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Bachelor thesis project proposal

Numerical issues of the elastic wave equation in highly heterogenous media

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1. Background and general goal

In this project we consider the multiscale elastic wave equations. In the scalar case (where it is typically called the acoustic wave equation) we seek a wave function $u(x,t)$ (with $x \in \mathbb{R}^3$ and $t \in [0,T]$) with initial values $u(x,0) = f(x)$ and $\partial_t u(x,0) = g(x)$ that solves

$$\partial_{tt} u(x,t) - \nabla \cdot (A(x)\nabla u(x,t)) = F(x,t). \tag{1}$$

The wave speed $A \in L^\infty(\mathbb{R}^3; \mathbb{R}^{3\times 3})$ is rapidly varying on a very fine scale due to structural variations in the medium. Typically, A is also highly heterogeneous and discontinuous. A source term is given by F.

Elastic wave equations have a significant relevance for determining subsurface formations in the earth's crust. For instance, in order to recover oil or petroleum from geologic formations, to sequestrate carbon dioxide (i.e. storing liquid CO2 in the subsurface) or to predict earthquakes, scientists need a clear image of the geologic underground, i.e. its composition concerning rock formations, soil, faults, groundwater, oil, petroleum and so on. Practically, an image of the underground is determined by generating an energy impulse (or vibrating source) that sends a seismic wave into the ground. A fraction of the wave is reflected, where the reflection pattern depends on the multiscale structures in the subsurface. When the reflected wave is measured by microphones, so called seismometers, it is possible to reconstruct the subsurface structures from the measured data by solving an inverse problem. Solving this inverse problem requires to solve the elastic wave equation (forward problem) several times. The spatial resolution required for a corresponding realistic simulation of the forward problem can result in up to one billion degrees of freedom for a conventional scheme. This number is multiplied by the number of required time steps and the number of forward simulations. In order to overcome these issues, numerical multiscale methods can be applied. In this project we shall study such methods and their performance. One issue of high relevance is how the choice of initial values (in the wave equation) might trigger fast temporal variations or dispersive effects. We shall investigate the sensitivity of the numerical multiscale methods with respect to these phenomena.

2. Tasks

The following list is preliminary and can change upon the students personal interests and preferences.

- Description of the model and its physical background.
- Stating numerical multiscale methods for solving the wave equation.
- Implementing one (or more) of the numerical methods.
- Simulating wave propagation in a multiscale setting.
- Investigating the accuracy of the method over (possibly long) time.
- Studying how the choice of the initial values influences the performance and the accuracy of the method. Stating explanations for the observed effects.

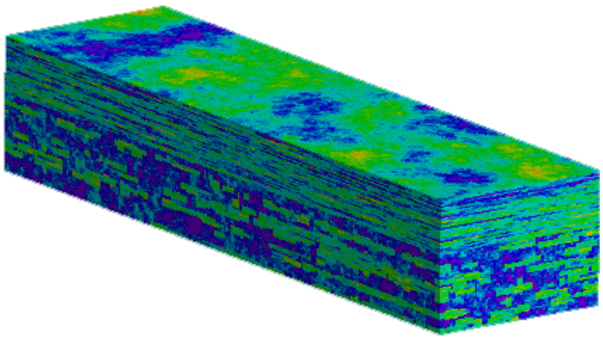

Figure 1: Geological structure of a subsurface shallow-marine sandstone formation taken from the SPE10 dataset (*www.spe.org*).

References

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