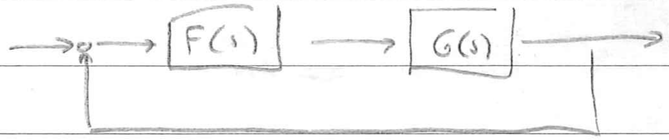
Exercize session #8 21 Nov 2014

Loopshaping - Lead-lag compensator

- Loopshaping is a control design technique based on the Bode diagram
- A common controller used in loopshaping is the lead-lag compensator



Aim: fulfil certain requirements on:

on close-loop $G_c(s)$		Bode diagram of $G_o(s)$
• bandwidth ω_B		• cross over frequency ω_c ($\omega_c \simeq \omega_B$)
• oscillation and overshoot	$\Rightarrow$	• phase margin φ_m bounds
• steady-state error		• static gain $G_o(0)$ ($e(\infty) = \lim \frac{1}{1+G_o(0)}$ for step)

How? designing the lead lag compensator

Lead compensator

Needed to achive a desired crossover frequency and phase margin

$$\boxed{F_{lead} = K \frac{\tau_D s+1}{\beta \tau_D s+1}}$$

- Combination of zero and pole.
  - For $\beta < 1$, zero before pole $\longrightarrow$ increase in phase where we want
  - τ_D defines where, β how much
- K change the modulus of $G_o(j\omega)$, not the phase.
  - increasing K, we increase ω_c

Therefore: K allows to obtained desidered ω_c

$\frac{\tau_D s+1}{\beta \tau_D s+1}$ to increase the phase at desired ω_c, to obtain desired ϕ_m

See fig 5.14 and fig 5.13

- $|F_{lead}|$ multiply to $|G|$
- $\arg(F_{lead})$ sum to $\arg(G)$
- β defines the distance between zero and pole (further they are bigger is φ_{max})

$$\phi_{max} = \arctan \frac{1-\beta}{2\sqrt{\beta}}$$

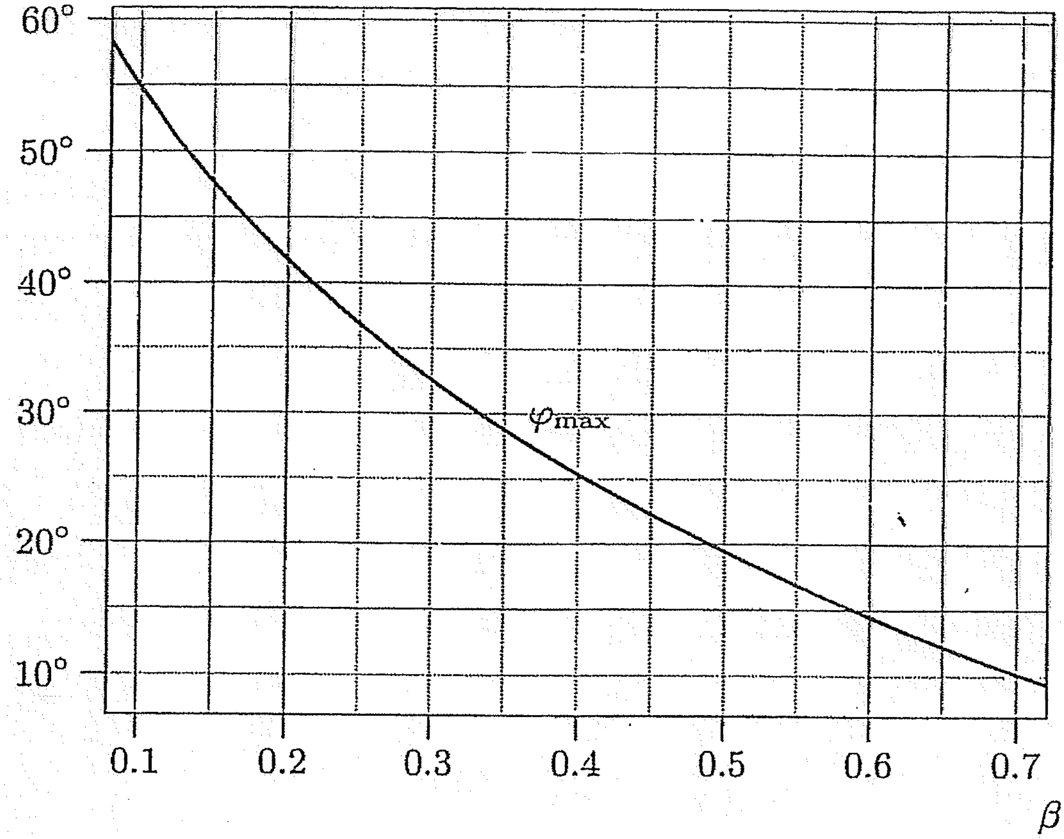
How to choose parameters?

