propose LASCOPF-ru$_k$. Here, given the sequence of contingencies $(c_1, \ldots, c_s)$ the contingency scenario dispatch in interval t is $(v_{n,t}^{\{c_1,\ldots,c_s\}}, p_{g,t}^{\{c_1,\ldots,c_s\}}, q_{g,t}^{\{c_1,\ldots,c_s\}} | n \in \mathcal{N}; g \in \mathcal{G})$. LASCOPF-ru$_k$ is LASCOPF-r$_k$ with the additional constraint

$$\begin{aligned}&\left(v_{n,t}^{(c_1,\ldots,c_s)}, p_{g,t}^{(c_1,\ldots,c_s)}, q_{g,t}^{(c_1,\ldots,c_s)} | n \in \mathcal{N}; g \in \mathcal{G}\right)\\ &\quad = \left(v_{n,t}^{\{c_1,\ldots,c_s\}}, p_{g,t}^{\{c_1,\ldots,c_s\}}, q_{g,t}^{\{c_1,\ldots,c_s\}} | n \in \mathcal{N}; g \in \mathcal{G}\right)\\ &\quad \forall s \in \mathbb{N}; s \le k; \forall c_1, \ldots, c_s \in \mathcal{C}; c_1 \ne \ldots \ne c_s; \forall t \in \mathbb{N}; t \le T,\end{aligned} \tag{8f}$$

and with $\mathcal{S} = (v_{n,t}^{\{0\}}, p_{g,t}^{\{0\}}, q_{g,t}^{\{0\}}, v_{n,t}^{\{c_1,\ldots,c_s\}}, p_{g,t}^{\{c_1,\ldots,c_s\}}, q_{g,t}^{\{c_1,\ldots,c_s\}} | s \in \mathbb{N}; s \le k; c_1, \ldots, c_s \in \mathcal{C}; c_1 \ne \ldots \ne c_s; n \in \mathcal{N}; g \in \mathcal{G}; t \in \mathbb{N}; t \le T)$.

Observation 3. If LASCOPF-ru$_k$ is feasible, then LASCOPF-r$_k$ and consequently, LASCOPF$_k$ are feasible.

Lemma 3. *For every feasible solution of* LASCOPF-r$_k$, *the feasible regions defined by (8d) and (8e) for* $(v_{n,t}^{\{c_1,\ldots,c_s\}}, p_{g,t}^{\{c_1,\ldots,c_s\}}, q_{g,t}^{\{c_1,\ldots,c_s\}} | n \in \mathcal{N}; g \in \mathcal{G}; \forall t \in \mathbb{N}; t \le T)$ *intersect* $\forall s \in \mathbb{N}; s \le k; \forall c_1, \ldots, c_s \in \mathcal{C}; c_1 \ne \ldots \ne c_s$.

Proof. See Appendix C. □

Since separate contingency scenario dispatches are defined only for every contingency set $\{c_1, \ldots, c_s\}$ rather than sequence in every interval t, the number of contingency scenario dispatches are $\sum_{k'=1}^{k} {}^{|\mathcal{C}|}C_{k'} \times T$ where ${}^{n}C_k$ represents the number of k combinations of n. Accordingly, LASCOPF-ru$_k$ is more computationally feasible than LASCOPF-r$_k$ for large values of k.

3.4. Recovering from an outage

The LASCOPF$_k$ formulation can be extended to include recovery of the failed components from outages. Recovery of a failed component entails bringing the component back online and accordingly dispatching generators. Recovery can be accommodated in the LASCOPF$_k$ formulation by removing the requirement that $c_1 \ne \ldots \ne c_s$ in the definition of the set of decision variables, $\mathcal{S}$ such that $\mathcal{S} = (v_{n,t}^{\{0\}}, p_{g,t}^{\{0\}}, q_{g,t}^{\{0\}}, v_{n,t}^{(c_1,u_1,\ldots,c_s,u_s)}, p_{g,t}^{(c_1,u_1,\ldots,c_s,u_s)}, q_{g,t}^{(c_1,u_1,\ldots,c_s,u_s)} | s \in \mathbb{N}; s \le k; c_1, \ldots, c_s \in \mathcal{C}; n \in \mathcal{N}; g \in \mathcal{G}; t, u_1, \ldots, u_s \in \mathbb{N}; u_1 \le \ldots \le u_s < t \le T)$. Let $d \in \mathbb{N}; d \le k$ and consider the contingency scenario dispatch $(v_{n,t}^{(c_1,u_1,\ldots,c_d,u_d)}, p_{g,t}^{(c_1,u_1,\ldots,c_d,u_d)}, q_{g,t}^{(c_1,u_1,\ldots,c_d,u_d)} | n \in \mathcal{N}; g \in \mathcal{G})$ in interval t where $u_d < t \le T$. If a single contingency c' appears twice in the string of contingencies $(c_1, \ldots, c_d)$, then it represents a dispatch in a system that was previously affected by a corresponding outage but has since recovered. If c' appears thrice, it represents that the outage has recurred since its first recovery and so on.

Recall that constraints (7b) and (7c) are static, i.e., would apply to a given contingency scenario dispatch $(v_{n,t}^{(c_1,u_1,\ldots,c_d,u_d)}, p_{g,t}^{(c_1,u_1,\ldots,c_d,u_d)}, q_{g,t}^{(c_1,u_1,\ldots,c_d,u_d)} | n \in \mathcal{N}; g \in \mathcal{G})$ only based on its *active* contingencies $\{c_1, \ldots, c_s\}$ where $s \le d$. The set of active contingencies $\{c_1, \ldots, c_s\} \ni c'$ if and only if c' is contained in the tuple $(c_1, \ldots, c_d)$ an odd number of times. If $s = 0$, then the base-case dispatch constraints would apply.

In order to define LASCOPF-r$_k$, (7g) would apply as defined. However, to define LASCOPF-ru$_k$ (8f) would have to be modified as

$$\begin{aligned}&\left(v_{n,t}^{(c_1,\ldots,c_d)}, p_{g,t}^{(c_1,\ldots,c_d)}, q_{g,t}^{(c_1,\ldots,c_d)} | n \in \mathcal{N}; g \in \mathcal{G}\right)\\ &\quad = \left(v_{n,t}^{\{c_1,\ldots,c_s\}}, p_{g,t}^{\{c_1,\ldots,c_s\}}, q_{g,t}^{\{c_1,\ldots,c_s\}} | n \in \mathcal{N}; g \in \mathcal{G}\right)\\ &\quad \forall d \in \mathbb{N}; s \le d \le k; \forall c_1, \ldots, c_d \in \mathcal{C}; \forall t \in \mathbb{N}; t \le T.\end{aligned} \tag{9}$$

Note that if $s = 0$, instead of using the base-case dispatch $(v_{n,t}^{\{0\}}, p_{g,t}^{\{0\}}, q_{g,t}^{\{0\}} | n \in \mathcal{N}; g \in \mathcal{G})$ on the right hand side we define a second *base-case* dispatch with the superscript $\{0'\}$ as $(v_{n,t}^{\{0'\}}, p_{g,t}^{\{0'\}}, q_{g,t}^{\{0'\}} | n \in \mathcal{N}; g \in \mathcal{G})$ so that recovery does not impose any constraints on the original base-case dispatch. This new base-case dispatch will satisfy the same constraints as the original base-case dispatch but will not factor into the objective function. This will allow Lemma 3 to hold and will also allow some results that we will present in the following sections.

4. LASCOPF with DC power flow

In what follows, we introduce a particular formulation for LASCOPF$_1$ under the DC power flow model, DC-LASCOPF$_1$ (Varawala et al., 2022). We consider the set of contingencies to be generator contingencies, i.e., $\mathcal{C} \subseteq \mathcal{G}$. Under an outage corresponding to a generator contingency, the failed generator cannot generate. Since DC power flow does not consider reactive power, the DC-LASCOPF$_1$ formulation is defined over the set of variables $\mathcal{S} = (p_{g,t}^{\{0\}}, p_{g,t}^{(c,u)} | c \in \mathcal{C}; g \in \mathcal{G}; t, u \in \mathbb{N}; u < t \le T)$ as

$$\min_{\mathcal{S}} \sum_{t=1}^{T} \sum_{g \in \mathcal{G}} C_{g,t}\left(p_{g,t}^{\{0\}}\right), \tag{10a}$$

subject to
(Base-case dispatch constraints:)

$$\sum_{g \in \mathcal{G}} p_{g,t}^{\{0\}} = \sum_{n \in \mathcal{N}} \tilde{D}_{n,t}, \tag{10b}$$

$$\underline{P}_g \le p_{g,t}^{\{0\}} \le \overline{P}_g, \tag{10c}$$

$$-\overline{K}_l \le \sum_{n \in \mathcal{N}} H_{ln} \left(\sum_{g \in \mathcal{G}} A_{ng} p_{g,t}^{\{0\}} - \tilde{D}_{n,t} \right) \le \overline{K}_l, \tag{10d}$$

$$-\overline{R}_g \le p_{g,t}^{\{0\}} - p_{g,t-1}^{\{0\}} \le \overline{R}_g, \tag{10e}$$

(Generator contingency scenario constraints on the corresponding dispatch:)

$$p_{g,t}^{(c,u)} = 0 \text{ if } g = c, \tag{10f}$$

$$\sum_{g \in \mathcal{G}} p_{g,t}^{(c,u)} = \sum_{n \in \mathcal{N}} \tilde{D}_{n,t}, \tag{10g}$$

$$\underline{P}_g \le p_{g,t}^{(c,u)} \le \overline{P}_g \text{ if } g \ne c, \tag{10h}$$

$$-\overline{R}_g \le p_{g,t}^{(c,u)} - p_{g,t-1}^{(c,u)} \le \overline{R}_g \text{ if } g \ne c; t > u+1, \tag{10i}$$

$$-\overline{R}_g \le p_{g,t}^{(c,u)} - p_{g,t-1}^{\{0\}} \le \overline{R}_g \text{ if } g \ne c; t = u+1, \tag{10j}$$

$\forall c \in \mathcal{C}; \forall g \in \mathcal{G}; \forall l \in \mathcal{L}; \forall t, u \in \mathbb{N}; u < t \le T$.

For every generator $g \in \mathcal{G}$ in interval $t \in \mathbb{N}; t \le T$, the cost of generating active power $p_{g,t} \in \mathbb{R}_{\ge 0}$ is $C_{g,t}(p_{g,t}) \in \mathbb{R}_{\ge 0}$. For every bus $n \in \mathcal{N}$ in interval $t \in \mathbb{N}; t \le T$, the active power demand is $\tilde{D}_{n,t} \in \mathbb{R}_{\ge 0}$. For every generator $g \in \mathcal{G}$, the minimum and maximum active power generation limits are $\underline{P}_g, \overline{P}_g \in \mathbb{R}_{\ge 0}$ respectively where $\underline{P}_g \le \overline{P}_g$. $A_{ng} = 1$ if generator g is located at bus n and $A_{ng} = 0$ otherwise $\forall n \in \mathcal{N}; g \in \mathcal{G}$. For transmission line $l \in \mathcal{L}$, the maximum power flow is $\overline{K}_l \in \mathbb{R}_{\ge 0}$. $H_{ln} \in \mathbb{R}$ is the power transfer distribution factor for transmission line l and bus n $\forall n \in \mathcal{N}; \forall l \in \mathcal{L}$.

Objective function (10a) is the total generation cost in the base-case dispatch over the entire planning horizon. For the base-case dispatch, the power balance, active power generation limits, transmission line limits and ramping constraints are (10b), (10c), (10d) and (10e), respectively. When an outage corresponding to a generator contingency takes place, the failed generator cannot generate for the remainder of the planning horizon which is enforced by constraint (10f). In addition, for the generator contingency scenario dispatch[7], the power balance and active power generation limits are (10g) and (10h), respectively. The ramping constraints are (10i) and (10j). Note here that since the contingency scenario dispatch is only defined for intervals after which the outage would take place, i.e., $t > u$, the constraints are accordingly only defined for these intervals. Under severe outages such as those corresponding to generator contingencies, transmission line limits are often relaxed to increase flexibility (Chapter 8, Wood, Wollenberg, & Sheblé, 2014). In what follows, we characterise the structure of (10b) and (10g).

Observation 4. If DC-LASCOPF$_1$ is feasible, then the feasible regions defined by (10b) for $(p_{g,t}^{\{0\}} | g \in \mathcal{G})$ and (10g) for $(p_{g,t}^{(c,u)} | g \in \mathcal{G})$ $\forall c \in \mathcal{C}; \forall u \in \mathbb{N}; u < t \le T$ are identical and convex. Furthermore, $\exists F_t : \mathbb{R}^{|\mathcal{G}|} \to R$ such that $\partial F_t(p_{g,t} | g \in \mathcal{G}) / \partial p_{g,t} \ge 0 \ \forall g \in \mathcal{G} \ \ \forall t \in \mathbb{N}$ such that the feasible regions can be represented by $F_t(p_{g,t} | g \in \mathcal{G}) = 0$.

