Aggregate Interference from Secondary Users with Heterogeneous Density

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***Abstract*—This paper presents an analytical model to approximate the probability distribution function of the aggregate interference that a primary user receives from multiple secondary transmitters. In particular, we consider heterogeneity in spatial distribution of secondary users such that there are several sites with densely populated secondary users in the whole area. The concentration of secondary users is modeled as an annulus sector with higher user density, which is termed *hot zone*. The mathematical framework presented in this paper can readily be adapted to various existing interference models. It is observed that the heterogenous user distribution has a considerable impact on the aggregate interference if the hot zone is near the primary receiver, while hot zones over a certain distance is well approximated by a homogeneous secondary user distribution. The aggregate interference also depends on the shape of the hot zone and the interference threshold imposed on the secondary users.**

***Index Terms*—Aggregate interference, secondary spectrum access, heterogeneous density, hot zone model**

I. Introduction

Radio spectrum has become a fundamental resource for wireless communication services with the dramatic increase in the wireless traffic. The traditional way to manage spectrum has been assigning fixed and exclusive frequency bands to different systems for long periods of time. Problem that has emerged under the fixed spectrum allocation is that there is not enough spectrum available for the increasing demand of wireless services. On the other hand, measurement results indicate that the allocated spectrum is being under-utilized [1]. A possible solution to utilize the allocated spectrum more efficiently is secondary spectrum access which is envisioned by cognitive radio [2]. This allows secondary users to access the spectrum that has already been assigned to primary users. The basic idea is that the secondary users should be capable of detecting opportunities for using the allocated spectrum without causing harmful interference to the primary users.

The protection of the primary users from interference is crucial for allowing the secondary users to access the spectrum. Accurate modeling of the aggregate interference is of importance in addressing the impact of multiple interfering secondary users. Mathematical models for the probability distribution of aggregate interference in secondary access have recently been investigated in [3]–[7]. The models in [3]–[6] considered an exclusion region of a circle with a fixed radius in order to offer the protection to the primary user from detrimental interference. Homogeneous Poisson point process was employed to spread the secondary users outside the exclusion region. In [7], secondary users are uniformly distributed in a large circle and the transmissions of secondary users are regulated by an individual interference threshold under the assumption that the secondary users know the propagation loss to the primary user. The model in [7] has been applied to practical secondary access scenarios where meteorological radars and aeronautical equipments are considered to be the potential primary users [8], [9].

In spite of the extensive research, the existing work has a limitation that the secondary users are assumed to be distributed in a homogeneous manner. In practical environments, it is usual that the secondary users have heterogeneity in spatial user distribution. For example, let us consider low power secondary users such as WLAN devices. The towns or cities will have higher concentration of secondary users than the rural areas. In this regard, the following research question should be addressed:

- How is the aggregate interference affected if heterogeneous densities of secondary users are considered?

In this paper, we aim at answering the question. Our main contribution is to propose a mathematical framework to derive the probability distribution function (pdf) of the aggregate interference generated by multiple secondary users but including heterogeneous density. We consider a situation that there are several zones with different levels of concentration in the whole area of a homogeneous or uniform background user density. Each zone is modeled as an annulus sector, which is termed *hot zone*. We obtain the pdf of the aggregate interference based on our previous work [7]. Note that the framework presented in this work can also be easily applied to other existing models, e.g. [3]–[6].

Our hot zone model has an advantage that various shapes of hot zones can be considered by adjusting the parameters of the annulus sector. We investigate the impact of shaping parameters of the hot zone such as the distance from the primary receiver, density difference from the background, and size of the hot zone.

The rest of the paper is organized as follows. Section II details the system model, the basic assumptions, and the hot zone model. Section III presents the analytic model for the aggregate interference. Section IV shows the numerical results obtained from mathematical analysis and simulation. Finally, we close with the conclusion in section V.

II. System Model

A. Modeling of secondary spectrum access

We consider a reference scenario that a primary receiver is interfered by multiple secondary transmitters spreading in a large area. Secondary access to 5.6 GHz radar spectrum can be regarded as a practical example of our scenario [8]. However, we do not take into account radar specific features such as the radar antenna pattern. Instead, we focus on developing a mathematical framework to reflect the heterogeneous user density. A mechanism to protect the primary user is considered that resembles dynamic frequency selection (DFS) specified in the IEEE 802.11h standard [10]. Each secondary user is allowed to transmit if its individual interference to the primary user is less than a certain threshold. We assume that the secondary users can have an accurate estimate of the propagation loss to the primary receiver. This assumption is reasonable if the primary user is the radar or the secondary users are assisted by a beacon signal from the primary receiver. Detailed description of the considered interference protection scheme can be found in [7].