- β chosen s.t. it produces the needed increase of phase $\Delta\phi$ at the desired ω_c to reach desired φ_M
  from fig 5.13 $\boxed{\Delta\phi_d \longrightarrow \beta}$
- τ_D chosen s.t φ_{MAX} increase happens at ω_c

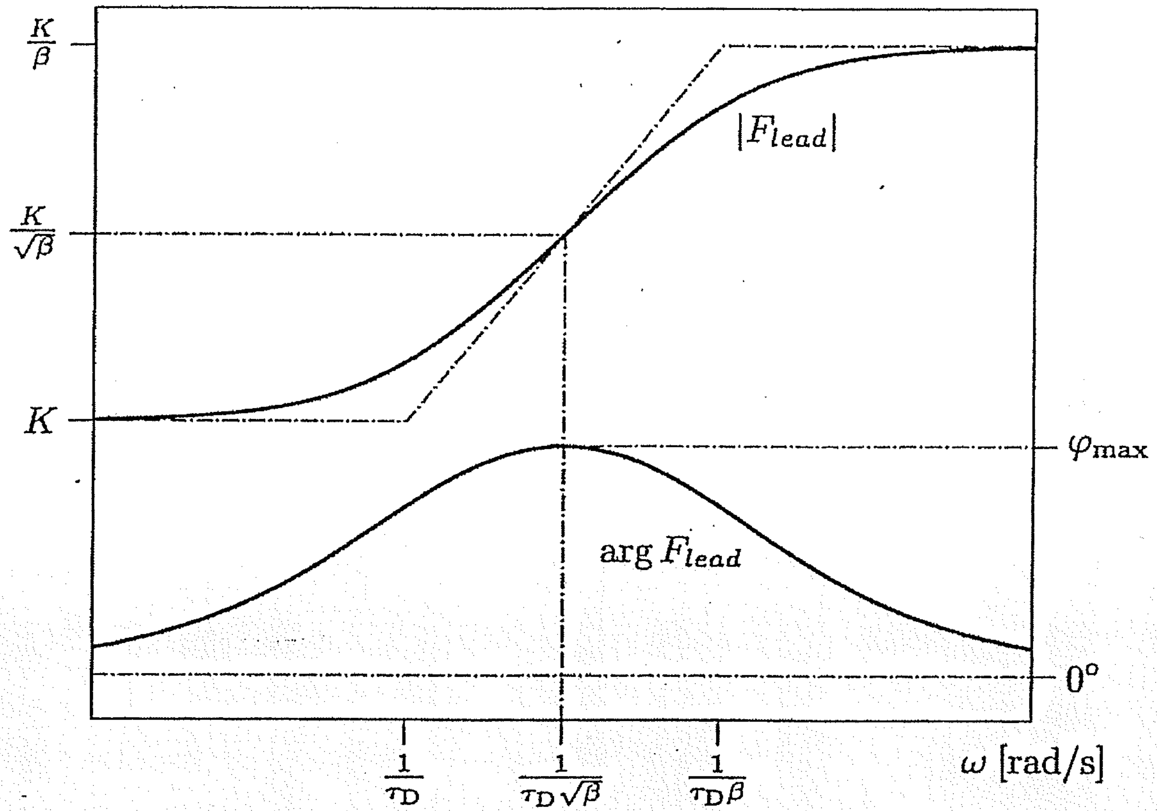
$$\frac{1}{\tau_D\sqrt{\beta}} = \omega_c \implies \tau_D = \frac{1}{\omega_c\sqrt{\beta}}$$

