Impact of Densification on Energy Efficiency in Wireless Access Networks

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***Abstract*—Mobile communication networks alone consume 0.5 percent of the global energy today. Rapidly growing demand for capacity will further increase the energy consumption. Thus, improving energy efficiency has recently gained great interest within the research community not only for environmental awareness but also to lower the operational cost of network operators. Base station deployment strategy is one of the key challenges to be addressed for fulfilling the future capacity demand with energy efficiency. In this paper, we investigate the relationship between energy efficiency and densification with regard to an area capacity requirement. To this end, we refine the base station power consumption model such that the parameters are determined by the maximum transmit power and develop a simple analytical framework to derive the optimum transmit power that maximizes energy efficiency for a certain capacity target. Our framework takes into account interference, noise and backhaul power consumption. Numerical results show that deployment of smaller cells significantly reduces the base station transmit power, and thus shifts the key elements of energy consumption to idling and backhauling power. Network densification can only be justified when capacity expansion is anticipated.**

***Index Terms*—Energy Efficiency, Backhaul, Densification, Area Capacity, Power Consumption.**

I. Introduction

The cellular networks of today provide good coverage and services in many countries, in both urban and rural areas. The key challenge for the industry is the rapid proliferation of smartphones, laptops, and tablet PCs with built-in cellular access that is rapidly driving the demand for capacity increase. This exponentially growing data traffic will require the deployment of several orders of magnitude more base stations (BSs) which will significantly increase the energy consumption, apart from infrastructure investment and acquisition of spectrum [1], [2]. Currently, wireless access networks alone consume 0.5% of the global energy consumption and the cost of energy constitutes almost 50% of the operational expenditures (OPEX) of mobile operators in some countries [3], [4]. With the increasing rate of BS deployment and soaring energy prices, we foresee that energy will be the main design criteria for the next generation wireless access networks [3].

Motivated by these factors, main operators and their equipment suppliers started to address the arising concern of excessive energy consumption by making efforts to improve the state-of-the-art wireless broadband networks. An overview of the different proposed methods to increase the energy efficiency on all levels of the network, including hardware design, network management, network deployment, and resource allocation, is given in [5]. From network deployment perspective, the use of small BSs is believed to reduce the BS energy consumption because the BS closer to mobile users can lower the required transmit power due to advantageous path loss conditions [6]. However, this comes with an increase in the number of BSs, which demands more energy for running the BSs. This tradeoff has been investigated in the literature and the effect of cell size on energy efficiency is shown in [7]–[10]. Area power consumption of different deployment scenarios are evaluated in [7] for a minimum received power target at the cell edge considering different idle power consumption figures and it is stated that large cell deployments are more energy efficient due to the high idle power of existing BSs. On the contrary, in [8]–[10] small-cell based mobile communication systems are claimed to be an effective solution to accommodate high data rates with low energy consumption.

The main reasons of having contradictory conclusions are due to the differences in power consumption model (i.e., only considering radio transmit power, total power consumption), selected network performance metric (i.e., coverage, network capacity, cell capacity) and considered energy efficiency metric (i.e., bit/joule, W/km^2). Besides, neither the impact of interference level nor the dependency of power consumption parameters on cell size has been taken into account in previous studies. The increase in system throughput via deployment of short-range BSs has also been ignored.

We believe that to be able to answer the question of ”which type of network deployment is more energy efficient” both network performance metrics, coverage and capacity, should be considered and kept independent of the selected cell size for the deployment. With this in mind, this paper investigates the relation between energy efficiency, area capacity and cell size by considering the impact of both interference and noise, cell size dependent idle power consumption and backhaul power consumption which is often ignored in the literature. We target to answer the following questions:

- What is the effect of network densification on energy efficiency?
- How shall the operators deploy their network to cope with expected capacity demand from the energy efficiency point of view?

For this purpose, we develop a simple analytical model to derive the optimum transmission power which maximizes energy efficiency under a network capacity target, for a given cell range. We proposed a refined power consumption model where the parameters are determined by the maximum transmit power of BSs. This allows us to adjust the power consumption of hypothetical BSs in accordance with the densification level. We demonstrate how different assumptions about capacity requirement affect the energy efficiency in different levels of network densification.

II. Definitions and Problem Formulation

The focus of this paper is the homogenous network deployment with short-range and low-power BSs on a given network area, A. We assume that base stations have omni-directional antennas, so that each BS covers a single cell.

