Updates to: Scale-Space Theory in Computer Vision

This document contains corrections and additional remarks to:[1]

> *Scale-Space Theory in Computer Vision* by Tony Lindeberg
> published by Kluwer Academic Publishers,
> Dordrecht, The Netherlands, 1994.

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Cover

The illustration on the front page shows a model of the receptive fields in a foveal scale-space representation. Details about this model (indicated in footnote 9 on page 54) can be found in (Lindeberg & Florack 1994).

Chapter 1

Figures 1.3 and 1.5 (pages 12-13): The horizontal axis in both these figures is the x-axis (the spatial domain) and the vertical axis is the t-axis (the scale domain).

Chapter 2

Complementary reviews of linear scale-space theory can be found in (Lindeberg 1994d), (Lindeberg & ter Haar Romeny 1994) and (Lindeberg 1996e).

Section 2: A monograph on pyramid representations has been written by (Jolion & Rozenfeld 1994) after the completion of this manuscript.

Section 2.3.2 (page 35): The unimodality condition should read:

- unimodality: $c(|n|) \geq c(|n| + 1)$

Section 2.5.4: A modified necessity proof from scale invariance is given in (Lindeberg 1994b). In this article it is also made explicit how scale invariance combines with semi-group structure if the assumption about separability in Cartesian coordinates is relaxed.

[1] This document can be fetched on the WWW via the home page of the author http://www.nada.kth.se/~tony. Please, report corrections and suggestions to tony@nada.kth.se or Tony Lindeberg, KTH, NADA, S-100 44 Stockholm, Sweden.

Section 2.5.4: Footnote 7 (page 49): The undetermined parameter gives rise to a one-parameter family of smoothing kernels. A treatment of this subject is presented in (Pauwels, Fiddelaers, Moons & van Gool 1995).

Section 2.9.2: A monograph on non-linear diffusion has been written after the completion of this manuscript (ter Haar Romeny 1994).

Chapter 4

Page 104: One arrow is missing in figure 4.2.

Chapter 6

Page 151: Explicit expressions for directional derivatives in (p, q) coordinates can be found in (Lindeberg 1994d).

Page 161: A "ridge detector" expressed within the same framework is described in (Lindeberg 1994d).

Chapter 13

Extended descriptions of this material is given in (Lindeberg 1994c, Lindeberg 1996a, Lindeberg 1996b, Lindeberg 1996c).

Page 330: Lines 4–8 and equations (13.43) and (13.44) should read:

> At the origin it holds that $L_{x_1x_1} = 0$ and $L_{x_2x_2} = 0$. Hence, since $\Phi(0;\, t) = 1/2$ for all t and $g(0;\, t) = 1/\sqrt{2\pi t}$, it follows that
>
> $$|\tilde{\kappa}_{norm}(0, 0;\, t)| = |2 L_{x_1} L_{x_2} L_{x_1x_2}| = \frac{t^2}{8\pi^2(t_0 + t)^2}, \qquad (13.43)$$
>
> and
>
> $$\partial_t |\tilde{\kappa}_{norm}(0, 0;\, t)| = \frac{t_0 t}{4\pi^2(t_0 + t)^3} > 0 \quad (t > 0), \qquad (13.44)$$
>
> showing that the magnitude of $\tilde{\kappa}_{norm}(0, 0;\, t)$ increases *monotonically* with scale. Moreover, $\tilde{\kappa}_{norm}$ is small when t is small compared to t_0.

Page 345: Equation (13.79) should read

$$\int_{-\infty}^{\infty} |g_{\xi\xi}(u;\, t)| \, du = -2t \int_{-\sqrt{t}}^{\sqrt{t}} g_{xx}(u;\, t) \, du = 4t \, g_x(-\sqrt{t};\, t) = 2\sqrt{\frac{2}{\pi e}}. \qquad (13.79)$$

Section 13.8: An application of the junction detector to deriving features for object recognition is given in (Lindeberg & Li 1995a, Lindeberg & Li 1995b) and an application to feature tracking in (Bretzner & Lindeberg 1996).

Chapter 14

Section 14.1: After the completion of this chapter, a "stationarity assumption" has been introduced for computing shape-from-texture by (Malik & Rosenholtz 1993). In principle, the type of approach developed in this chapter can be applied under that assumption as well, although the details remain to be worked out.

Page 355: A factor of 1/2 is missing in equation (14.9):

$$\lambda_{1,2} = \frac{P \pm Q}{2} = \frac{P(1 \pm \tilde{Q}}{2} \qquad (14.9)$$

Pages 381-382: An extension of the shape-from-texture method to shape-from-disparity-gradients is described in (Gårding & Lindeberg 1994, Gårding & Lindeberg 1996).

A description of how to compute image correspondence and more general image deformations using this overall framework can be found in (Lindeberg 1994a, Lindeberg 1995, Lindeberg 1996d).

Chapter 15

Section 15.4: An extended description of the material in this chapter can be found in (Lindeberg & Gårding 1994).

A monograph on non-linear diffusion (edited by (ter Haar Romeny 1994)) has been written after the completion of this manuscript.

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