- K chose s.t. $|G_o(j\omega)| = |F_{lead}(j\omega)||G(j\omega)| = 1$ at $\omega = \omega_c$

Note: if the needed increase of phase $\Delta\phi_d$ is greater than $40°$ we design to identical lead compensators each adding half of $\Delta\phi_d$



Figur 5.13: *Maximal fasavancering som funktion av β i intervallet* $0.1 - 0.7$.



Figur 5.14: *PD-regulator (fasavancerande länk). Asymptoterna visas streckade.*

Lag compensator → low frequency

Needed to achive a desired static error by increasing $G_a(0)$ $\left(e(\infty) = \frac{1}{1+G(0)} \right)$ step case

$$\boxed{F_{lag}(s) = \frac{\tau_I s + 1}{\tau_I s + \gamma}}$$

ex step case $\quad e(\infty) = \dfrac{1}{1 + \underbrace{F_{lag}(0)}_{= \frac{1}{\gamma}} \underbrace{F_{lead}(0)}_{= k} \underbrace{G(0)}_{\text{given}}}$

See fig 5.15

$\downarrow \gamma \Rightarrow F_{lag}(0) \uparrow$

reduce the phase (1 decade after phase reduced by $5{,}7^\circ$

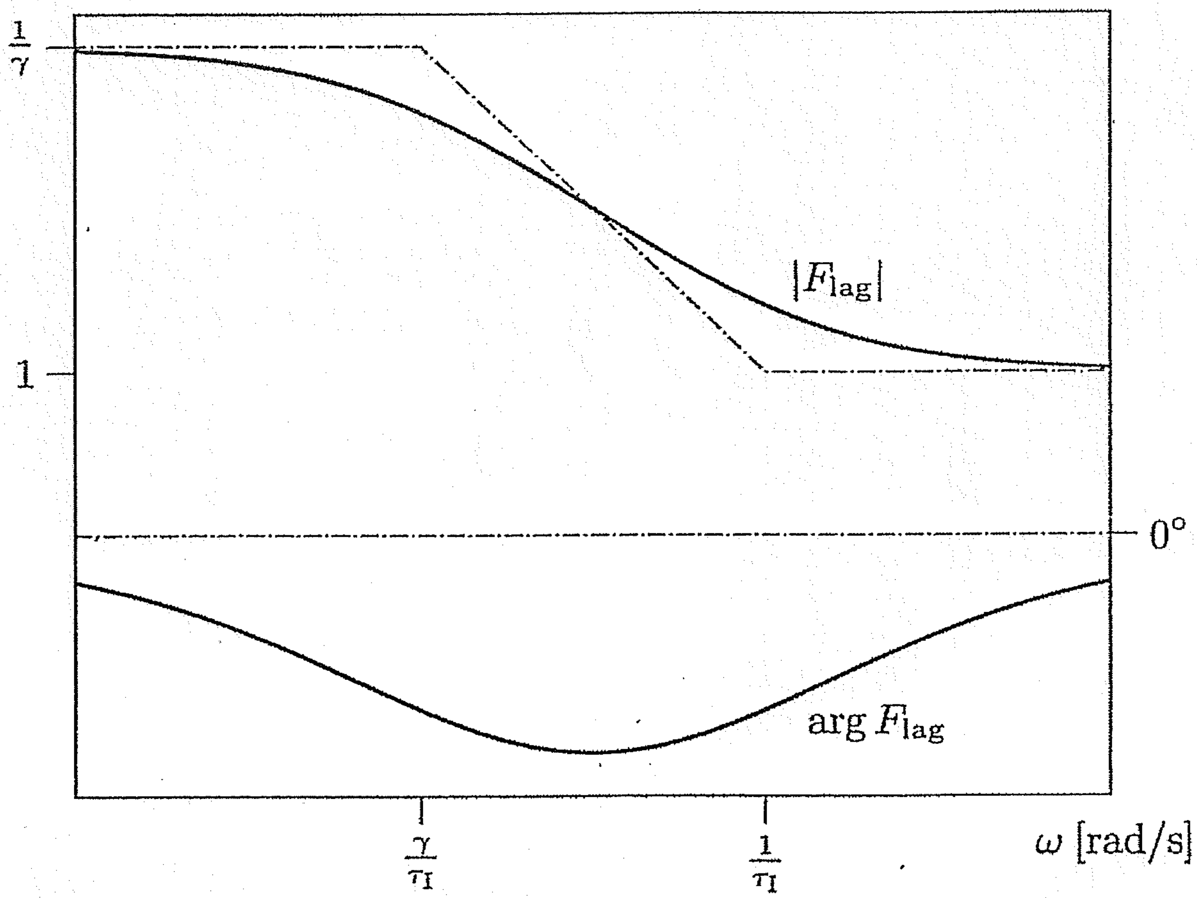
How to choose parameters?