Let ξ_j be the interference that the primary user will receive from the transmission of a secondary user j. Since the transmission of j is regulated by the interference protection mechanism, the user j stops transmission if it causes interference greater than the threshold I_{thr}. Thus, the actual interference from user j, namely I_j, is given by:

$$I_j = \begin{cases} \xi_j, & \text{if } \xi_j \le I_{thr}, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

The interference threshold I_{thr} is assumed to be predetermined. We consider an instantaneous moment that N secondary users in the system desire to transmit. Let us define I_a as the aggregate interference at the primary user caused from the secondary users. Then, I_a is computed as the sum of the individual interference of N secondary users.

$$I_a = \sum_{j=1}^{N} I_j. \tag{2}$$

B. Modeling of the heterogeneous secondary user density

Modeling of heterogeneity in the spatial distribution of secondary users should be done first in order to obtain the pdf of I_a. There would be various models to describe the heterogeneity depending on specific geographical locations and the types of primary and secondary systems. Note that we consider a large area consisting of cities, towns, and rural areas. It is usual that the population density of a city or town is much higher than that of rural area. It is also reasonable to consider that the number of secondary users is proportional to the population density. We further assume that the secondary users are homogeneously distributed within each city or town with it own density. Then, an annulus sector, namely hot zone, is employed to describe the crowded region as depicted in Fig. 1. For the rest of the paper, we focus on one hot zone for brevity and for better investigation of its impact on the aggregate interference. Extending our work to several hot zones with different densities is trivial.

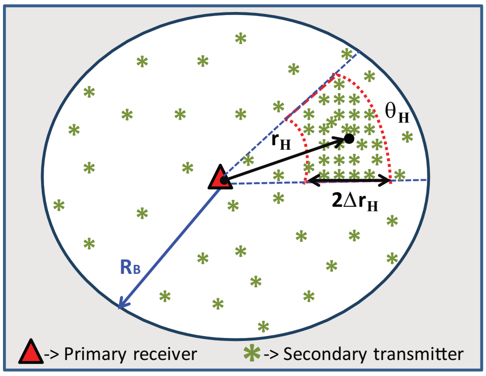

Fig. 1. Representation of the scenario with the hot zone model.

The use of annulus sector in the modeling of the hot zone is inspired by recent work in the field of secondary spectrum access. In [11], non-circular area is described by the aggregation of infinitesimal annulus sectors. In [12], the impact of secondary field size is investigated by assuming the annulus sector area. The annulus sector has an advantage that it can be molded to various shapes according to the three shaping parameters: r_H, Δ_{r_H}, and θ_H. As illustrated in Fig. 1, r_H is the distance between the center of the hot zone and the primary user, the length of the hot zone (depth) is $2\Delta_{r_H}$, and the central angle (width) is given by θ_H.

The distance, depth, and width characterize the hot zone along with the density of the zone. We assume that N_B secondary users are homogeneously distributed in a circle of radius R_B (background) and N_H secondary users are homogeneously distributed within a hot zone. Let ρ_B and ρ_H denote the densities of secondary users in the background and the hot zone, respectively ($\rho_H > \rho_B$). The primary receiver is located at the middle of the background circle.

The hot zone model is roughly demonstrated in near Stockholm area as illustrated in Fig. 2 where a meteorological radar is considered to be the primary receiver. The primary user is about 35 Km away from the Stockholm city where population density is significantly higher than surrounding areas.

III. Probability Distribution of Aggregate Interference

A. Distribution of the secondary users

Let us consider an annulus sector with inner radius R_1, outer radius R_2, and central angle θ_c. A circle can be regarded as a special case of the annulus sector. Thus, it can represent both the background area and the hot zone. For the case of the background area, the following parameters are applied: $R_1 = 0$, $R_2 = R_B$, and $\theta_c = 2\pi$. As for the hot zone, the parameters are $R_1 = r_H - \Delta_{r_H}$, $R_2 = r_H + \Delta_{r_H}$, and $\theta_c = \theta_H$. Note that the primary user is located at the origin of the background circle.

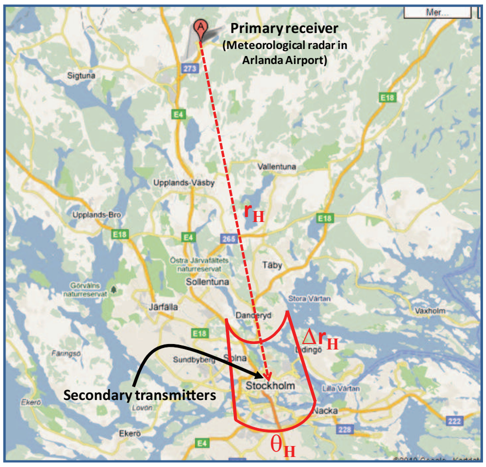

Fig. 2. An application of the hot zone model to Stockholm area. The map is captured from http://www.eniro.se/.

The location of an arbitrary secondary user j is denoted by (r_j, θ_j), where the random variable (RV) r_j is the distance from the primary user to the user j, and the RV θ_j is the its angle. Since the user j is assumed to have the uniform distribution, the pdf the location is given by:

$$f_{r_j,\theta_j}(y,\theta) = \frac{2y}{(R_2^2 - R_1^2)\theta}, \;\; R_1 < y \leq R_2, \;\; 0 \leq \theta \leq \theta_c. \quad (3)$$

By assuming the primary and secondary users have omni-directional antennas, (3) can be simplified as

$$f_{r_j}(y) = \frac{2y}{(R_2^2 - R_1^2)}, \;\; R_1 < y \leq R_2. \quad (4)$$

Notice that we defined (3) to model $N_B + N_H$ secondary users heterogeneously distributed within an area of radius R_B. However the same distribution can also model $N_B + N_H$ secondary users homogeneously distributed if the hot spot is not considered.