This observation is instrumental for obtaining the results in the rest of the section and also towards developing the theorems for AC-LASCOPF$_1$ in the following section. We now continue with proposing DC-LASCOPF-r$_1$ which is essentially DC-LASCOPF$_1$ with the additional constraint

$$\left(p_{g,t}^{(c,u)} | g \in \mathcal{G}\right) = \left(p_{g,t}^{(c)} | g \in \mathcal{G}\right) \ \forall c \in \mathcal{C}; \forall t, u \in \mathbb{N}; u < t \le T, \tag{10k}$$

and with $\mathcal{S} = (p_{g,t}^{\{0\}}, p_{g,t}^{(c)} | c \in \mathcal{C}; g \in \mathcal{G}; t \in \mathbb{N}; t \le T)$.

It is intuitive to expect that, barring borderline cases, an optimal solution for DC-LASCOPF$_1$ exists, noting that there may be other optimal solutions, for which the contingency scenario active power generation would all not be less than the base-case active power generation for all healthy generators based on the net loss of generation. In what follows, we show that if this holds, then solving DC-LASCOPF$_1$ and DC-LASCOPF-r$_1$ are equivalent.

Theorem 1. *If* DC-LASCOPF$_1$ *is feasible and has a solution for which either* $p_{g,t}^{(c,u)} \ge p_{g,t}^{\{0\}} \ \forall g \in \mathcal{G}; g \ne c$ *or* $p_{g,t}^{(c,u)} \le p_{g,t}^{\{0\}} \ \forall g \in \mathcal{G}; g \ne c \ \ \forall c \in \mathcal{C}; \forall t, u \in \mathbb{N}; u < t \le T$, *then* DC-LASCOPF-r$_1$ *is feasible. Furthermore, if* DC-LASCOPF$_1$ *has such a solution that is optimal, then the optimal objective value is equal for both* DC-LASCOPF$_1$ *and* DC-LASCOPF-r$_1$.

Proof. The proof follows that of Theorem 4 presented in the next section. □

Note here that the condition that there be a feasible solution of LASCOPF$_1$ such that the contingency scenario active power generation meets the given requirements is only required to prove the equivalence between the formulations. The condition is not explicitly included as a constraint in either formulation and accordingly, either formulation may have solutions that do not satisfy this condition.

Now, consider the $N-k$ contingency criterion. We formulate the DC-LASCOPF$_k$, DC-LASCOPF-r$_k$ and DC-LASCOPF-ru$_k$ problems as extensions of DC-LASCOPF$_1$ according to the general LASCOPF$_k$, LASCOPF-r$_k$ and LASCOPF-ru$_k$ formulations respectively presented in Section 3. We obtain the following results for the formulations.

Theorem 2. *If* DC-LASCOPF$_k$ *is feasible and has a solution for which either* $p_{g,t}^{(c_1,u_1,\ldots,c_s,u_s)} \ge p_{g,t}^{\{0\}} \ \forall g \in \mathcal{G}; g \notin \{c_1, \ldots, c_s\}$ *or* $p_{g,t}^{(c_1,u_1,\ldots,c_s,u_s)} \le p_{g,t}^{\{0\}} \ \forall g \in \mathcal{G}; g \notin \{c_1, \ldots, c_s\} \ \ \forall s \in \mathbb{N}; s \le k; \forall c_1, \ldots, c_s \in \mathcal{C}; c_1 \ne \ldots \ne c_s; \forall t, u_1 \in \mathbb{N}; u_1 = \ldots = u_s < t \le T$ *and either* $p_{g,t}^{(c_1,u_1,\ldots,c_s,u_s)} \ge p_{g,t}^{(c_1,u_1,\ldots,c_r,u_r)} \ \forall g \in \mathcal{G}; g \notin \{c_1, \ldots, c_s\}$ *or* $p_{g,t}^{(c_1,u_1,\ldots,c_s,u_s)} \ge p_{g,t}^{(c_1,u_1,\ldots,c_r,u_r)} \ \forall g \in \mathcal{G}; g \notin \{c_1, \ldots, c_s\} \ \ \forall r, s \in \mathbb{N}; r \le s \le k; \forall c_1, \ldots, c_r \in \mathcal{C}; c_1 \ne \ldots \ne c_s; \forall t, u_1, \ldots, u_s \in \mathbb{N}; u_1 \le \ldots \le u_r \le u_{r+1} = \ldots = u_s < t \le T$, *then* DC-LASCOPF-r$_k$ *is feasible. Furthermore, if* DC-LASCOPF$_1$ *has such a solution that is optimal, then the optimal objective value is equal for both* DC-LASCOPF$_k$ *and* DC-LASCOPF-r$_k$.

Proof. The proof follows that of Theorem 1 can be generalised to the $N-k$ contingency criterion. □

Theorem 3. *If* DC-LASCOPF-r$_k$ *is feasible and has a solution for which either* $p_{g,t}^{(c_1,\ldots,c_s^1)} \ge p_{g,t}^{(c_1,\ldots,c_s^2)} \ \forall g \in \mathcal{G}; g \notin \{c_1, \ldots, c_{s-1}, c_s^1, c_s^2\}$ *or* $p_{g,t}^{(c_1,\ldots,c_s^1)} \le p_{g,t}^{(c_1,\ldots,c_s^2)} \ \forall g \in \mathcal{G}; g \notin \{c_1, \ldots, c_{s-1}, c_s^1, c_s^2\} \ \ \forall s \in \mathbb{N}; s \le$

[7] A theoretically optimal approach would consider load-shedding as a possible action and weigh its associated cost against the increased cost due to consideration of security constraints that prevent load-shedding. However, in practice, the cost of load-shedding is typically so high compared to generation costs that it is rarely the optimal outcome. If low cost load-shedding is available and set up to be triggered through dispatch signals, then, in principle, it could be considered in our LASCOPF formulation.

$k; \forall c_1, \ldots, c_{s-1}, c_s^1, c_s^2 \in \mathcal{C}; c_1 \neq \ldots \neq c_{s-1} \neq c_s^1 \neq c_s^2; \forall t \in \mathbb{N}; t \leq T,$ *then* DC-LASCOPF-ru$_k$ *is feasible. Furthermore, if* DC-LASCOPF$_1$ *has such a solution that is optimal, then the optimal objective value is equal for both* DC-LASCOPF-r$_k$ *and* DC-LASCOPF-ru$_k$.

Proof. The proof follows that of Theorem 1. □

5. LASCOPF with AC power flow

In what follows, we introduce LASCOPF$_1$ under AC power flow, which we refer to as AC-LASCOPF$_1$ and we show that due to similarities between DC-LASCOPF$_1$ and AC-LASCOPF$_1$ the analysis developed in the previous section would also apply to AC-LASCOPF$_1$. We consider the set of contingencies to be generator contingencies, i.e., $\mathcal{C} \subseteq \mathcal{G}$. The AC-LASCOPF$_1$ formulation is defined over the set $\mathcal{S} = (v_{n,t}^{\{0\}}, p_{g,t}^{\{0\}}, q_{g,t}^{\{0\}}, v_{n,t}^{(c,u)}, p_{g,t}^{(c,u)}, q_{g,t}^{(c,u)} |\ c \in \mathcal{C}; n \in \mathcal{N}; g \in \mathcal{G}; t, u \in \mathbb{N}; u < t \leq T)$ as

$$\min_{\mathcal{S}} \sum_{t=1}^{T} \sum_{g \in \mathcal{G}} C_{g,t}\left(p_{g,t}^{\{0\}}\right), \tag{11a}$$

subject to

(base-case dispatch constraints:)

$$\sum_g A_{ng}\left(p_{g,t}^{\{0\}} + \jmath q_{g,t}^{\{0\}}\right) - D_{n,t} = v_{n,t}^{\{0\}} \sum_{n'} Y_{nn'}^* v_{n',t}^{\{0\}*}, \tag{11b}$$

$$\underline{P}_g \leq p_{g,t}^{\{0\}} \leq \overline{P}_g, \tag{11c}$$

$$-\overline{Q}_g \leq q_{g,t}^{\{0\}} \leq \overline{Q}_g, \tag{11d}$$

$$\underline{V}_n \leq \left|v_{n,t}^{\{0\}}\right| \leq \overline{V}_n, \tag{11e}$$

$$\left|\sum_n T_{ln} v_{n,t}^{\{0\}} \sum_{n'} \tilde{Y}_{ln'}^* v_{n',t}^{\{0\}*}\right| \leq \overline{K}_l, \tag{11f}$$

$$\left|\sum_n F_{ln} v_{n,t}^{\{0\}} \sum_{n'} \tilde{Y}_{ln'}^* v_{n',t}^{\{0\}*}\right| \leq \overline{K}_l, \tag{11g}$$

$$-\overline{R}_g \leq p_{g,t}^{\{0\}} - p_{g,t-1}^{\{0\}} \leq \overline{R}_g, \tag{11h}$$

(Generator contingency scenario constraints on the corresponding dispatch:)

$$p_{g,t}^{(c,u)} = 0 \text{ if } g = c, \tag{11i}$$

$$q_{g,t}^{(c,u)} = 0 \text{ if } g = c, \tag{11j}$$

$$\sum_g A_{ng}\left(p_{g,t}^{(c,u)} + \jmath q_{g,t}^{(c,u)}\right) - D_{n,t} = v_{n,t}^{(c,u)} \sum_{n'} Y_{nn'}^* v_{n',t}^{(c,u)*}, \tag{11k}$$

$$\underline{P}_g \leq p_{g,t}^{(c,u)} \leq \overline{P}_g \text{ if } g \neq c, \tag{11l}$$

$$-\overline{Q}_g \leq q_{g,t}^{(c,u)} \leq \overline{Q}_g \text{ if } g \neq c, \tag{11m}$$

$$\underline{V}_n \leq \left|v_{n,t}^{(c,u)}\right| \leq \overline{V}_n, \tag{11n}$$

$$-\overline{R}_g \leq p_{g,t}^{(c,u)} - p_{g,t-1}^{(c,u)} \leq \overline{R}_g \text{ if } g \neq c; t > u+1, \tag{11o}$$

$$-\overline{R}_g \leq p_{g,t}^{(c,u)} - p_{g,t-1}^{\{0\}} \leq \overline{R}_g \text{ if } g \neq c; t = u+1, \tag{11p}$$

$\forall c \in \mathcal{C}; \forall n \in \mathcal{N}; \forall g \in \mathcal{G}; \forall l \in \mathcal{L}; \forall t, u \in \mathbb{N}; u < t \leq T.$

For every pair of buses $n, n' \in \mathcal{N}$, the bus admittance factor is $Y_{nn'} \in \mathbb{C}$. For every bus $n \in \mathcal{N}$ in interval $t \in \mathbb{N}; t \leq T$, $D_{n,t} \in \mathbb{C}$ represents the complex power demand where $\text{Re}(D_{n,t}) \in \mathbb{R}_{\geq 0}$ is the active power demand and $\text{Im}(D_{n,t}) \in \mathbb{R}$ is the reactive power demand. For every generator $g \in \mathcal{G}$, the reactive power generation limit is $\overline{Q}_g \in \mathbb{R}_{\geq 0}$. For every bus $n \in \mathcal{N}$, the minimum and maximum voltage magnitude limits are $\underline{V}_n, \overline{V}_n \in \mathbb{R}_{\geq 0}$ respectively where $\underline{V}_n \leq \overline{V}_n$. $Y_{ln} \in \mathbb{C}$ is the bus branch admittance factor for transmission line l and bus n $\forall n \in \mathcal{N}; \forall l \in \mathcal{L}$. T_{ln} and F_{ln} take on the value 1 if transmission line l end at bus n or originates at bus n respectively and are 0 otherwise.

For the base-case dispatch, the power balance, reactive power generation limits, voltage magnitude limits and transmission line limits are (11b), (11d), (11e), and (11f) and (11g), respectively (Chapter 8, Wood et al., 2014). When an outage corresponding to a generator contingency takes place, the failed generator cannot generate for the remainder of the planning horizon which is enforced by constraints (11i) and (11j). Accordingly, for the generator contingency scenario dispatch, the power balance, reactive power generation limits and voltage magnitude limits are (11k), (11m) and (11n), respectively. The illustrative example in Appendix D highlights the effect of considering generator contingency scenario dispatches on the base-case dispatch.

We make the following Conjecture for AC-LASCOPF$_1$ similar to Observation 4 for DC-LASCOPF$_1$.

Conjecture 1. *If* AC-LASCOPF$_1$ *is feasible, then there exists a convex set of values of* $(p_{g,t}^{(c,u)} | g \in \mathcal{G})$ *that are feasible w.r.t. (11k) and (11n) such that a feasible* $(v_{n,t}^{(c,u)} | g \in \mathcal{G})$ *exists, given feasible* $(q_{g,t}^{(c,u)} | g \in \mathcal{G})$ $\forall c \in \mathcal{C}; \forall u \in \mathbb{N}; u < t \leq T$. *Furthermore,* $\exists F_t : \mathbb{R}^{|\mathcal{G}|} \to R$ *such that* $\partial F_t(p_{g,t} | g \in \mathcal{G}) / \partial p_{g,t} \geq 0\ \forall g \in \mathcal{G}\ \forall t \in \mathbb{N}$ *such that the feasible regions can be represented by* $F_t(p_{g,t} | g \in \mathcal{G}) = 0$.