- τ_I is chosen such that the correrponding frequency $\frac{1}{\tau_I}$ is at least 1 decade before the desired ω_c)

  if $\dfrac{1}{\tau_I} = \dfrac{\omega_{c,d}}{10}$ → decrease of phase at $\omega_{c,d}$ of $5{,}7^\circ$

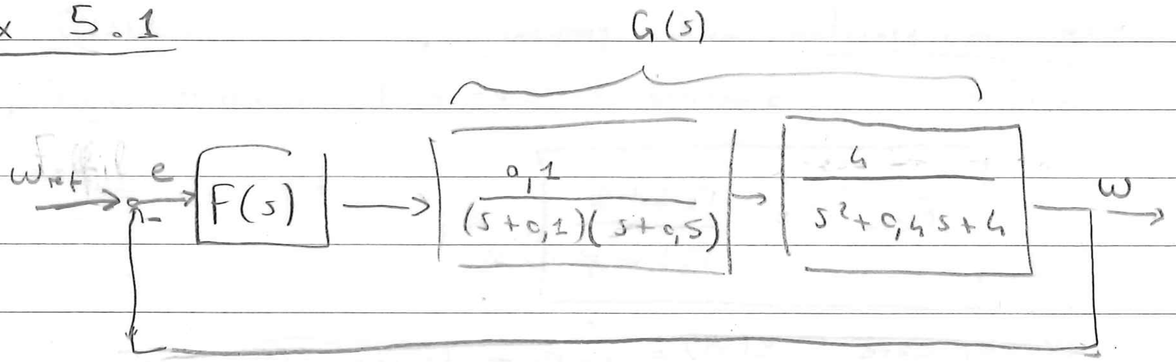
  Rule ⇒ increase $\Delta\phi_d$ of 5.7°

- γ chosen s.t. steady state error respects requirements



Figur 5.15: *Fasretarderande länk. De streckde kurvorna visar det asymptotiska diagrammet.* $\gamma = 0$ *ger PI-regulator.*

Ex 5.1



Fulfil certain requirement:

- twice as fast and same damping compare to $F(s)=1$ case $\Longrightarrow$ $\omega_{c,d} = \omega_{c,1}$; $\varphi_{M,d} = \varphi_{M,1}$
- $\frac{|w_{ref} - w|}{w_{ref}} \leq 5\%$ $\Longrightarrow$ unit step response steady-state error $< 0,05$
- $G_c(s)$ small for high frequency, $G_o(s)$ not too big for low frequency $\Longrightarrow$ $G_c = \frac{G_o}{1+G_o} \simeq G_o$ for high frequency
  $\rightarrow \begin{cases} G_o \text{ small for high frequency} \\ G_o \text{ not too big for small frequency} \end{cases}$