A. Network Capacity and Energy Efficiency

Energy efficiency (Ψ) is defined as the ratio of the total number of bits that were correctly delivered in the network during the observation period (T) over the network energy consumption during the same time where the unit is bits/Joule [11]. Since the network energy consumption is the multiplication of the power consumption with the observation period, this metric can equivalently be written as the ratio of the throughput, (which is equal to network capacity, C_{net} for considered fully loaded system), over the network power consumption, P_{net}, which can be expressed in bps/W as below:

$$\Psi(P_{tx}) = \frac{C_{net}}{P_{net}}. \tag{1}$$

Here, C_{net} is defined as the summation of cell capacity C_{cell} within the network area, whereas P_{net} is the total power consumption of the network including the sum of power consumption figures of individual base stations as well as backhaul network which are introduced in Section V and IV respectively. Under the assumption that the network is fully loaded regardless of the cell size, i.e., each base station has at least one mobile requesting data with all resources allocated, network capacity is shown below:

$$C_{net} = N_{BS} C_{cell}, \tag{2}$$

where $N_{BS} = \frac{A}{\pi R^2}$ is the number of base stations and R is the cell radius.

B. Problem Formulation

The objective is to find the optimum transmit power, P_{tx}^{opt} within the range of maximum allowed RF output power of the base station, P_{tx}^{max}, which maximizes the energy efficiency under the constraint that network capacity will stay constant. Note that both Ψ and C_{net} are the functions of cell range. The optimization problem can be mathematically expressed as follows:

$$\begin{aligned} \underset{P_{tx}}{\textbf{Maximize}} \quad & \Psi \\ \textbf{subject to} \quad & C_{net} = C_{target}, \\ & P_{tx} \leq P_{tx}^{max}. \end{aligned} \tag{3}$$

We aim to see the relationship between $\Psi(P_{tx}^{opt})$, C_{target}, and R to analyze the impact of network densification.

It should be noted that highly dense deployment scenario results in low P_{tx}^{opt} figure. Therefore, there will always be a transmit power $P_{tx}^{*} \geq P_{tx}^{opt}$ which increases the network capacity with a comparably low increase in total network consumption and thus improve the energy efficiency. However, it is doubtful that providing more capacity than required is meaningful even if it gives higher energy efficiency. Therefore, we considered equality constraint for the capacity target in (3).

III. SINR Modeling

In downlink direction of wireless communication, the received power at distance d from a base station can be modeled by

$$P_{rx}(d) = \frac{cGP_{tx}}{d^{\alpha}}, \tag{4}$$

where P_{tx} is the transmit power, c and α are the path loss coefficient and exponent respectively, and G is the antenna gain.

Lets us consider a system with no interference arising from the serving cell. The downlink SINR denoted by Γ, at a given distance d can be obtained by

$$\Gamma(d) = \frac{P_{rx}(d)}{\sum_{j=1}^{L} I_j + N}, \tag{5}$$

where I_j denotes the received interference power from the j-th interfering BS, N is the noise power on the total bandwidth $(N_0 W)$ and N_0 is the power spectral density of additive white gaussian noise (AWGN). The derivation of the probability distribution for received signal, interference, and SINR for the downlink of a cellular network have been extensively investigated in the literature [12]–[15]. The proposed semi-analytical [15] and approximate [14] approaches for SINR characterization obviate the need for time-consuming simulations and obtain the main characteristics of the network. Here, we consider the fluid model [14] which is based on the idea of replacing a given fixed finite number of interfering base stations by an equivalent continuum of transmitters. The model assume that mobiles and base stations are uniformly distributed in the area, thus the network has constant mobile station (MS) density ρ_{MS} and cochannel BS density ρ_{BS}.

For an OFDMA or TDMA like system with universal frequency reuse where there is no internal interference because of the perfect orthogonality between users, the sum of the external interference powers for the mobile at the distance d, $p_{ext,d}$, is approximated as follows [14]:

$$p_{ext,d} = \frac{2\pi\rho_{BS} P_{tx} cG}{(\alpha-2)} \left[(2R-d)^{2-\alpha} - (R_{area}-d)^{2-\alpha} \right], \tag{6}$$

where R_{area} is the radius of the network area, and $\rho_{BS} = \frac{1}{\pi R^2}$.