We propose AC-LASCOPF-r$_1$ which is essentially AC-LASCOPF$_1$ with the additional constraint

$$\left(v_{n,t}^{(c,u)}, p_{g,t}^{(c,u)}, q_{g,t}^{(c,u)} | n \in \mathcal{N}; g \in \mathcal{G}\right) = \left(v_{n,t}^{(c)}, p_{g,t}^{(c)}, q_{g,t}^{(c)} | n \in \mathcal{N}; g \in \mathcal{G}\right)$$
$$\forall c \in \mathcal{C}; \forall t, u \in \mathbb{N}; u < t \leq T, \tag{11q}$$

and with $\mathcal{S} = (v_{n,t}^{\{0\}}, p_{g,t}^{\{0\}}, q_{g,t}^{\{0\}}, v_{n,t}^{(c)}, p_{g,t}^{(c)}, q_{g,t}^{(c)} | c \in \mathcal{C}; n \in \mathcal{N}; g \in \mathcal{G}; t \in \mathbb{N}; t \leq T)$. In what follows, we show that if AC-LASCOPF$_1$ has a solution for which active power generation for all healthy generators in the contingency scenario is either not less than or not greater than that in the base-case for all healthy generators, then solving AC-LASCOPF$_1$ and AC-LASCOPF-r$_1$ are equivalent.

Theorem 4. *If Conjecture 1 holds, then if* AC-LASCOPF$_1$ *is feasible and has a solution for which either* $p_{g,t}^{(c,u)} \geq p_{g,t}^{\{0\}}\ \forall g \in \mathcal{G}; g \neq c$ *or* $p_{g,t}^{(c,u)} \geq p_{g,t}^{\{0\}}\ \forall g \in \mathcal{G}; g \neq c\quad \forall c \in \mathcal{C}; \forall t, u \in \mathbb{N}; u < t \leq T$, *then* AC-LASCOPF-r$_1$ *is feasible. Furthermore, if* AC-LASCOPF$_1$ *has such a solution that is optimal, then the optimal objective value is equal for both AC-LASCOPF$_1$ and* AC-LASCOPF-r$_1$.

Proof. Consider a feasible instance of AC-LASCOPF$_1$ with a solution for which either $p_{g,t}^{(c,u)} \geq p_{g,t}^{\{0\}}\ \forall g \in \mathcal{G}; g \neq c$ or $p_{g,t}^{(c,u)} \geq p_{g,t}^{\{0\}}\ \forall g \in \mathcal{G}; g \neq c\quad \forall c \in \mathcal{C}; \forall t, u \in \mathbb{N}; u < t \leq T$. We begin by observing that the objective (11*a*) is a function only of the base-case dispatch $(p_{g,t}^{\{0\}} | g \in \mathcal{G}; t \in \mathbb{N})$ and thus, whether or not the solution is optimal depends only upon the value of $(p_{g,t}^{\{0\}} | g \in \mathcal{G}; t \in \mathbb{N})$. Therefore, to prove the theorem, it is sufficient to show that given *any* set of base-case dispatch $(v_{n,t}^{\{0\}}, p_{g,t}^{\{0\}}, q_{g,t}^{\{0\}} | n \in \mathcal{N}; g \in \mathcal{G}; t \in \mathbb{N})$, since the contingency scenario dispatch $(v_{n,t}^{(c,u)}, p_{g,t}^{(c,u)}, q_{g,t}^{(c,u)} | n \in \mathcal{N}; g \in \mathcal{G}; \forall t, u \in \mathbb{N}; u < t \leq T)$ exists satisfying (11i) to (11p), there exists a reduced set $(v_{n,t}^{(c)}, p_{g,t}^{(c)}, q_{g,t}^{(c)} | n \in \mathcal{N}; g \in \mathcal{G}; \forall t, u \in \mathbb{N}; u < t \leq T)$ satisfying (11q) $\forall c \in \mathcal{C}$. Then, if the selected set of base-case dispatch $(v_{n,t}^{\{0\}}, p_{g,t}^{\{0\}}, q_{g,t}^{\{0\}} | n \in \mathcal{N}; g \in \mathcal{G}; t \in \mathbb{N})$ is optimal, the entire solution is optimal. Consider interval $t = 2$. It is straightforward to see that u can only take on one value and consequently, (11q) is trivially satisfied.

Now, consider contingency $c = c'$ and interval $t = 3$. Here, $u \in \{1, 2\}$. In order to show that $\exists\ (v_{n,3}^{(c')}, p_{g,3}^{(c')}, q_{g,3}^{(c')} | n \in \mathcal{N}; g \in \mathcal{G})$ satisfying (11q), it is sufficient to show that the feasible region defined by (11i) to (11o) when $u = 1$ for $(v_{n,3}^{(c',1)}, p_{g,3}^{(c',1)}, q_{g,3}^{(c',1)} | n \in \mathcal{N}; g \in \mathcal{G})$ intersect with that defined by (11i) to (11n) and (11p) when $u = 2$ for $(v_{n,3}^{(c',1)}, p_{g,3}^{(c',2)}, q_{g,3}^{(c',2)} | n \in \mathcal{N}; g \in \mathcal{G})$, allowing us to choose equal values for both sets. From Lemma 1, the feasible regions defined by (11o) when $u = 1$, $\mathcal{Y} = \mathbb{C}^{|\mathcal{N}|} \times (\prod_{g\in\mathcal{G}; g\neq c'}[-\overline{R}_g + p_{g,2}^{(c,1)}, \overline{R}_g + p_{g,2}^{(c,1)}]) \times \mathbb{R} \times \mathbb{R}^{|\mathcal{G}|}$ and (11p) when $u = 2$, $\mathcal{Z} = \mathbb{C}^{|\mathcal{N}|} \times (\prod_{g\in\mathcal{G}; g\neq c'}[-\overline{R}_g + p_{g,2}^{\{0\}}, \overline{R}_g + p_{g,2}^{\{0\}}]) \times \mathbb{R} \times \mathbb{R}^{|\mathcal{G}|}$ intersect, i.e., $\mathcal{Y} \cap \mathcal{Z} \neq \emptyset$. Let us consider the case where $p_{g,2}^{\{0\}} \leq p_{g,2}^{(c,1)}\ \forall g \in \mathcal{G}; g \neq c'$. Therefore, it is left to show that the feasible region defined by (11i) to (11n), which is identical for all values of u, intersects with $\mathbb{C}^{|\mathcal{N}|} \times (\prod_{g\in\mathcal{G}; g\neq c'}[-\overline{R}_g + p_{g,2}^{(c,1)}, \overline{R}_g + p_{g,2}^{\{0\}}]) \times \mathbb{R} \times \mathbb{R}^{|\mathcal{G}|}$.

Let the feasible region defined by (11i), (11j), (11l) and (11m) be $\mathcal{X} = \mathbb{C}^{|\mathcal{N}|} \times (\prod_{g\in\mathcal{G}; g\neq c'}[\underline{P}_g, \overline{P}_g]) \times \{0\} \times (\prod_{g\in\mathcal{G}; g\neq c'}[\underline{Q}_g, \overline{Q}_g]) \times \{0\}$. From Conjecture 1, given feasible $(q_{g,3} | g \in \mathcal{G})\ \exists\ F_3 : \mathbb{R}^{|\mathcal{G}|} \to \mathbb{R}$ such that $\partial F_3(p_{g,3} | g \in \mathcal{G})/\partial p_{g,3} \geq 0\ \forall g \in \mathcal{G}$ where $F_3(p_{g,3} | g \in \mathcal{G}) = 0$ represents the feasible region defined by (11k) and (11n) projected onto $(p_{g,t}^{(c,u)} | g \in \mathcal{G})$. Since AC-LASCOPF$_1$ is feasible, $\exists\ (v_{n,3}^{(c',1)}, p_{g,3}^{(c',1)}, q_{g,3}^{(c',1)} | n \in \mathcal{N}; g \in \mathcal{G}) \in \mathcal{X} \cap \mathcal{Y}$ for $u = 1$ such that $F_3(p_{g,3}^{(c',1)} | g \in \mathcal{G}) = 0$. Since $\partial F_3(p_{g,3} | g \in \mathcal{G})/\partial p_{g,3} \geq 0\ \forall g \in \mathcal{G}$, $F_3(\max\{\underline{P}_g, -\overline{R}_g + p_{g,2}^{(c,1)}\}, 0 | g \in \mathcal{G}; g \neq c') \leq 0$. Now, consider $(v_{n,3}^{(c',2)}, p_{g,3}^{(c',2)}, q_{g,3}^{(c',2)} | n \in \mathcal{N}; g \in \mathcal{G})$ for $u = 2$ and let $(q_{g,3}^{(c',2)} | g \in \mathcal{G}) = (q_{g,3}^{(c',1)} | g \in \mathcal{G})$. Then, since $\mathcal{Y}$ and $\mathcal{Z}$ place no constraints on $(q_{g,3}^{(c',1)} | g \in \mathcal{G})$, $(v_{n,3}^{(c',2)}, p_{g,3}^{(c',2)}, q_{g,3}^{(c',2)} | n \in \mathcal{N}; g \in \mathcal{G}) \in \mathcal{X} \cap \mathcal{Z}$ such that $F_3(p_{g,3}^{(c',2)} | g \in \mathcal{G}) = 0$ and $F_3(\max\{\overline{P}_g, \overline{R}_g + p_{g,2}^{\{0\}}\}, 0 | g \in \mathcal{G}; g \neq c') \geq 0$. Since the feasible region defined by (11k) and (11n), $\mathcal{X}$, $\mathcal{Y}$ and $\mathcal{Z}$ are convex when projected onto $(p_{g,t}^{(c,u)} | g \in \mathcal{G})$, $\exists\ (v_{n,3}^{(c')}, p_{g,3}^{(c')}, q_{g,3}^{(c')} | n \in \mathcal{N}; g \in \mathcal{G}) \in (\prod_{n\in\mathcal{N}}[\underline{V}_n, \overline{V}_n]) \times (\prod_{g\in\mathcal{G}; g\neq c'}[\max\{\underline{P}_g, -\overline{R}_g + p_{g,2}^{(c,1)}\}, \min\{\overline{P}_g, \overline{R}_g + p_{g,2}^{\{0\}}\}]) \times \{0\} \times \mathbb{R}^{(|\mathcal{G}|-1)} \times \{0\}$ such that $F_3(p_{g,3}^{(c')} | g \in \mathcal{G}) = 0$. Consequently, the feasible regions defined by (11i) to (11n) intersect with $\mathbb{C}^{|\mathcal{N}|} \times (\prod_{g\in\mathcal{G}; g\neq c'}[-\overline{R}_g + p_{g,2}^{(c,1)}, \overline{R}_g + p_{g,2}^{\{0\}}]) \times \mathbb{R} \times \mathbb{R}^{|\mathcal{G}|}$. We can con conduct a similar analysis for the case where $p_{g,2}^{\{0\}} \geq p_{g,2}^{(c,1)}\ \forall g \in \mathcal{G}; g \neq c'$ and show that the feasible regions defined by (11i) to (11n) intersect with $\mathbb{C}^{|\mathcal{N}|} \times (\prod_{g\in\mathcal{G}; g\neq c'}[-\overline{R}_g + p_{g,2}^{\{0\}}, \overline{R}_g + p_{g,2}^{(c,1)}]) \times \mathbb{R} \times \mathbb{R}^{|\mathcal{G}|}$. We can repeat the above analysis for interval $t = t'$ starting with $t' = 3$ up to $t' = T$ in increasing order and then for all contingencies $c \in \mathcal{C}$. This concludes the proof. □

Note that nowhere in Conjecture 1 or in the proof above have we considered the specific form of the objective function. Therefore, Conjecture 1, Theorem 4 and the subsequent results would hold for any objective function that only depend on the base-case dispatch, such as (1a). E.g., one could use the objective function of SCOPF, where costs are minimised only for $t = 1$, but with the entire set of LASCOPF constraints over the planning horizon.

Now, consider the $N - k$ contingency criterion. We formulate the AC-LASCOPF$_k$, AC-LASCOPF-r$_k$ and AC-LASCOPF-ru$_k$ problems as extensions of AC-LASCOPF$_1$ according to the general LASCOPF$_k$, LASCOPF-r$_k$ and LASCOPF-ru$_k$ formulations respectively presented in Section 3. We obtain the following results for the formulations.