② $\boxed{\text{Good exercise: draw Bode diagram for } F(s)=1}$

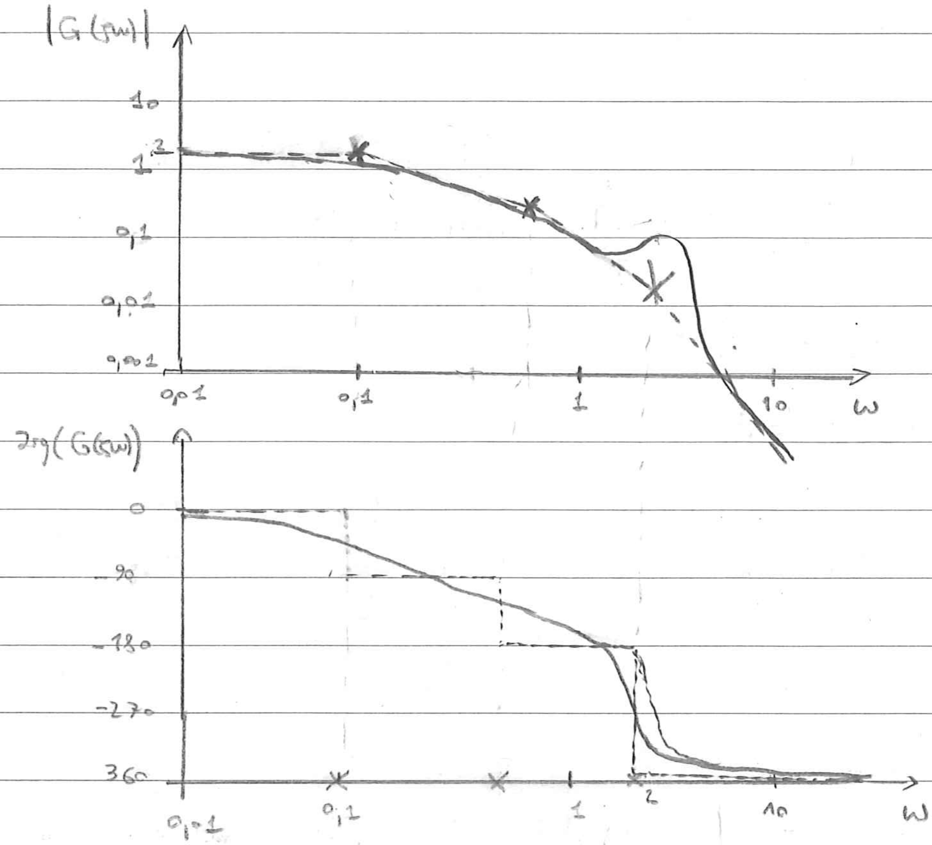
$$G(s) = \frac{2}{\left(1+\frac{s}{0,1}\right)\left(1+\frac{s}{0,5}\right)\left(1+2\cdot 0,1\cdot\frac{1}{2}s+\frac{s^2}{2^2}\right)}$$

- no zeros ; 4 poles: 2 real, 2 complex
- low frequency $\boxed{\omega \to 0}$

  $G(j\omega) \to 2 \Longrightarrow \begin{cases} |G| = 2 \ (\text{slope} = 0) \\ \arg(G) = 0^\circ \end{cases}$
- high frequency $\boxed{\omega \to \infty}$

  $G(j\omega) \to \frac{2\cdot 0,1\cdot 0,5\cdot 2^2}{(j\omega)^4} = \frac{0,4}{(j\omega)^4} \Longrightarrow \begin{cases} |G| \to \frac{0,4}{\omega^4} \ (\text{slope} = -4) \\ \arg(G) \to -360^\circ \end{cases}$

frequency		P $\omega = 0,1$	P $\omega = 0,5$	P_2 (complex) $\omega = 2$ $\xi = 0,1$
modulus slope	0	-1	-2	-4
phase	0°	-90°	-180°	-360°



With test point for the modulus around complex poles and for argument for different frequency we get a more correct shape