Based on the model above, SINR experienced by the user can be expressed as

$$\Gamma(d) = \frac{1}{\frac{2d^\alpha}{(\alpha-2)R^2}\left[(2R-d)^{2-\alpha} - (R_{area}-d)^{2-\alpha}\right] + \frac{N_0 W}{cGP_{tx}} d^\alpha}. \tag{7}$$

IV. Power Consumption Model

A. Power Consumption Elements

In the literature, the average power consumption of a base station is modeled as a linear function of average transmit power and is given by $P = aP_{tx} + b_{radio}$ [16]. Here, P and P_{tx} denote the average total power per base station and the power fed to the antenna, respectively. The coefficient a accounts for the part of the power consumption that is proportional to the transmitted power (e.g., radio frequency (RF) amplifier power including feeder losses), while b_{radio} denotes the power that is consumed independent of the average transmit power (e.g., signal processing, site cooling). The total power consumption of the network is calculated based on the deployment strategy, i.e. type and the number of base stations in the network and the backhaul power consumption is usually ignored in the literature. However, the deployment strategy will indeed affect the implementation of the backhaul and consequently its power consumption. We have shown in [17] that the relative impact of backhaul power consumption is non-negligible for dense network deployment. Therefore, in order to obtain consistent and realistic results to assess the the impact of network densification on energy efficiency, backhaul power consumption (P_{bh}) should be incorporated into the analysis.

Total power consumption of the network with N_{BS} number of base stations including the fiber optic based mobile backhaul can be written as [17]:

$$P_{net} = N_{BS}[aP_{tx} + b_{radio}] + P_{bh}. \tag{8}$$

The backhaul power P_{bh} includes not only the downlink and the uplink power consumption (i.e., from a base station to the aggregation switch(es) and from the switch(es) to the aggregation network, respectively) but also the power consumed at the aggregation switch(es), which is proportional to the total traffic backhauled from the mobile network. A detailed expression for P_{bh} is given by

$$P_{bh} = N_{BS}\left[b_{backhaul} + \frac{(1-\tau)P_{switch}^{max}}{n_{ports}C_{switch}^{max}} Ag_{switch} + \frac{\tau P_{switch}^{max}}{n_{ports}}\right]. \tag{9}$$

Here $b_{backhaul}$ represents the power consumed by the backhaul transceiver, and the uplink interface, P_{switch}^{max} is the maximum power consumption of the switch, Ag_{switch} is the aggregate traffic traversing the switch, while τ, C_{switch}^{max} and n_{ports} are percentage of the switch power that is independent of the network traffic, $\tau \in [0,1]$, maximum capacity of a switch and number of ports of the switch respectively. A more detailed explanation of these parameters can be found in [17].

It should be noted that in this paper we have considered two cases to calculate the total power consumption: 1) same type of BSs with identical power consumption parameters are chosen for network deployment which are are fixed, regardless of the cell range. In this case, values of the parameters, a and b_{radio} are selected for micro type base stations; 2) BS type, and thus power consumption parameters depend on the cell range where a and b_{radio} vary with the maximum BS transmit power which will be explained in the next subsection in detail.

B. Transmit Power Dependent Idle Power Consumption Modeling

The power consumption of different type of base stations has been modeled in European FP7 project EARTH [11] where it is stated that while power amplifier power consumption dominates the total power consumption of Macro BSs, idle power consumption becomes the main consumer in smaller BSs like pico and femto as it is shown in Table I.

TABLE I
POWER CONSUMPTION PARAMETERS FOR BASE STATION [6], [11]

Base Station Type	$P_{max}[W]$	a	$b_{radio}[W]$
Micro	6.3	3,1	53
Pico	0.13	4.2	6.8
Femto	0.05	7.5	4.8

The significant effect of power consumption parameters on area power consumption has been assessed in [1] which shows that completely contradictory conclusions can be drawn about the cell size impact if we consider different ratios between load dependent power consumption, aP_{tx} and the idle power consumption, b_{radio}. Therefore, we propose to write these parameters as a function of maximum transmit power of the hypothetical BSs to fairly asses the impact of network densification. We have created an approximating functions that capture important patterns in the data in Table I as below:

$$a = \mu - \eta \log_2 (P_{max}), \tag{10}$$

$$b_{radio} = \kappa P_{max} + \psi. \tag{11}$$

Then, we have used nonlinear regression [18] which is a procedure for fitting data to any selected equation and define the the best set of parameters which gives the least squares solution, which are $\mu = 4.15$, $\eta = 0.5$, $\kappa = 7.6$ and $\psi = 5.1$.