Theorem 5. *If Conjecture 1 holds, then if* AC-LASCOPF$_k$ *is feasible and has a solution for which either* $p_{g,t}^{(c_1,u_1,\ldots,c_s,u_s)} \geq p_{g,t}^{\{0\}}\ \forall g \in \mathcal{G}; g \notin \{c_1, \ldots, c_s\}$ *or* $p_{g,t}^{(c_1,u_1,\ldots,c_s,u_s)} \leq p_{g,t}^{\{0\}}\ \forall g \in \mathcal{G}; g \notin \{c_1, \ldots, c_s\}\ \forall s \in \mathbb{N}; s \leq k; \forall c_1, \ldots, c_s \in \mathcal{C}; c_1 \neq \ldots \neq c_s; \forall t, u_1 \in \mathbb{N}; u_1 = \ldots = u_s < t \leq T$, *and either* $p_{g,t}^{(c_1,u_1,\ldots,c_s,u_s)} \geq p_{g,t}^{(c_1,u_1,\ldots,c_r,u_r)}\ \forall g \in \mathcal{G}; g \notin \{c_1, \ldots, c_s\}$ *or* $p_{g,t}^{(c_1,u_1,\ldots,c_s,u_s)} \leq p_{g,t}^{(c_1,u_1,\ldots,c_r,u_r)}\ \forall g \in \mathcal{G}; g \notin \{c_1, \ldots, c_s\}\quad \forall r, s \in \mathbb{N}; r \leq s \leq k; \forall c_1, \ldots, c_r \in \mathcal{C}; c_1 \neq \ldots \neq c_s; \forall t, u_1, \ldots, u_s \in \mathbb{N}; u_1 \leq \ldots \leq u_r \leq u_{r+1} = \ldots = u_s < t \leq T$, *then* AC-LASCOPF-r$_k$ *is feasible. Furthermore, if* AC-LASCOPF$_k$ *has such a solution that is optimal, then the optimal objective value is equal for both* AC-LASCOPF$_k$ *and* AC-LASCOPF-r$_k$.

Proof. The proof follows that of Theorem 4. □

Theorem 6. *If Conjecture 1 holds, then if* AC-LASCOPF-r$_k$ *is feasible and has a solution for which either* $p_{g,t}^{(c_1,\ldots,c_s^1)} \geq p_{g,t}^{(c_1,\ldots,c_s^2)}\ \forall g \in \mathcal{G}; g \notin \{c_1, \ldots, c_{s-1}, c_s^1, c_s^2\}$ *or* $p_{g,t}^{(c_1,\ldots,c_s^1)} \leq p_{g,t}^{(c_1,\ldots,c_s^2)}\ \forall g \in \mathcal{G}; g \notin \{c_1, \ldots, c_{s-1}, c_s^1, c_s^2\}\quad \forall s \in \mathbb{N}; s \leq k; \forall c_1, \ldots, c_{s-1}, c_s^1, c_s^2 \in \mathcal{C}; c_1 \neq \ldots \neq c_{s-1} \neq c_s^1 \neq c_s^2; \forall t \in \mathbb{N}; t \leq T$, *then* AC-LASCOPF-ru$_k$ *is feasible. Furthermore, if* AC-LASCOPF-r$_k$ *has such a solution that is optimal, then the optimal objective value is equal for both* AC-LASCOPF-r$_k$ *and* AC-LASCOPF-ru$_k$.

Proof. The proof of Theorem 4 can be generalised to prove this theorem. □

6. Numerical results

In this section, we present numerical results that demonstrate the computational advantage of the proposed reduced formulations. To do so, we consider systems of various size: the IEEE 14 bus, 30 bus and 300 bus systems (Christie, 1999) and the 1354 bus part of the European power system (Fliscounakis, Panciatici, Capitanescu, & Wehenkel, 2013; Josz, Fliscounakis, Maeght, & Panciatici, 2016). For every system, we chose $(p_{g,0}^{\{0\}} | g \in \mathcal{G})$ to be the active power generation provided in the case data. In addition, in order to impose ramping constraints, we set $\overline{R}_g = (\overline{P}_g - \underline{P}_g)/2\ \forall g \in \mathcal{G}$ where $\overline{P}_g$ and $\underline{P}_g$ are as provided in the case data. We only consider contingencies in the first two generators, i.e., $\mathcal{C} = \{1, 2\}$, for illustrative purposes. Furthermore, let $(d_n | n \in \mathcal{N})$ be the active power demand provided in the case data. For each system, we consider the following demand scenarios.

1. $\mathrm{Re}(D_{n,t}) = r_{n,t} d_n$ where $r_{n,t}$ is a random number uniformly distributed over [0,1] $\forall n \in \mathcal{N}, \forall t \in \mathbb{N}, t \leq T$, i.e., the demand at every bus varies randomly in time independent of other buses.
2. $\mathrm{Re}(D_{n,t}) = r_t d_n\ \forall n \in \mathcal{N}$ where r_t is a random number uniformly distributed over [0,1] $\forall t \in \mathbb{N}, t \leq T$, i.e., the demands at all buses vary by the same factor randomly in time. We do this because, in practice, demands would vary with factors that are almost equal to each other. This correlated variation of demand over time would mean that the ramping constraints between most adjacent intervals in the system are likely binding.
3. $\mathrm{Re}(D_{n,t}) = d_n/1.5$ if t is odd and $\mathrm{Re}(D_{n,t}) = d_n$ if t is even $\forall c \in \mathcal{C}$. We do this in order to deterministically simulate large changes in demand causing the ramping constraints in the system to be binding.

All other parameters were as provided in the case data. Our simulations were performed on an Intel(R) Core(TM) i7-6700 processor with 32 gigabyte RAM using MATPOWER (Zimmerman, Murillo-Sanchez, & Thomas, 2011) version 7.1[8] with the native MATLAB 2021a interior point solver, MIPS.

[8] R. D. Zimmerman, C. E. Murillo-Sanchez (2020). MATPOWER (Version 7.1) [Software]. Available: https://matpower.org

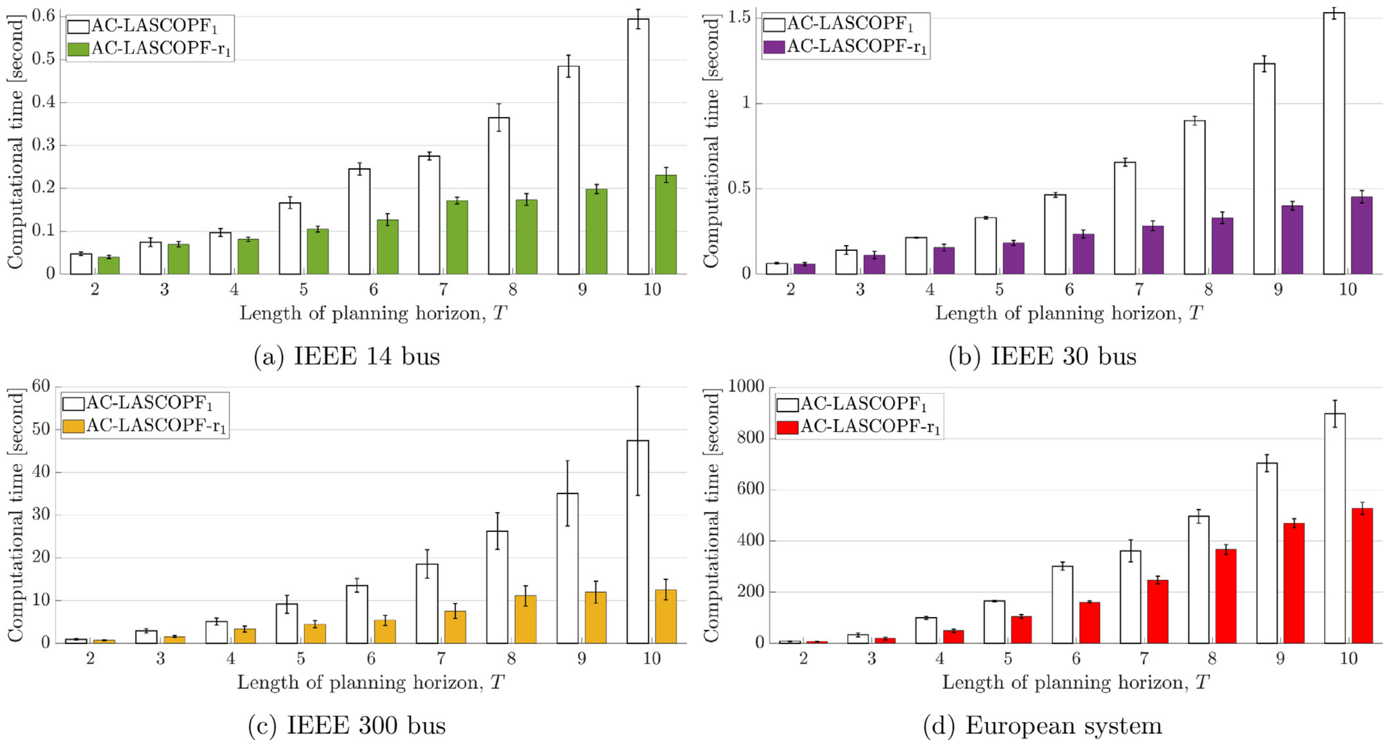

Fig. 3. Computational times for AC-LASCOPF$_1$ and AC-LASCOPF-r$_1$ for demand scenario 1.

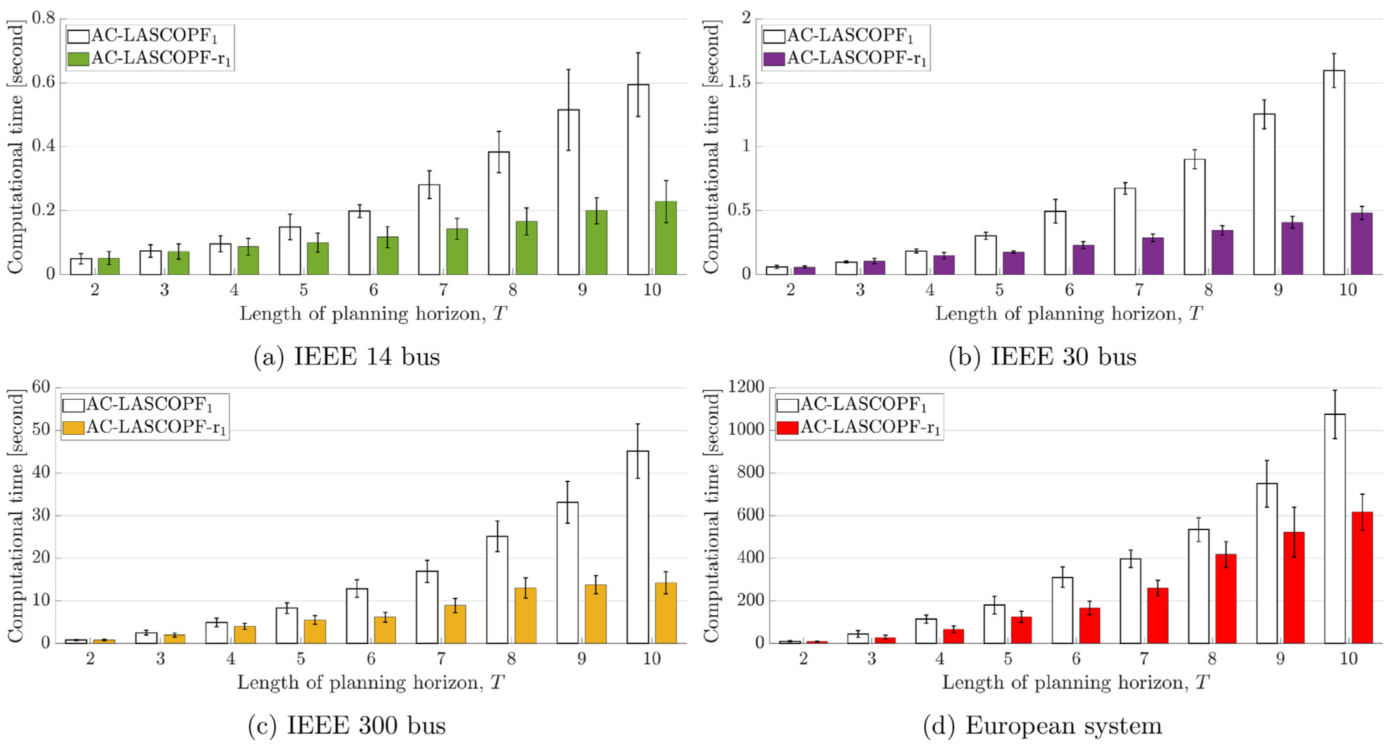

Fig. 4. Computational times for AC-LASCOPF$_1$ and AC-LASCOPF-r$_1$ for demand scenario 2.