$\boxed{\omega_{c,1} = 0,16}$ $\boxed{\varphi_{M,1} = -78° + 180° = 102°}$

(b) **Design lead-lag compensator**

$\omega_{c,d} = 0,32$ rad/s $\quad \varphi_{M,d} = 102°$

(b.1) Lead part $\quad F_{lead} = K \dfrac{\tau_D s + 1}{\beta \tau_D s + 1}$

$\arg\left(G(j\omega_{c,d})\right) = -108°$

we want $\arg\left(G(j\omega_{c,d}) F_{lead}(j\omega_{c,d})\right) = -78°$

$\arg\left(G(j\omega_{c,d})\right) + \arg\left(F_{lead}(j\omega_{c,d})\right) = -78°$

$$\Rightarrow \arg\left(F_{lead}(j\omega_{c,d})\right) = -78^\circ - (-108^\circ) = 30^\circ = \Delta\phi_d$$

note: since we will add a lag-part we need also to compensate for the decrease of phase given by it.

- If the lag-part is put 1 decade before we increase $\Delta\phi_d$ of $5{,}7^\circ$

$$\Delta\phi_d = 35.7$$

Check on fig. 5.13 to have $\Delta\phi_d = 35{,}7^\circ \rightarrow \boxed{\beta = 0{,}275}$

$$\frac{1}{\tau_D\sqrt{\beta}} = \omega_{c,d} \rightarrow \boxed{\tau_D = \frac{1}{\omega_{c,D}\sqrt{\beta}} = 5{,}9 \text{ s}}$$

We want that $|F_{lead}(j\omega)\, G(j\omega)| = 1$ at $\omega = \omega_{c,d} = 0{,}32$ rad/s

$$\left|F_{lead}(j0{,}32)\right|\left|G(j0{,}32)\right| = \frac{K}{\sqrt{\beta}} \cdot 0{,}52 = 1$$

see fig 5.14

$$\Rightarrow \boxed{K = \frac{\sqrt{0{,}275}}{0{,}52} \approx 1}$$

$$\boxed{F_{lead}(s) = \frac{1 + \frac{s}{0{,}16}}{1 + \frac{s}{0{,}61}}}$$

(b.2) Lag part

$$F_{lag} = \frac{\tau_I s + 1}{\tau_I s + \gamma}$$

$$\frac{1}{\tau_I} = \frac{\omega_{c,d}}{10} \Rightarrow \boxed{\tau_I = \frac{10}{0{,}32} = 31{,}25}$$

$$E(s) = \frac{1}{1 + G(s)\,F(s)} W_{ref}(s)$$

- stable so we use final value theorem

$$e(\infty) = \lim_{s \to 0} \frac{1}{\cancel{s}} \frac{1}{1+G(s)F(s)} \cancel{s} = \frac{1}{1+G(0)F(0)}$$

$$= \frac{1}{1 + G(0)\,F_{lead}(0)\,F_{lag}(0)} = \frac{1}{1 + 2K\frac{1}{\gamma}} = \frac{1}{1+\frac{2}{\gamma}}$$