It should be noted that each combination of a and b_{radio} represents a hypothetical BS customized to P_{max}.

V. Energy Efficiency Optimization

In this section, we derive the closed form expression of optimum transmit power that maximizes energy efficiency under the constraint that network capacity in the considered area is guaranteed. To this end, interference model in the previous section is used for the worst case scenario where the

$$\Psi = \frac{\frac{R_{area}^2}{R^2} W \log_2\left(1+\frac{1}{\frac{2}{(\alpha-2)}\left[1-(\frac{R_{area}}{R}-1)^{2-\alpha}\right]+\frac{N_0 W}{cGP_{tx}}R^\alpha}\right)}{\frac{R_{area}^2}{R^2}\left[aP_{tx}+b_{radio}+b_{backhaul}+\frac{(1-\tau)P_{switch}^{max}}{n_{ports}C_{switch}^{max}}Ag_{switch}+\frac{\tau P^{max}_{switch}}{n_{ports}}\right]}. \quad (12)$$

user stands at the cell edge. Then, we have calculated network power consumption based on our proposed model where backhaul power consumption is incorporated by considering fixed and variable BS power consumption parameters.

Cell capacity in the worst case scenario can be evaluated using Shannon's formula, such as $C_{cell} = W \log_2(1 + \Gamma(R))$ where $\Gamma(R)$ is the SINR of the cell edge user. Inserting $d = R$ in (7) and using the definition in (1), we can rewrite energy efficiency as in (12). Under a minimum achievable network throughput target C_{target}, required transmit power, P_{tx}^C can be calculated as below:

$$P_{tx}^C = \frac{N_0 W R^\alpha / cG}{\left[\frac{1}{2^{\left(\frac{R}{R_{area}}\right)^2 \frac{C_{target}}{W}}-1}\right] - \frac{2}{(\alpha-2)}\left(1-(\frac{R_{area}}{R}-1)^{2-\alpha}\right)}. \quad (13)$$

By considering the fact that network capacity is a non-decreasing function of transmit power, the solution of the optimization problem given in (3) is calculated as $P_{tx}^{opt} = P_{tx}^C$.

VI. Numerical Results

In this section, numerical results are presented to demonstrate the relationships between area network capacity, energy efficiency and cell size for fixed (Case 1) and variable (Case 2) BS power consumption parameters.

We consider an area with a radius of R_{area}=10 km covered by small base stations where the cell range varies between 50-500m. Base stations are assumed to be equipped with one omni-directional antenna. It should be noted that for Case 1, micro type base stations are considered for the network power consumption calculation where $a = 3.1$ and $b_{radio} = 53$ [6], [11]. System and backhaul power consumption parameters are listed in Table II.

Fig. 1 shows the energy efficiency figures as a function of cell range, R, for different area capacity requirements under the assumption that BS power consumption parameters a and b_{radio} are fixed regardless of P_{tx}. We can clearly observe that energy efficiency is maximized by the deployment of the largest feasible cell size which can satisfy the given network performance requirement C_{target}. This simply results from the fact that dense deployment of the smaller cells reduces the transmit power and thus shifts the key problem of the energy consumption from the transmit power to idle power which constitutes more than 60% of the total power consumption. Therefore, reduced transmit power can not compensate the additional required idle power consumption, leading to lower energy efficiency.

TABLE II
Experimental Parameters

System and Path Loss Parameters	Value
Frequency (f)	2GHz
Bandwidth (W)	5 MHz
Path loss constant (c)	10^{-3}
Antenna Gain (G)	2 dBi
Path Loss exponent (α)	4
Thermal Noise (N_0)	-174 dBm/Hz
Noise figure	10 dB
Backhaul Power Consumption Parameters	**Value**
$b_{backhaul}$	3W
P_{switch}^{max}	300W
τ	0.8
C_{switch}^{max}	10 Gb/s
n_{ports}	24