First, for the $N-1$ contingency criterion we compare the comprehensive AC-LASCOPF$_1$ to the reduced AC-LASCOPF-r$_1$ for all the cases and scenarios. Figs. 3, 4 and 5 show, for demand scenarios 1, 2 and 3, respectively, the computational time for AC-LASCOPF$_1$ and AC-LASCOPF-r$_1$ for different lengths of the planning horizon, T. For scenarios 1 and 2, we only considered the computational time of the instances that converged, i.e., had a feasible solution. The results reflect that for AC-LASCOPF-r$_1$ the number of dispatches is linearly dependent on T, as opposed to quadratic for AC-LASCOPF$_1$ and thus, AC-LASCOPF-r$_1$ is computationally more efficient, with an increasing advantage as the length of the planning horizon increases. Also, note that for $T=2$, LASCOPF$_1$ and LASCOPF-r$_1$ are

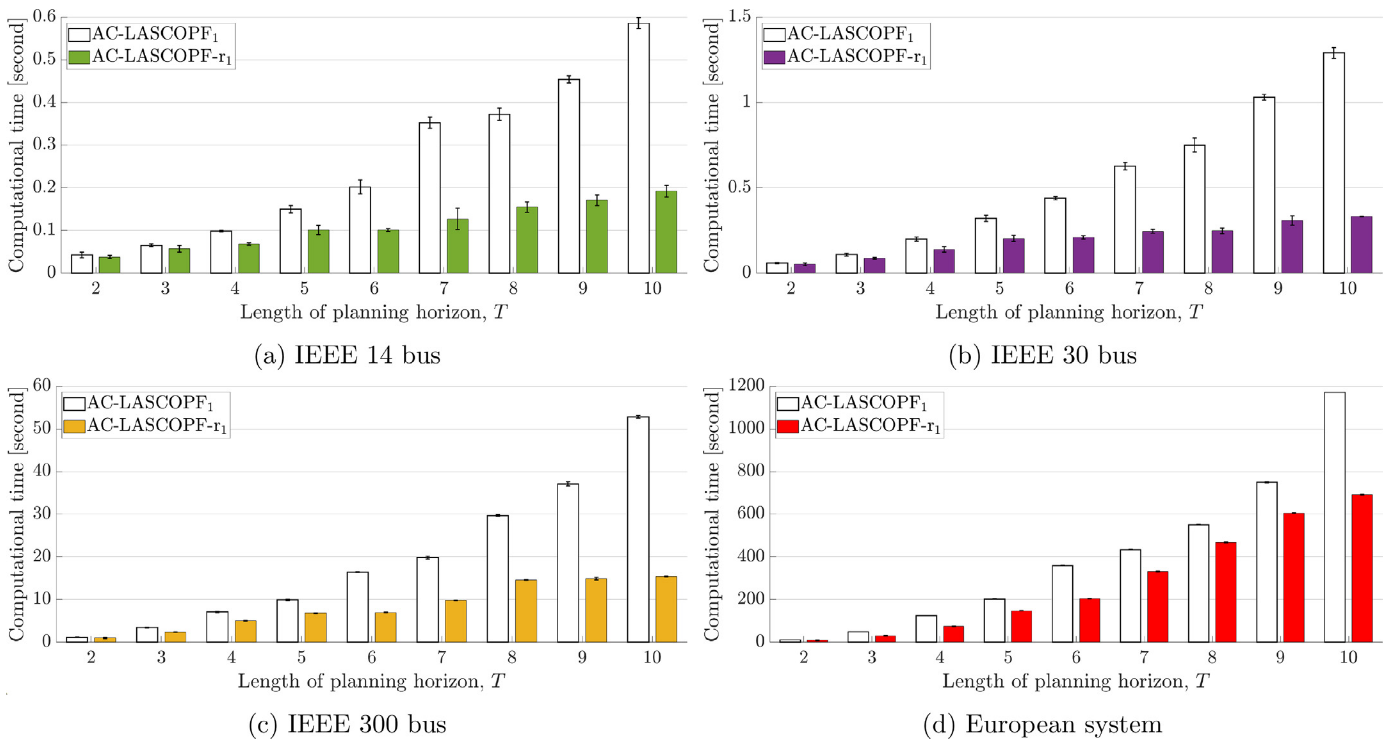

Fig. 5. Computational times for AC-LASCOPF$_1$ and AC-LASCOPF-r$_1$ for demand scenario 3.

Table 1
Number of decision variables and constraints for $T = 10$.

		Base-case	Contingency scenario			
		All	AC-LASCOPF$_1$	AC-LASCOPF-r$_1$	AC-LASCOPF-r$_2$	AC-LASCOPF-ru$_2$
Variables	IEEE 14 bus	380	3420	684	1368	1026
	IEEE 30 bus	720	6480	1296	2592	1944
	IEEE 300 bus	7380	66420	13284	26568	19926
	European system	32280	290520	58104	116208	87156
Inequality constraints	IEEE 14 bus	1380	5220	1204	2568	1966
	IEEE 30 bus	2600	8640	1920	4032	3072
	IEEE 300 bus	26580	91260	20460	43128	32898
	European system	122320	384120	85144	178608	136036
Equality constraints	IEEE 14 bus	280	2700	540	1080	810
	IEEE 30 bus	600	5580	1116	2232	1674
	IEEE 300 bus	6000	54180	10836	21672	16254
	European system	27080	243900	48780	97560	73170

identical since u can only take on one value such that $u \in \mathbb{N}; u < 2$. Therefore, as expected, the confidence intervals for the computational time of AC-LASCOPF$_1$ and AC-LASCOPF-r$_1$ intersect for $T = 2$.

Second, we consider the $N-2$ contingency criterion since it is the most tractable example of the $N-k$ contingency criterion for which we can compare the computational time of AC-LASCOPF-r$_2$ with ordered contingencies to AC-LASCOPF-ru$_2$ with unordered contingencies. The number of contingency scenario dispatches for AC-LASCOPF-r$_2$ depends on $\sum_{k'=1}^{k} {}^{|\mathcal{C}|}P_{k'} \sim 4$ given $|\mathcal{C}| = k = 2$. On the contrary, for AC-LASCOPF-ru$_2$ the number of contingency scenario dispatches depends on $\sum_{k'=1}^{k} {}^{|\mathcal{C}|}C_{k'} \sim 3$. Figs. 6, 7 and 8 show, for demand scenarios 1, 2 and 3, respectively, the computational time for AC-LASCOPF-r$_2$ and AC-LASCOPF-ru$_2$ for different lengths of the planning horizon, T. The results confirm that AC-LASCOPF-ru$_2$ is computationally more efficient. Also, we note that for all the cases above the pairs AC-LASCOPF$_1$ and AC-LASCOPF-r$_1$, and AC-LASCOPF-r$_2$ and AC-LASCOPF-ru$_2$ resulted in the same optimal objective value.

In Table 1, we tabulate the number of decision variables and constraints for the IEEE 14 bus, 30 bus and 300 bus systems, and the 1354 bus part of the European power system, under the comprehensive and the reduced formulations for the $N-1$ and $N-2$ contingency criteria when $T = 10$. Note that the bus voltages are complex and thus they count as two variables each. Also, recall that for a given system, the base-case is identical for all problem formulations and therefore, the number of decision variables in $(v_{n,t}^{\{0\}}, p_{g,t}^{\{0\}}, q_{g,t}^{\{0\}} | n \in \mathcal{N}; g \in \mathcal{G}, t \in \mathbb{N}, t \leq T)$ and number of constraints in (1b) to (1d) are the same. In Fig. 9, we plot the computational time for every scenario in each of the 16 cases in Table 1 against the number of decision variables. We can see that the computational time for each problem formulation increases with the number of decision variables and the computational time for AC-LASCOPF$_1$ for a given number of decision variables is lower compared to the other formulations. We attribute this to the fact that it is the decision variables in the base-case that are optimised, while for the contingency scenario variables only a feasi-

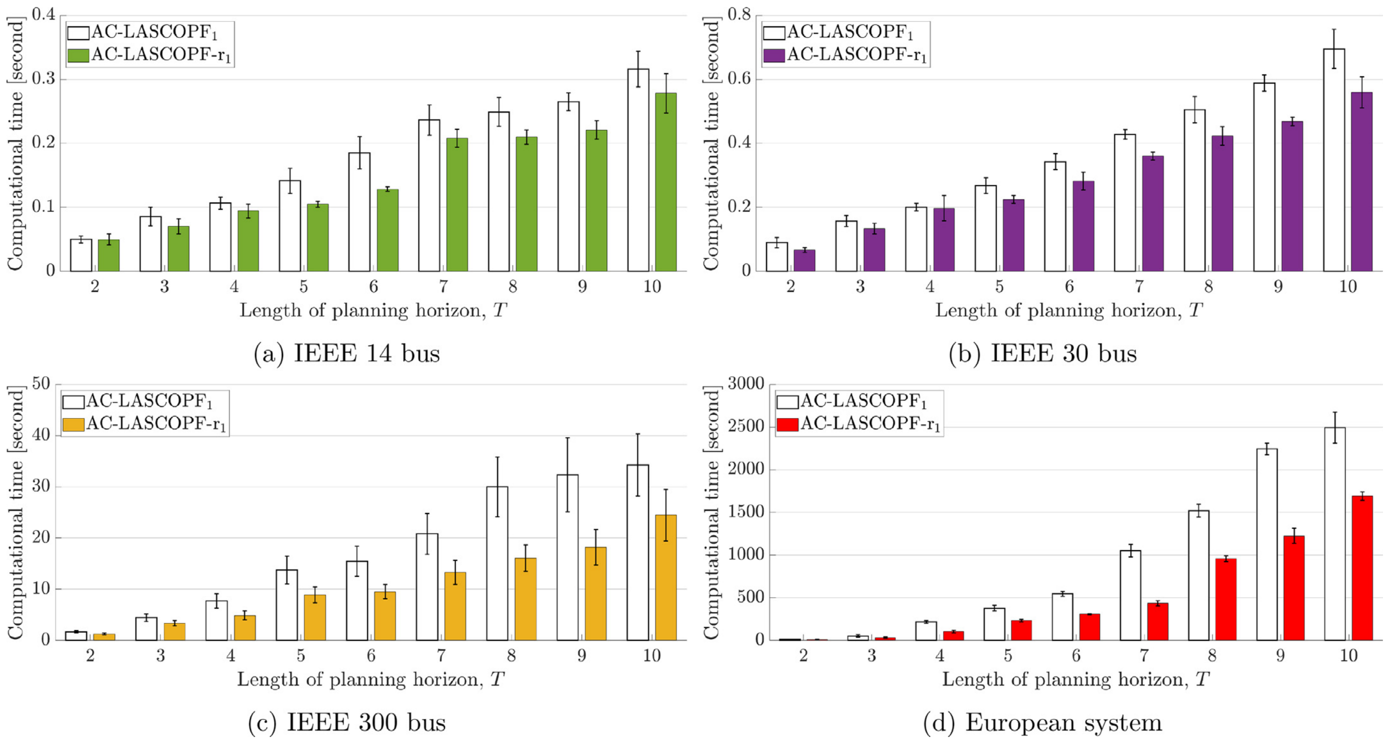

Fig. 6. Computational times for AC-LASCOPF-r_2 and AC-LASCOPF-ru_2 for demand scenario 1.

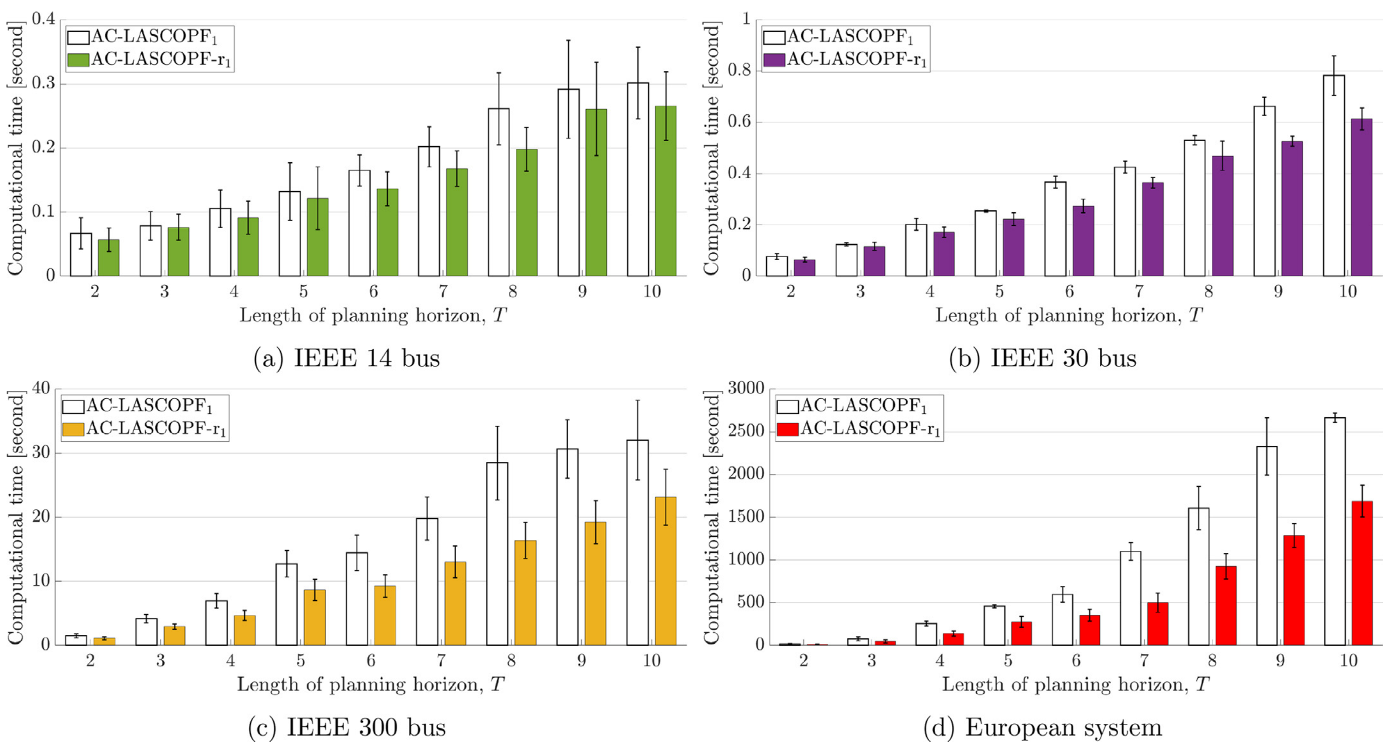

Fig. 7. Computational times for AC-LASCOPF-r_2 and AC-LASCOPF-ru_2 for demand scenario 2.