$$e(\infty) \leq 0{,}05 \implies \frac{1}{1+\frac{2}{\gamma}} \leq 0{,}05$$

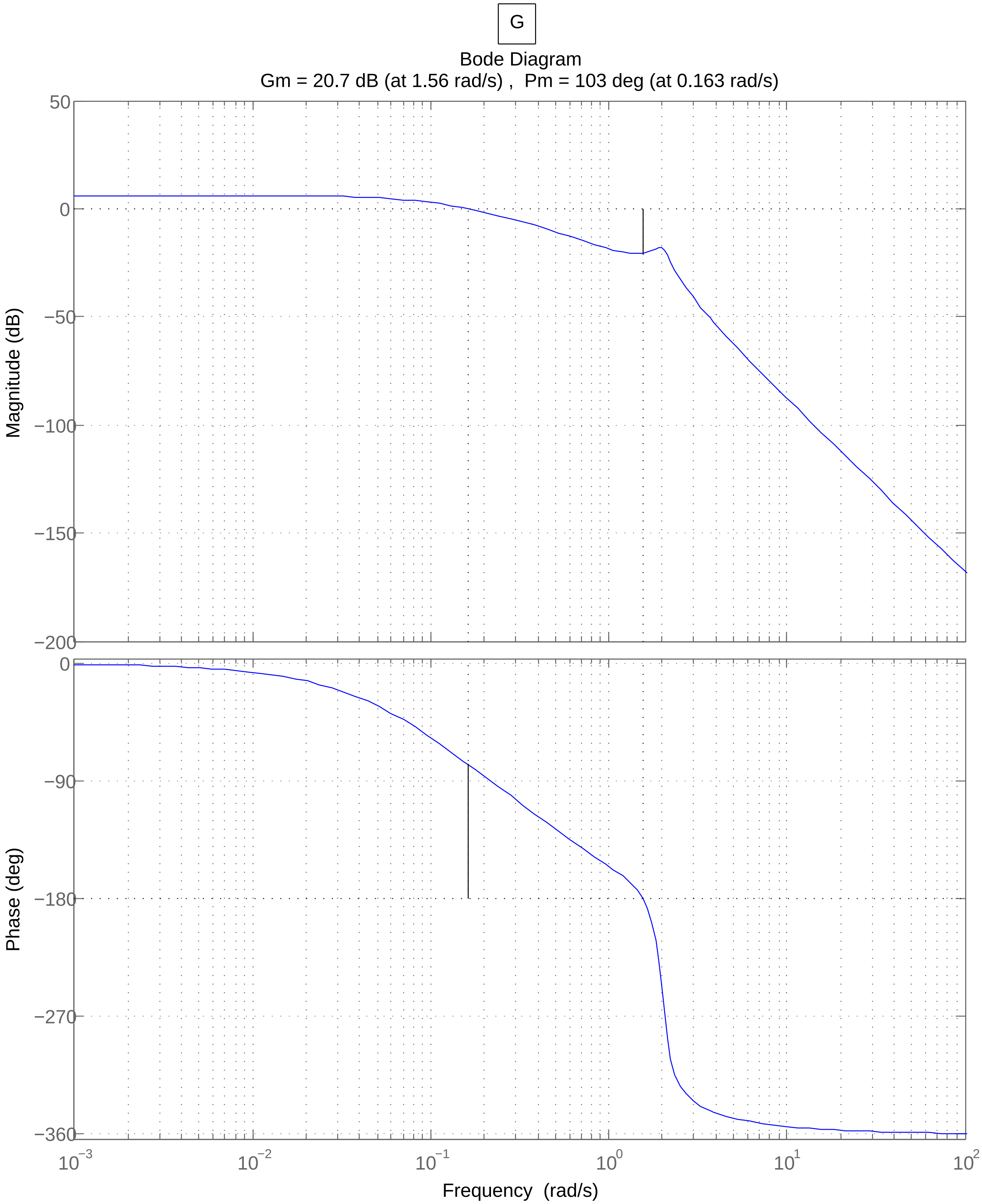
$$\implies 1 \leq 0{,}05 + \frac{2}{\gamma} 0{,}05$$