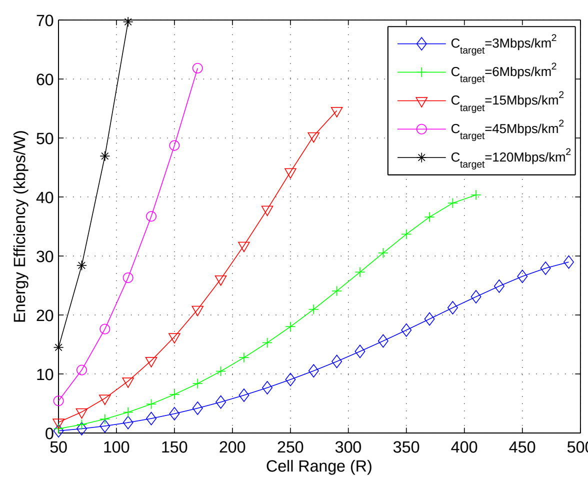
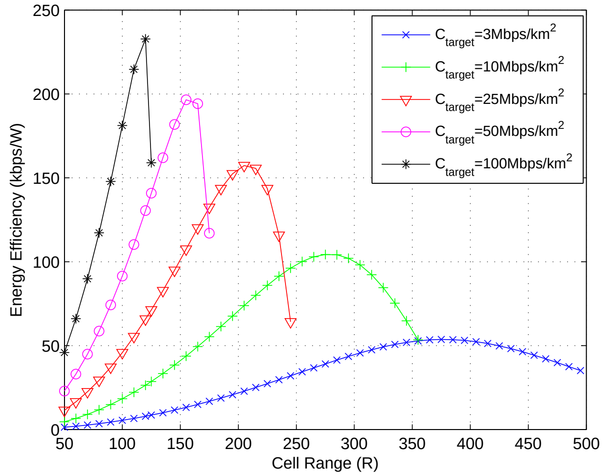
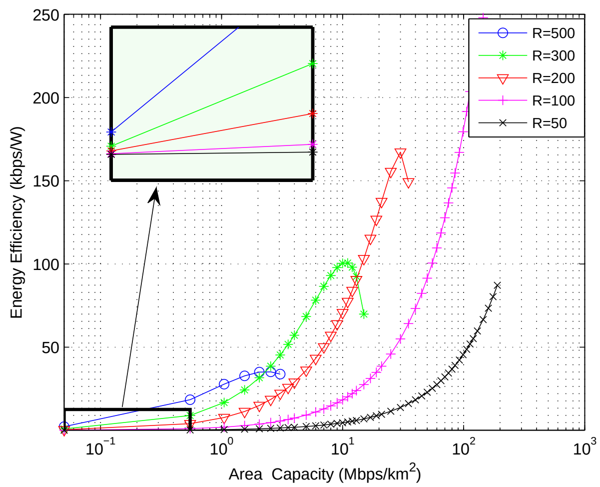


Fig. 1. Energy efficiency as a function of cell range for fixed power consumption parameter (a, b_{radio}).

When we consider variable power consumption parameters as in (10) and (11), the optimal energy efficiency is achieved before the cells reach the largest feasible size as it is illustrated in Fig. 2(a). It is due to the fact that impact of load dependent power consumption (aP_{tx}) becomes dominant for large cell size deployment because of the increase in both optimum transmit power, P_{tx}^{opt}, and the coefficient a which accounts for the part of the power consumption that is proportional to the transmitted power as it is shown in (10). This generates an optimal cell size that maximizes the energy efficiency for all considered area capacity targets. It may come no surprise that higher area capacity requirement lowers the optimum cell



(a) Optimum energy efficiency versus cell range.



(b) Optimum energy efficiency versus network capacity requirement.

Fig. 2. Relationship between energy efficiency, area capacity and network densification for variable BS power consumption parameters (a, b_{radio}).

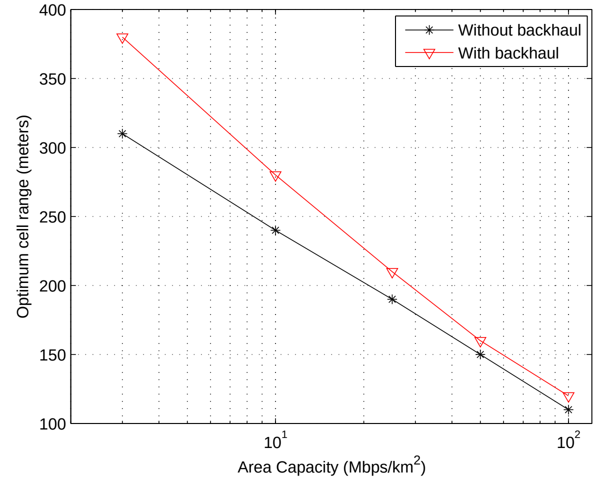


Fig. 3. Optimum cell range vs. required network capacity for variable BS power consumption parameters (a, b_{radio}).