ble value is sought and therefore the computation time per base-case variable is expected to be longer. Among the problem formulations it is AC-LASCOPF$_1$ that has the lowest proportion of base-case decision variables ($\mathcal{O}(T)$) compared to contingency scenario decision variables ($\mathcal{O}(T^2)$), which explains the observation. Note that this does not indicate that AC-LASCOPF$_1$ is faster to solve than AC-LASCOPF-r_1 since, for a given case, the number of decision variables and hence the computational time is significantly greater in the former as compared to the latter.

Finally, note that in all the cases of the $N-1$ contingency criterion considered above, we obtained the same optimal objective value while solving AC-LASCOPF$_1$ and AC-LASCOPF-r_1. Similarly, for all the cases of the $N-2$ contingency criterion we considered, AC-LASCOPF-r_2 and AC-LASCOPF-ru_2 resulted in the same

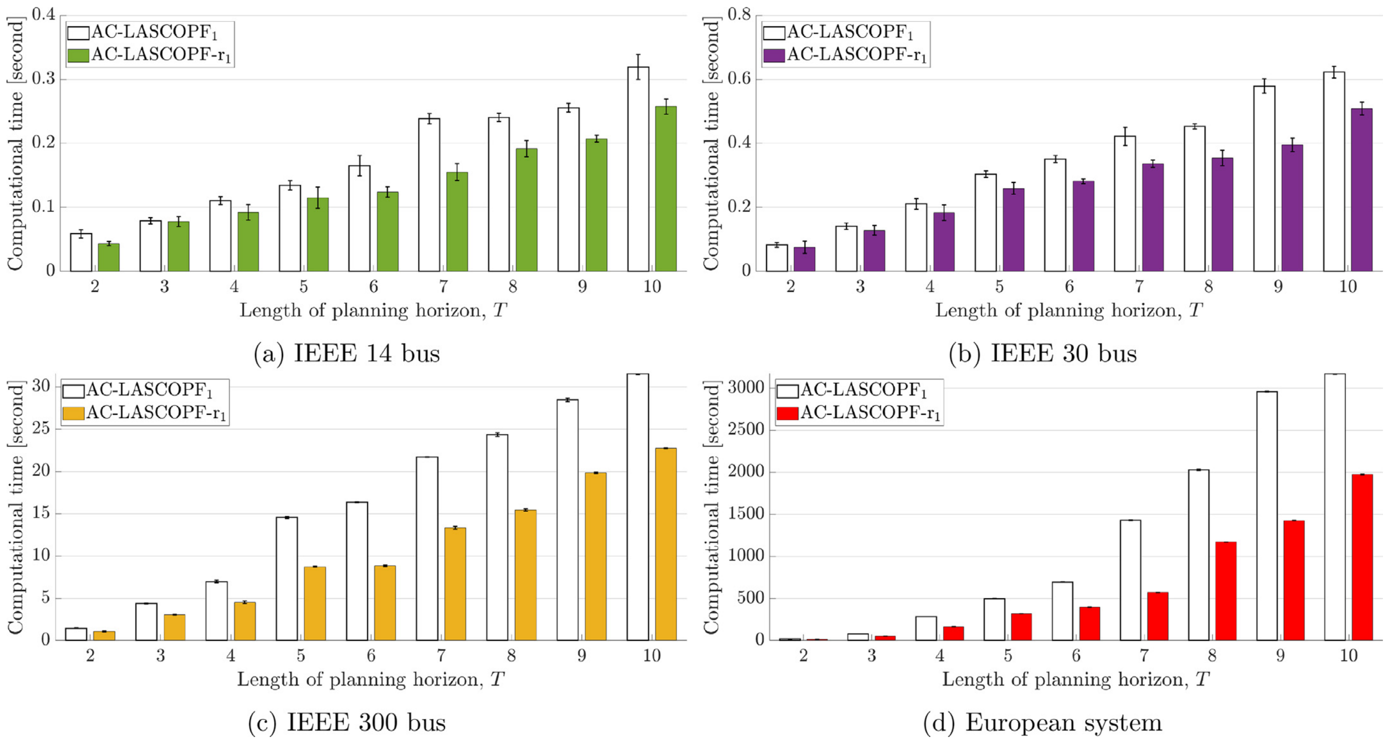

Fig. 8. Computational times for AC-LASCOPF-r$_2$ and AC-LASCOPF-ru$_2$ for demand scenario 3.

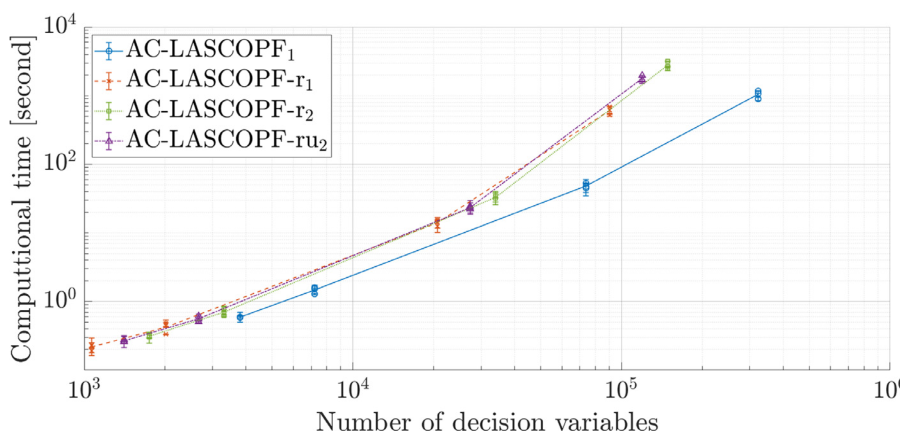

Fig. 9. Scatter plot of computational times vs. number of decision variables for $T = 10$.

optimal objective value. These are in accordance with the claims of Theorems 4 and 6, respectively.

7. Conclusion

We considered LASCOPF under the $N-1$ contingency criterion. We presented a generalised, comprehensive LASCOPF$_1$ formulation and observed that the dependence of the decision variables on u resulted in a $\mathcal{O}(T^2)$ dependence of the number of decision variables. To make the problem scalable, we proposed the reduced LASCOPF-r$_1$ formulation, which is independent of u and hence has only an $\mathcal{O}(T)$ dependence. Similarly, for the $N-k$ contingency criterion, we presented LASCOPF$_k$ with its $\mathcal{O}(T^{k+1})$ dependence and proposed the corresponding reduced LASCOPF-r$_k$ with its $\mathcal{O}(T)$ dependence. Furthermore, we observed that in LASCOPF-r$_k$, the contingency scenario dispatches depend upon the tuple $(c_1, \dots, c_s)$ and hence the number of contingency scenario dispatches varies with $\sum_{k'=1}^{k} {}^{|\mathcal{C}|}P_{k'} \times T$. We proposed LASCOPF-ru$_k$ in which the contingency scenario dispatches only depend upon the set $\{c_1, \dots, c_s\}$ and hence the dependence reduces to $\sum_{k'=1}^{k} {}^{|\mathcal{C}|}C_{k'} \times T$. Then, we proposed DC-LASCOPF$_1$ and AC-LASCOPF$_1$ under generator contingencies. We proved that, barring borderline cases, solving AC-LASCOPF$_1$ and AC-LASCOPF-r$_1$ are equivalent. Similarly, solving AC-LASCOPF$_k$, AC-LASCOPF-r$_k$ and AC-LASCOPF-ru$_k$ are equivalent. Finally, we presented numerical results on the IEEE 14 bus, IEEE 30 bus and IEEE 300 bus test cases, and the 1354 bus part of the European power system to demonstrate the computational advantage of the reduced formulations under the $N-1$ and $N-2$ contingency criteria.

An interesting supplement to this work could be to empirically validate Conjecture 1 for AC-LASCOPF$_1$ on typical test systems. One may also find exact conditions on the problem parameters given which the reduced formulations would always equivalent to the comprehensive formulations.

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Declaration of Competing Interest

None.

Appendix

Appendix A. Notation

Sets:

$\mathbb{N}_0$	non-negative integers
$\mathbb{N}$	positive integers
$\mathbb{R}$	real numbers
$\mathbb{R}_{\geq 0}$	non-negative real numbers
$\mathbb{C}$	complex numbers

Indices:

t	$t \in \mathbb{N}_0, t \leq T$	dispatch interval
u	$u \in \mathbb{N}, u < t$	dispatch interval in which outage corresponding to contingency would take place
n	$n \in \mathcal{N}$	bus
g	$g \in \mathcal{G}$	generator
c	$c \in \mathcal{C}$	contingency
s	$s \in \mathbb{N}, s \leq k$	number of contingencies of considered simultaneously
u_i	$u_i \in \mathbb{N}, u_1 \leq \ldots \leq u_s < t$	dispatch interval in which outage corresponding to ith contingency would take place
c_i	$c_i \in \mathcal{C}, c_1 \neq \ldots \neq c_s$	ith contingency

Parameters:

$\mathcal{N}$		set of buses
$\mathcal{G}$		set of generators
$\mathcal{C}$		set of contingencies
T	$T \in \mathbb{N}$	length of planning horizon
k	$k \in \mathbb{N}, k > 1$	number of contingencies considered simultaneously
$\overline{R}_g$	$\overline{R}_g, \overline{R}_g \in \mathbb{R}_{\geq 0}$	ramping limits for generator g
$\underline{S}_g, \overline{S}_g$	$\underline{S}_g, \overline{S}_g \in \mathbb{R}_{\geq 0}$	lower and upper contingency reserve limits for generator g

Scenarios:

$\{0\}$	base-case
$\{c\}, (c)$	contingency scenario for contingency c
(c, u)	contingency scenario for contingency c in interval u
$\{c_1, \ldots, c_s\}$	contingency scenario for unordered contingencies $c_1, \ldots, c_s$
$(c_1, u_1 \ldots, c_s, u_s)$	contingency scenario for contingencies $c_1, \ldots, c_s$ in intervals $u_1, \ldots, u_s$ respectively
$(c_1 \ldots, c_s)$	contingency scenario for ordered contingencies $c_1, \ldots, c_s$
$\{0'\}$	base-case after recovery

Decision variables:

$v_{n,t}^{<>}$	$v_{n,t}^{<>} \in \mathbb{C}$	voltage at bus n in interval t
$p_{g,t}^{<>}$	$p_{g,t}^{<>} \in \mathbb{R}_{\geq 0}$	active power generation by generator g in interval t
$q_{g,t}^{<>}$	$q_{g,t}^{<>} \in \mathbb{R}$	reactive power generation by generator g in interval t
$\mathcal{S}$		set of decision variables which varies with formulation

Generalised functions:

$f(v_{n,t}^{\{0\}}, p_{g,t}^{\{0\}}, q_{g,t}^{\{0\}} \mid n \in \mathcal{N}; g \in \mathcal{G}; t \in \mathbb{N}; t \leq T)$	objective function
$h_t^{<>}(v_{n,t}^{<>}, p_{g,t}^{<>}, q_{g,t}^{<>} \mid n \in \mathcal{N}; g \in \mathcal{G})$	function representing inequality constraints
$\tilde{h}_t^{<>}(v_{n,t}^{<>}, p_{g,t}^{<>}, q_{g,t}^{<>} \mid n \in \mathcal{N}; g \in \mathcal{G})$	function representing equality constraints

The decision variables and generalised functions above are defined for various scenarios, e.g. the base-case $\{0\}$ and the contingency scenario (c) for contingency c. The scenario to which the entity pertains is specified in the superscript. For brevity, we have defined these entities only once and used $<>$ as a placeholder for the scenario.