$$\implies 0{,}95 \leq \frac{0{,}1}{\gamma}$$

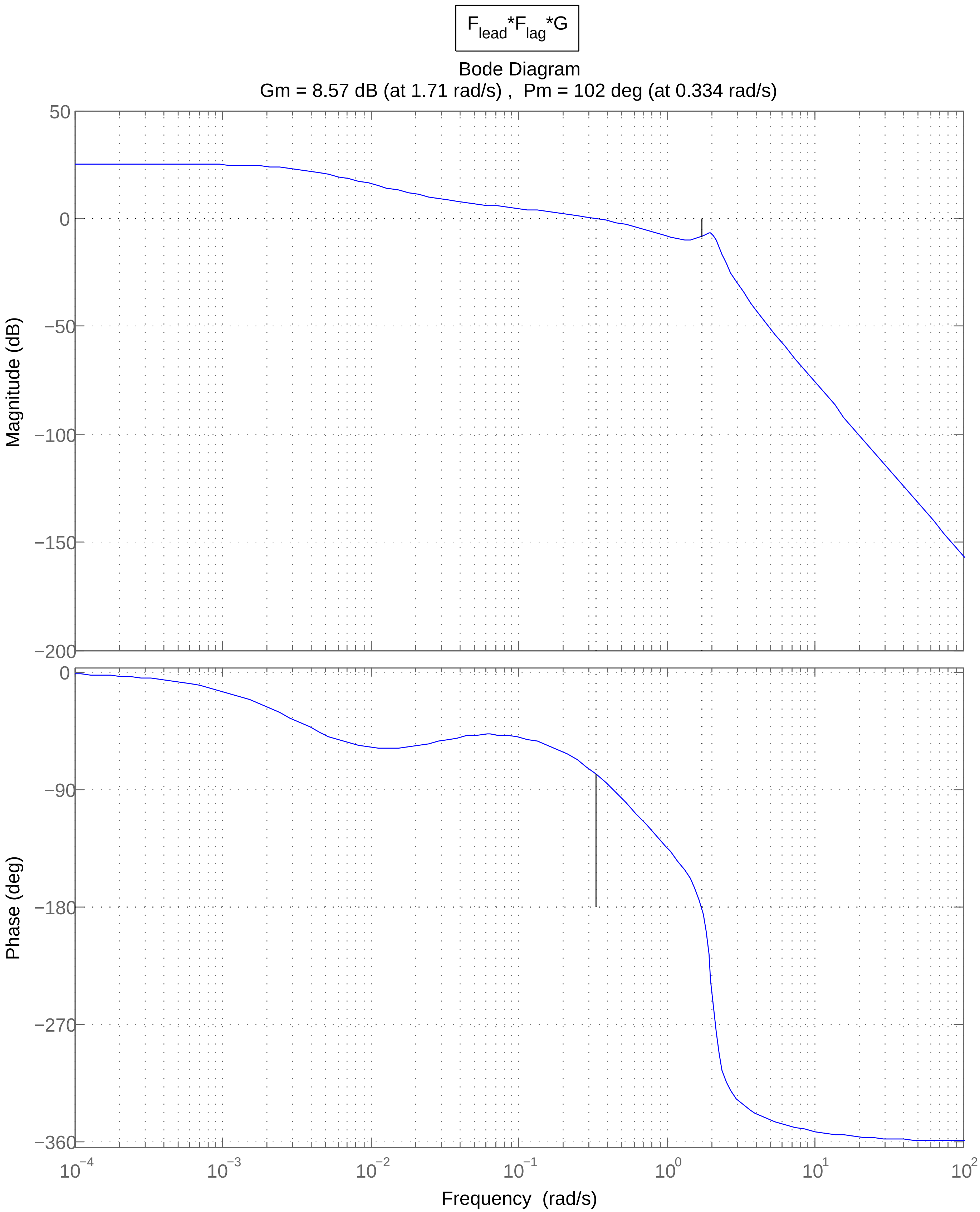
$$\implies \gamma \leq 0{,}11$$

- Since we don't want to have to high $|G(j\omega)|$ for low frequency we choose $\gamma = 0{,}11$

$$\boxed{G_{lag}(s) = \frac{31{,}35s + 1}{31{,}35s + 0{,}11}}$$



G
Bode Diagram
Gm = 20.7 dB (at 1.56 rad/s) , Pm = 103 deg (at 0.163 rad/s)
Magnitude (dB)
50
0
−50
−100
−150
−200
Phase (deg)
0
−90
−180
−270
−360
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/s)



F_lead*F_lag*G
Bode Diagram
Gm = 8.57 dB (at 1.71 rad/s) , Pm = 102 deg (at 0.334 rad/s)
Magnitude (dB)
50
0
−50
−100
−150
−200
Phase (deg)
0
−90
−180
−270
−360
10^−4
10^−3
10^−2
10^−1
10^0
10^1
10^2
Frequency (rad/s)