1 Dec 2014

Exercise session #11

- Solve 5.13, 6.10, 9.13 ab
- New functions: • sys = ss(G) — convert G(s) in state-space form
  - [A, B, C, D] = ssdata(G) — as object or as matrixes
  - K = place(A,B,P) → chose K s.t. eigenvalues of A-BK are the element of P
- Old functions: • [Am, φ_m, ω_c, ω_p] = margin(G)
  - bode(G)
  - step(G) → step response for G(s)
    - step($\frac{1}{s}$ · G) → ramp response for G(s)
  - dcgain(G) → steady-state gain of G (i.e. G(0))

Ex 5.13

point a

matlab

```
s = tf('s');
G = 725/((s+1)·(s+2,5)·(s+25));
F = 1;
[Am, φm, ωc, ωp] = margin(F·G);
```

result

$A_m = 3{,}5$; $\varphi_m = 27°$; $\omega_c = 5$ rad/s ; $\omega_p = 9{,}5$ rad/s point b, c

Design lead-lag compensator such that:

$\omega_{c,d} = 5\,rad/s$; $\varphi_{M,d} \geq 60°$; $e(\infty) = 0$ for step input

Lead part

$$\boxed{F_{lead}(s)\ K\ \frac{\tau_D s + 1}{\beta \tau_D s + 1}}$$

$$\arg(G(j\omega_{c,d})) = \arg(G(5j)) = 27°$$

phase increase $\Delta\varphi \geq 60° - 27° + 5.7° = 38.7°$

$\Delta\varphi = 40°$

From fig 5.13, Ljung book $\boxed{\beta = 0{,}21}$

$$\tau_D = \frac{1}{\omega_{c,d}\sqrt{\beta}} = 0{,}43\,s$$

K s.t. $\frac{K}{\sqrt{\beta}}|G(5j)| = 1 \Rightarrow \boxed{K = 0{,}46}$

Lag part

$$\boxed{F_{lag}(s) = \frac{\tau_I s + 1}{\tau_I s + \gamma}}$$

For $e(\infty) = 0$ for input step $\boxed{\gamma = 0}$

$\tau_I = \frac{10}{\omega_{c,d}} \rightarrow \boxed{\tau_I = 2}$

Matlab

```
F = Flead . Flag
margin (F, G)
```

```
Gc = feedback(F·G, 1)
step(Gc)

Gc1 = feedback(G, 1)
bode(Gc1, Gc)
```
← closed-loop Bode diagrams

<u>point d</u>

$\boxed{\text{Matlab}}$

$$Se = \text{minreal}\ \frac{1}{1+F\cdot G}$$

$$\text{step}\left(\frac{1}{s}\cdot Se\right);$$

$\boxed{\text{Ex } 6.10}$

$$G^{\circ}(s) = G(s)\,\frac{1}{s+1} = G(s)\left[1 + \underbrace{\frac{-s}{s+1}}_{\Delta_G(s)}\right]$$

For robustness we want to compare

$$|T(j\omega)| = \left|\frac{F(j\omega)\,G(j\omega)}{1+F(j\omega)\,G(j\omega)}\right| \quad \text{and} \quad \left|\frac{1}{\Delta_G(j\omega)}\right| = \left|\frac{j\omega+1}{-j\omega}\right|$$

$$\text{if } \boxed{|T(j\omega)| < \left|\frac{1}{\Delta_G(j\omega)}\right| \quad \forall \omega} \Longrightarrow \text{system is } \underline{\text{ROBUST}}$$

$\boxed{\text{Matlab}}$

```
T = feedback(F·G, 1);
IDG = (s+1)/(-s);
bode(T, 'b', IDG, 'r');
```

check if $\boxed{}$ is satisfied

$\boxed{\text{Ex 9.14}}$

point a

```
s = tf('s');
G = 1/(s.(s+1));
[A,B,C,D] = ssdata(G)
L=place(A,B,[p1,p2]');

Gco = ss(A-BL, B, C, 0);

Lo = 1/dcgain(Gco);

Gc = Gco Lo     <- note the order

[y,t,x]=step(Gc)
u = Lo - x.L';
plot(t,y,'b',t,u,'r');
```