Notation for LASCOPF with DC power flow:

$\mathcal{L}$		set of transmission lines
l	$l \in \mathcal{L}$	transmission line
$C_{g,t}$	$C_{g,t} : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$	cost of active power generation by generator g in interval t
$\tilde{D}_{n,t}$	$\tilde{D}_{n,t} \in \mathbb{R}_{\geq 0}$	active power demand at bus n in interval t
$\underline{P}_g, \overline{P}_g$	$\underline{P}_g, \overline{P}_g \in \mathbb{R}_{\geq 0}, \underline{P}_g \leq \overline{P}_g$	active power generation limits for generator g
A_{ng}	$A_{ng} \in \{0, 1\}$	$A_{ng} = 1$ if generator g is at n, 0 otherwise
H_{ln}	$H_{ln} \in \mathbb{R}$	power transfer distribution factor for line l and bus n
$\overline{K}_l$	$\overline{K}_l \in \mathbb{R}_{\geq 0}$	capacity for line l

Notation for LASCOPF with AC power flow:

$\mathcal{L}$		set of transmission lines
l	$l \in \mathcal{L}$	transmission line
$C_{g,t}$	$C_{g,t} : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$	cost of active power generation by generator g in interval t
$D_{n,t}$	$D_{n,t} \in \mathbb{C}, \mathrm{Re}(D_{n,t}) \in \mathbb{R}_{\geq 0}$	power demand at bus n in interval t where $\mathrm{Re}(D_{n,t})$ is the active and $\mathrm{Im}(D_{n,t})$ is the reactive power demand
A_{ng}	$A_{ng} \in \{0, 1\}$	$A_{ng} = 1$ if generator g is at n, 0 otherwise
$Y_{nn'}$	$Y_{nn'} \in \mathbb{C}$	bus admittance factor for buses n and n'
$\underline{P}_g, \overline{P}_g$	$\underline{P}_g, \overline{P}_g \in \mathbb{R}_{\geq 0}, \underline{P}_g \leq \overline{P}_g$	active power generation limits for generator g
$\overline{Q}_g$	$\overline{Q}_g \in \mathbb{R}_{\geq 0}, \underline{P}_g \leq \overline{P}_g$	reactive power generation limit for generator g
$\underline{V}_n, \overline{V}_n$	$\underline{V}_n, \overline{V}_n \in \mathbb{R}_{\geq 0}, \underline{V}_n \leq \overline{V}_n$	voltage magnitude limits for bus n
Y_{ln}	$Y_{ln} \in \mathbb{C}$	bus branch admittance factor for line l and bus n
T_{ln}, F_{ln}	$T_{ln}, F_{ln} \in \{0, 1\}$	$(T_{ln}, F_{ln}) = (1, 0)$ if line l ends at bus n, (0,1) if line l originates atbus n, (0,0) otherwise
$\overline{K}_l$	$\overline{K}_l \in \mathbb{R}_{\geq 0}$	capacity for line l

Appendix B. Number of constraints in LASCOPF$_1$

In LASCOPF$_1$, the number of base-case dispatches $(v_{n,t}^{\{0\}}, p_{g,t}^{\{0\}}, q_{g,t}^{\{0\}} \mid n \in \mathcal{N}; g \in \mathcal{G})$ is T. Since constraint sets (1b) and (1c) are defined for each base-case dispatch, there are T such constraint sets each. Note that each set of constraints may consist of multiple constraints. In addition, there are $|\mathcal{G}| \times T$ ramping constraints in (1d), defined for every generator $g \in \mathcal{G}$ in every interval $t \in \mathbb{N}, t \leq T$.

Besides the base-case dispatch, we also have $|\mathcal{C}| \times T(T-1)/2$ contingency scenario dispatches $(v_{n,t}^{\{c,u\}}, p_{g,t}^{\{c,u\}}, q_{g,t}^{\{c,u\}} \mid n \in \mathcal{N}; g \in \mathcal{G})$. Since constraint sets (1e) and (1f) are defined separately for each contingency scenario dispatch, there will be $|\mathcal{C}| \times T(T-1)/2$ such constraint sets each. Ramping constraints in (1g) are defined for every healthy generator (in the case when one generator fails under a generator contingency, as is the case in the example of contingencies we describe in Sections 4 and 5). Thus, for every contingency $c \in \mathcal{C}$ in every interval $t \in \mathbb{N}, u+1 < t \leq T$, where $u \in \mathbb{N}, u < T$, there will be

$$(|\mathcal{G}| - 1) \times |\mathcal{C}| \times \sum_{u \in \mathbb{N}, u < T} (T - u - 1) = (|\mathcal{G}| - 1) \times |\mathcal{C}| \times (T-1)(T-2)/2 \quad (12)$$

constraints of type (1g). In addition, there will be $(|\mathcal{G}| - 1) \times |\mathcal{C}| \times (T-1)$ constraints of type (1h), since the constraint is defined for healthy generators when $t = u + 1 \; \forall u \in \mathbb{N}, u < T$, i.e., for $2 \leq t \leq T$. Therefore, the total number of constraints follows $\mathcal{O}(T^2)$.

Appendix C. Proof of Lemma 3

Consider a feasible instance of LASCOPF-r$_k$ and a feasible solution $(v_{n,t}^{\{0\}}, p_{g,t}^{\{0\}}, q_{g,t}^{\{0\}}, v_{n,t}^{(c_1,\ldots,c_s)}, p_{g,t}^{(c_1,\ldots,c_s)}, q_{g,t}^{(c_1,\ldots,c_s)} | s \in \mathbb{N}; s \leq k; c_1,\ldots,c_s \in \mathcal{C}; c_1 \neq \ldots \neq c_s; n \in \mathcal{N}; g \in \mathcal{G}; t \in \mathbb{N}; t \leq T)$. Given the base-case dispatch, $(v_{n,t}^{\{0\}}, p_{g,t}^{\{0\}}, q_{g,t}^{\{0\}} | n \in \mathcal{N}; g \in \mathcal{G}; t \in \mathbb{N}; t \leq T)$, observe that (8d) and (8e) together with (7g) only place constraints on $(v_{n,t}^{\{c_1,\ldots,c_s\}}, p_{g,t}^{\{c_1,\ldots,c_s\}}, q_{g,t}^{\{c_1,\ldots,c_s\}} | n \in \mathcal{N}; g \in \mathcal{G})$ $\forall s \in \mathbb{N}; s \leq k; \forall c_1,\ldots,c_s \in \mathcal{C}, c_1, \neq \ldots \neq, c_s$. First, consider interval $t = t'; t' > 1$, number of contingencies $s = 1$ and contingency $c_1 = c_1'$, and let $(p_{g,t'}^{(c_1')} | g \in \mathcal{G})$ be the feasible set of dispatch. Now consider the corresponding LASCOPF-ru$_k$ formulation and the set of dispatch $(p_{g,t'}^{\{c_1'\}} | g \in \mathcal{G})$. Since the tuple (c_1') contains only a single element, from (8f) $(p_{g,t'}^{\{c_1'\}} | g \in \mathcal{G}) = (p_{g,t'}^{(c_1')} | g \in \mathcal{G})$. As a result, $(p_{g,t'}^{\{c_1'\}} | g \in \mathcal{G})$ satisfies (8e). Observe that (8d) does not constrain $(p_{g,t'}^{\{c_1'\}} | g \in \mathcal{G})$.

Let us now consider $s = 2$ and contingencies c_1', c_2', and let $(p_{g,t'}^{(c_1',c_2')} | g \in \mathcal{G})$ and $(p_{g,t'}^{(c_2',c_1')} | g \in \mathcal{G})$ be the feasible sets of dispatch for LASCOPF-r$_k$ for the contingency tuples (c_1', c_2') and (c_1', c_2') respectively such that both sets satisfy (8e) for the same parameters. For LASCOPF-ru$_k$, let $(p_{g,t'}^{\{c_1',c_2'\}} | g \in \mathcal{G})$ be the corresponding set of dispatch which would then also satisfy (8e) for the same parameters allowing us to choose $(p_{g,t'}^{\{c_1',c_2'\}} | g \in \mathcal{G}) = (p_{g,t'}^{(c_1',c_2')} | g \in \mathcal{G}) = (p_{g,t'}^{(c_2',c_1')} | g \in \mathcal{G})$. If $t' = 2$, observe that (8d) does not constrain $(p_{g,2}^{\{c_1'\}} | g \in \mathcal{G})$. Now consider $t' > 2$. The feasible sets $(p_{g,t'}^{(c_1',c_2')} | g \in \mathcal{G})$ and $(p_{g,t'}^{(c_2',c_1')} | g \in \mathcal{G})$ would also satisfy (8d) when $r = 1$ and $r = 2$ for $c_1 = c_1', c_2 = c_2'$ and $c_2 = c_1', c_1 = c_2'$ respectively. From (8f), $(p_{g,t'}^{\{c_1',c_2'\}} | g \in \mathcal{G})$ is constrained by all the four constraints. Let $\mathcal{W}$ be the feasible region defined by (8e) for $c_1 = c_1', c_2 = c_2'$. We can decompose $\mathcal{W} = \prod_{g \in \mathcal{G}} \mathcal{W}_g$, where

$$\mathcal{W}_g = \begin{cases} \left[-\overline{R}_g + p_{g,t'-1}^{\{0\}}, \overline{R}_g + p_{g,t'-1}^{\{0\}}\right] & \text{if } g \notin \{c_1', c_2'\}, \\ \mathbb{R} & \text{otherwise.} \end{cases} \tag{13}$$

Similarly, let $\mathcal{X}$ be the feasible region defined by (8d) for $r = 1$ and $c_1 = c_1', c_2 = c_2'$. We can decompose $\mathcal{X} = \prod_{g \in \mathcal{G}} \mathcal{X}_g$, where

$$\mathcal{X}_g = \begin{cases} \left[-\overline{R}_g + p_{g,t'-1}^{\{c_1'\}}, \overline{R}_g + p_{g,t'-1}^{\{c_1'\}}\right] & \text{if } g \notin \{c_1', c_2'\}, \\ \mathbb{R} & \text{otherwise.} \end{cases} \tag{14}$$

Similarly, let $\mathcal{Y}$ be the feasible region defined by (8d) for $r = 1$ and $c_1 = c_2', c_2 = c_1'$, which can also be decomposed as $\mathcal{Y} = \prod_{g \in \mathcal{G}} \mathcal{Y}_g$, where

$$\mathcal{Y}_g = \begin{cases} \left[-\overline{R}_g + p_{g,t'-1}^{\{c_2'\}}, \overline{R}_g + p_{g,t'-1}^{\{c_2'\}}\right] & \text{if } g \notin \{c_1', c_2'\}, \\ \mathbb{R} & \text{otherwise.} \end{cases} \tag{15}$$

Finally, consider $r = 2$ and observe that our choice of $(p_{g,t'}^{\{c_1',c_2'\}} | g \in \mathcal{G}) = (p_{g,t'}^{(c_1',c_2')} | g \in \mathcal{G}) = (p_{g,t'}^{(c_2',c_1')} | \ g \in \mathcal{G})$ renders the feasible regions for $c_1 = c_1', c_2 = c_2'$ and $c_2 = c_1', c_1 = c_2'$ identical. Let $\mathcal{Z}$ represent this feasible region, which can be decomposed as $\mathcal{Z} = \prod_{g \in \mathcal{G}} \mathcal{Z}_g$, where

$$\mathcal{Z}_g = \begin{cases} \left[-\overline{R}_g + p_{g,t'-1}^{\{c_1',c_2'\}}, \overline{R}_g + p_{g,t'-1}^{\{c_2',c_1'\}}\right] & \text{if } g \notin \{c_1', c_2'\}, \\ \mathbb{R} & \text{otherwise.} \end{cases} \tag{16}$$

In the next step, we show that for generator $g = g'$, we have $\mathcal{W}_{g'} \cap \mathcal{X}_{g'} \cap \mathcal{Y}_{g'} \cap \mathcal{Z}_{g'} = \emptyset$. If $g' \in \{c_1', c_2'\}$, then $\mathcal{W}_{g'} \cap \mathcal{X}_{g'} \cap \mathcal{Y}_{g'} \cap \mathcal{Z}_{g'} = \mathbb{R} \neq \emptyset$. If $g' \notin \{c_1', c_2'\}$, $\mathcal{W}_{g'} \cap \mathcal{X}_{g'} \cap \mathcal{Y}_{g'} \cap \mathcal{Z}_{g'} = [-\overline{R}_{g'} + \max\{p_{g,t'-1}^{\{0\}}, p_{g,t'-1}^{\{c_1'\}}, p_{g,t'-1}^{\{c_2'\}}, p_{g,t'-1}^{\{c_1',c_2'\}}\}, \overline{R}_{g'} + \min\{p_{g,t'-1}^{\{0\}}, p_{g,t'-1}^{\{c_1'\}}, p_{g,t'-1}^{\{c_2'\}}, p_{g,t'-1}^{\{c_1',c_2'\}}\}]$. To show that the intersection above is non-empty, let us first consider (8d) for $g = g'$, which is satisfied by $p_{g,t'-1}^{\{0\}}$. After rearrangement we obtain

$$-\overline{R}_{g'} + p_{g,t'-1}^{\{0\}} \leq p_{g',t'-2}^{\{0\}} \leq \overline{R}_{g'} + p_{g,t'-1}^{\{0\}}. \tag{17}$$