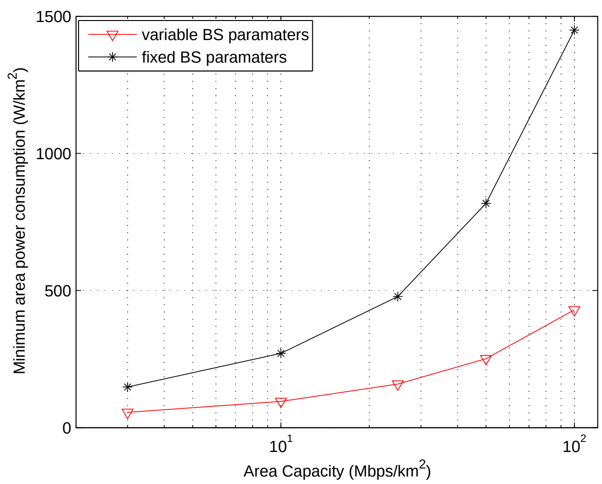


Fig. 4. Minimum area power consumption vs. required area capacity; backhaul power consumption is considered.

range and favors for smaller cell size which is shown more clearly in Fig. 2(b). We observe that, even though energy efficiency is maximized by the deployment of larger cell size in low capacity demand region, it quickly loses its efficiency, and even becomes infeasible to fulfill the capacity demand.

Fig. 3 depicts the trend of optimum cell range as a function of area capacity demand, C_{target}. The impact of backhaul power consumption on the optimum network deployment is clearly noticeable in the figure. For the cases where small low power base stations are deployed, the relative impact of backhaul gets more influential which indicates the trade-off between the power saved using low power base stations and the excess power that spent for backhaul. This situation gives rise to larger optimum cell size to maximize the energy efficiency compared to the case where the total power consumption of the network is restricted to the sum of the all base stations. Hence, when assessing the benefits of a deployment strategies, the backhaul power consumption should not be simply ignored.

Finally, the minimum achievable area power consumption when the network is deployed with optimal cell size is shown in Fig. 4, for fixed (Case 1) and variable (Case 2) BS power consumption parameters. It is observed that area power consumption is lower for all area capacity targets when we consider variable BS parameters. This is because Case 2 considers that fact that idle power consumption will depend on cell size and thus it will be lower for the dense deployment, thanks to its lower transmission power requirement. Therefore, a proper choice of BS type significantly improves the energy efficiency particularly when the capacity demand is high.

VII. Conclusion

In this paper, we investigated the relation between energy efficiency, area capacity and cell size by considering the impact of the interference, noise, backhaul and cell size dependent idle power consumption. We demonstrated how different assumptions about capacity requirement and base stations types affect the energy efficiency in different levels of network densification. To this end, we proposed a refine base power consumption model where the parameters are determined by the maximum transmit power and developed a simple analytical framework to derive the optimum transmission power which maximizes energy efficiency under the constraint that network capacity in the considered area is guaranteed. Numerical results show that deployment of smaller cells significantly reduces the transmit power and thus shifts the key elements of the energy consumption from the transmit power to idling and backhauling power. Therefore, energy efficiency is maximized by the largest feasible cell size that can satisfy the given network capacity target. However, considering the doubling capacity demand every year, we observe that larger cell deployment quickly loses its efficiency, and even becomes infeasible to fulfill the capacity requirement. Consequently, careful prediction of capacity demand has been identified as a key challenge for the energy efficient deployment of wireless networks.

It should be noted that the presented results are based on the assumption that the network is fully loaded regardless of the cell size. However, in reality the traffic in cellular mobile networks is typically unbalanced and dynamic both in the time and spatial domains. Thus, adapting the BS deployment and operation to the spatial and temporal heterogeneity of the traffic demand will play an important role for the energy efficiency. Its impact will be investigated as a future work.

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