Now consider (8e) for $g = g'$, $s = 1$ and $c_1 = c_1'$, which is satisfied by $p_{g,t'-1}^{\{c_1'\}}$. After rearrangement we obtain

$$-\overline{R}_{g'} + p_{g,t'-1}^{\{c_1'\}} \leq p_{g',t'-2}^{\{0\}} \leq \overline{R}_{g'} + p_{g,t'-1}^{\{c_1'\}}. \tag{18}$$

Now consider (8e) for $g = g'$, $s = 1$ and $c_1 = c_2'$, which is satisfied by $p_{g,t'-1}^{\{c_2'\}}$. After rearrangement we obtain

$$-\overline{R}_{g'} + p_{g,t'-1}^{\{c_2'\}} \leq p_{g',t'-2}^{\{0\}} \leq \overline{R}_{g'} + p_{g,t'-1}^{\{c_2'\}}. \tag{19}$$

Finally, consider (8e) for $g = g'$, $s = 2$ and $c_1 = c_1', c_2 = c_2'$, which is satisfied by $p_{g,t'-1}^{\{c_1',c_2'\}}$. After rearrangement we obtain

$$-\overline{R}_{g'} + p_{g,t'-1}^{\{c_1',c_2'\}} \leq p_{g',t'-2}^{\{0\}} \leq \overline{R}_{g'} + p_{g,t'-1}^{\{c_1',c_2'\}}. \tag{20}$$

Since LASCOPF$_1$ is feasible, we know that $\exists\, p_{g',t'-2}^{\{0\}}$ and $\exists\, p_{g',t'-1}^{\{0\}}$ satisfying the above, and we can combine the above inequalities to obtain

$$\begin{aligned} -\overline{R}_{g'} + \max\left\{p_{g,t'-1}^{\{0\}}, p_{g,t'-1}^{\{c_1'\}}, p_{g,t'-1}^{\{c_2'\}}, p_{g,t'-1}^{\{c_1',c_2'\}}\right\} \\ \leq \overline{R}_{g'} + \min\left\{p_{g,t'-1}^{\{0\}}, p_{g,t'-1}^{\{c_1'\}}, p_{g,t'-1}^{\{c_2'\}}, p_{g,t'-1}^{\{c_1',c_2'\}}\right\}, \end{aligned} \tag{21}$$

which implies $\mathcal{W}_g \cap \mathcal{X}_g \cap \mathcal{Y}_g \cap \mathcal{Z}_g \neq \emptyset$ for $g' \notin \{c_1', c_2'\}$. Therefore, the feasible regions defined by (8d) and (8e) for $(p_{g,t'}^{\{c_1',c_2'\}} | g \in \mathcal{G})$ intersect.

Let us now consider number of contingencies $s = 3$. The above analysis can first be repeated for contingencies pairwise and then can be extended to consider three contingencies at a time. Similarly, we can repeat the analysis for all $c_1, \ldots, c_{s'} \in \mathcal{C}$ for any number $s = s'$ starting with $s' = 4$ up to $s' = k$ in increasing order. Finally, we can repeat the analysis for all intervals $t' \in \mathbb{N}; t \leq T$. This concludes the proof.

Appendix D. Illustrative example for AC-LASCOPF$_1$

In what follows, we provide an example which illustrates the theory developed in this article. Consider the following one-bus $\mathcal{N} = \{1\}$ three-generator $\mathcal{G} = \{1, 2, 3\}$ system with parameters as shown in the following table.

g	$\partial C_{g,t}(x)/\partial x$	$\underline{P}_g$	$\overline{P}_g$	$-\underline{Q}_g = \overline{Q}_g$	$\overline{R}_g$
1	0	0	200	100	30
2	1	0	200	100	20
3	2	0	200	100	20

Here, we neglect the shunt admittance $Y_{11} = 0$, resulting the right hand side of (11b) and (11k) equalling zero. Also, since there are no transmission lines, we can ignore constraints (11f) and (11g). Since the objective function depends only upon the active power generation, we can eliminate consideration of bus voltage $(v_{1,t}^{\{0\}}, v_{1,t}^{(c,u)} | \forall c \in \mathcal{C}; \forall t, u \in \mathbb{N}; u < t \leq T)$ from the formulation. The predicted demand $D_{1,t}$ is as follows.

	$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$
$\mathrm{Re}(D_{1,t})$	0	10	20	30	50	70	100
$\mathrm{Im}(D_{1,t})$	50	50	50	50	50	50	50

Observe that none of the generators has a non-zero minimum generation limit, $\underline{P}_g = 0\ \forall g \in \mathcal{G}$ and therefore, they may each not generate at all. Therefore, the problem is feasible at $t = 0$ and it must be that $p^{\{0\}}_{1,0} = p^{\{0\}}_{2,0} = p^{\{0\}}_{3,0} = 0$.

First, we solve AC-LAOPF without any security constraints (formally equivalent to AC-LASCOPF$_1$ over $\mathcal{C} = \emptyset$, i.e., with an empty set of contingencies for this system given $T = 6$. Then, observe that generator 1 is cheaper than generator 2 which is cheaper than generator 3, $\partial C_{1,t}(x)/\partial x < \partial C_{2,t}(x)/\partial x < \partial C_{3,t}(x)/\partial x\ \forall t \in \{1,\dots,6\}$ and the active and reactive power demands in every interval lie within the individual limits of every generator, $\mathrm{Re}(D_{1,t}) \le \overline{P}_g, \underline{Q}_g \le \mathrm{Re}(D_{1,t}) \le \overline{Q}_g\ \forall t \in \{1,\dots,6\}\ \forall g \in \mathcal{G}$. Also, the change in active power demand between every pair of adjacent intervals lies within the ramping limits of every generator, $-\overline{R}_g \le \mathrm{Re}(D_{1,t}) - \mathrm{Re}(D_{1,t-1}) \le \overline{R}_g\ \forall t \in \{1,\dots,6\}\ \forall g \in \mathcal{G}$. Therefore, generator 1 can serve the entire demand as follows.

	$p^{\{0\}}_{g,t}$						$q^{\{0\}}_{g,t}$
	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t \in \{1,\dots,6\}$
$g=1$	10	20	30	50	70	100	50
$g=2$	0	0	0	0	0	0	0
$g=3$	0	0	0	0	0	0	0

Now, we will contrast the results obtained above to those when we consider contingencies. Consider contingencies in all generators such that the generators may shutdown. In what follows, we solve AC-LASCOPF$_1$ for $\mathcal{C} = \{1,2,3\}$ for a planning horizon of $T = 5$. Here, in addition to the constraints already considered, we have to ensure that in the interval following an outage in any generator, the healthy generators can change their generation within their ramping limits to satisfy the demand. The generation of the healthy generators should have a net increase by the amount of generation of the contingent generator before the outage and change by the change in demand, i.e.,

$$-\overline{R}_2 \quad -\overline{R}_3 \le p^{\{0\}}_{1,t-1} + \mathrm{Re}\,(D_{1,t}) - \mathrm{Re}\,(D_{1,t-1}) \le \overline{R}_2 + \overline{R}_3\ \forall t \in \{2,\dots,5\} \text{ if generator 1 is contingent,} \tag{22}$$

$$-\overline{R}_1 \quad -\overline{R}_3 \le p^{\{0\}}_{2,t-1} + \mathrm{Re}\,(D_{1,t}) - \mathrm{Re}\,(D_{1,t-1}) \le \overline{R}_1 + \overline{R}_3\ \forall t \in \{2,\dots,5\} \text{ if generator 2 is contingent,} \tag{23}$$

$$-\overline{R}_1 \quad -\overline{R}_2 \le p^{\{0\}}_{3,t-1} + \mathrm{Re}\,(D_{1,t}) - \mathrm{Re}\,(D_{1,t-1}) \le \overline{R}_1 + \overline{R}_2\ \forall t \in \{2,\dots,5\} \text{ if generator 3 is contingent.} \tag{24}$$

Accordingly, the least-cost base-case dispatch is as follows.

	$p^{\{0\}}_{g,t}$						$q^{\{0\}}_{g,t}$	
	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t \in \{1,\dots,5\}$	$t=6$
$g=1$	10	20	20	20	50	-	50	-
$g=2$	0	0	10	30	20	-	0	-
$g=3$	0	0	0	0	0	-	0	-

Observe that, given $\mathrm{Re}(D_{1,t}) - \mathrm{Re}(D_{1,t-1}) = 20\ \forall t \in \{4,5\}$ and $\overline{R}_2 + \overline{R}_3 = 40$, from (22), $p^{\{0\}}_{1,t-1} \le 20\ \forall t \in \{4,5\}$. In the last interval $t = 5$, we do not have to prepare for an outage. Therefore, generator 1 may generate above 20. However, the ramping constraint $p^{\{0\}}_{1,5} - p^{\{0\}}_{1,4} \le \overline{R}_1$ must be obeyed. Naturally, the dispatch costs would be higher in this case since more expensive generators are used.

Observe that if we considered a planning horizon of $T = 6$, the problem would be infeasible. To see this, first observe that $\mathrm{Re}(D_{1,6}) - \mathrm{Re}(D_{1,5}) = 30$ and $\overline{R}_1 + \overline{R}_3 = \overline{R}_1 + \overline{R}_2 = 50$. From (22), (23) and (24), respectively we require that $p^{\{0\}}_{1,5} \le 10$ and $p^{\{0\}}_{2,5}, p^{\{0\}}_{3,5} \le 20$. There is no possible generation such that $\sum_{g\in\mathcal{G}} p^{\{0\}}_{g,5} = 70$.

In what follows, we illustrate the notion of the $N-2$ contingency criterion by solving AC-LASCOPF$_1$ for $\mathcal{C} = \{1,2,3\}$ for a planning horizon of $T = 3$. In addition to the contingency states considered before, we must consider the contingency state where 2 of the generators have faced an outage, $\{1,2\}$, $\{1,3\}$ or $\{2,3\}$. Assuming simultaneous outages, the healthy generator should have a net increase in its generation by the generation of the contingent generators before the outage and change in its generation by the change in demand. This change should be within its ramping limits, i.e.,

$$-\overline{R}_3 \le p^{\{0\}}_{1,t-1} + p^{\{0\}}_{2,t-1} + \mathrm{Re}\,(D_{1,t}) - \mathrm{Re}\,(D_{1,t-1}) \le \overline{R}_3\quad \forall t \in \{1,2\} \text{ if both generators 1 and 2 are contingent,} \tag{25}$$

$$-\overline{R}_2 \le p^{\{0\}}_{1,t-1} + p^{\{0\}}_{3,t-1} + \mathrm{Re}\,(D_{1,t}) - \mathrm{Re}\,(D_{1,t-1}) \le \overline{R}_2\quad \forall t \in \{1,3\} \text{ if both generators 1 and 3 are contingent,} \tag{26}$$

$$-\overline{R}_1 \le p^{\{0\}}_{2,t-1} + p^{\{0\}}_{3,t-1} + \mathrm{Re}\,(D_{1,t}) - \mathrm{Re}\,(D_{1,t-1}) \le \overline{R}_1\quad \forall t \in \{2,3\} \text{ if both generators 2 and 3 are contingent.} \tag{27}$$

Accordingly, the least-cost base-case dispatch is as follows.

	$p^{\{0\}}_{g,t}$				$q^{\{0\}}_{g,t}$	
	$t=1$	$t=2$	$t=3$	$t \in \{4,5,6\}$	$t \in \{1,2,3\}$	$t \in \{4,5,6\}$
$g=1$	10	0	30	-	50	-
$g=2$	0	10	0	-	0	-
$g=3$	0	10	0	-	0	-

To see how the dispatch above is obtained, observe that, given $\mathrm{Re}(D_{1,t}) - \mathrm{Re}(D_{1,t-1}) = 10\ \forall t \in \{2,3\}$ and $\overline{R}_2 = \overline{R}_3 = 20$, from (25) and (26) respectively, $(p^{\{0\}}_{1,t-1} + p^{\{0\}}_{2,t-1}), (p^{\{0\}}_{1,t-1} + p^{\{0\}}_{3,t-1}) \le 10\ \forall t \in \{2,3\}$. Also, from (27), $p^{\{0\}}_{2,t-1} + p^{\{0\}}_{3,t-1} \le 20\ \forall t \in \{2,3\}$. In the last interval $t = 3$, we do not have to prepare for an outage. Also, observe that if we considered a planning horizon of $T > 3$, the problem would be infeasible.

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