B. Interference from an arbitrary secondary user

Note that ξ_j is defined as the interference that the primary user will receive from the transmission of secondary user j. Then, ξ_j is given by:

$$\xi_j = G P_t L(r_j) X_j, \quad (5)$$

where P_t denotes the transmit power of the secondary user, X_j is a random variable modeling fading effect, and $L(r_j)$ is the distance depend path loss model defined as:

$$L(r_j) = C {r_j}^{-\alpha}, \quad (6)$$

where C and α are the path loss constant and exponent, respectively. The other gain and losses are accounted by G.

We consider a log-normal shadow fading for X_j and assume that X_j is independent of r_j. From the result of [7] and the pdf of the secondary user j in (4), it is straightforward to show that the pdf of ξ_j, $f_{\xi_j}(z)$, is derived as follows:

$$f_{\xi_j}(z) = h(z, R_2) - h(z, R_1), \quad (7)$$

where

$$h(z,y) = \Omega z^{\frac{-2}{\alpha}-1} \left[1 + \text{erf}\left(\frac{\ln\left(\frac{z}{G P_t L(y)}\right) - \frac{2\sigma_{X_j}^2}{\alpha}}{\sqrt{2\sigma_{X_j}^2}} \right) \right]. \quad (8)$$

In (8), $\sigma_{X_j}^{dB}$ denote the standard deviation of the shadowing in dB scale, and the constant Ω is given by:

$$\Omega = \frac{1}{(R_2^2 - R_1^2)\alpha} \left(\frac{1}{G P_t C} \right)^{\frac{-2}{\alpha}} \exp\left(2\sigma_{X_j}^2 / \alpha^2 \right). \quad (9)$$

In order to obtain the pdf of I_j from $f_{\xi_j}(z)$, we follow the steps in [7]. When I_{thr} is applied to user j, it stops transmission if ξ_j exceeds I_{thr} as depicted in (1). This means that a portion of secondary users have *zero* transmission power. That portion of users is given as $1 - F_{\xi_j}(I_{thr})$ where $F_{\xi_j}(.)$ denote de cumulative distribution function (CDF) of ξ_j. Thus, the pdf of I_j is as follows:

$$f_{I_j}(z) = \begin{cases} 1 - F_{\xi_j}(I_{thr}), & \text{if } z = 0 \\ f_{\xi_j}(z), & 0 < z \leq I_{thr} \\ 0, & \text{otherwise.} \end{cases} \quad (10)$$

The derived $f_{I_j}(z)$ can be directly applied to both the background and the hot zone secondary users by adjusting the parameters R_1 and R_2 as discussed in III-A.

C. pdf of the aggregate interference

Let us define I_j^B and I_i^H as the interference from the secondary user j in the background and the secondary user i in the hot zone, respectively. The pdf of I_j^B and I_i^H are given in (10). Since we assume that N_B secondary users are uniformly distributed within the background area and N_H uniform secondary users are within the hot zone, the total interference received at the primary user from the all secondary users I_a^T is computed as the sum of the aggregate interference from the users in the background I_a^B and the aggregate interference from the users in the hot zone I_a^H.

$$I_a^T = I_a^B + I_a^H = \sum_{j=1}^{N_B} I_j^B + \sum_{i=1}^{N_H} I_i^H, \quad (11)$$

We employ a cumulant-based approach to approximate the pdf of I_a^T. Note that the cumulants have an attractive property that the m^{th} cumulant of the sum of independent RVs is equal to the sum of the individual m^{th} cumulants [6]. Also, the first and second cumulants of a RV correspond to the mean and variance. Let $k_{I_a^B}(m)$ and $k_{I_a^H}(m)$ denote the m^{th} cumulant of I_a^B and I_a^H, respectively. Then,

$$\begin{aligned} k_{I_a^T}(m) &= k_{I_a^B}(m) + k_{I_a^H}(m) \quad (12) \\ &= \sum_{j=1}^{N_B} k_{I_j^B}(m) + \sum_{i=1}^{N_H} k_{I_i^H}(m). \end{aligned}$$

TABLE I
SIMULATION PARAMETER VALUES

Parameters	Values
1. Primary receiver (Meteorological radar)	
Frequency band [MHz]	5600
Antenna height [meter]	30
Transmission power [kW]	250
Bandwidth [MHz]	4
Antenna Gain [dBi]	40
2. Secondary transmitters (WLAN)	
Antenna height [meter]	1.5
Transmission power [P^t in W]	0.2
Bandwidth [MHz]	20
Antenna gain [dBi]	0
Radius of background area [R_B in km]	150
Shadowing standard deviation [$\sigma_{X_j}^{dB}$ in dB]	8

From the cumulants of the I_a^T, the pdf of I_a^T can be approximated as a known distribution by employing the method of moments. We use a log-normal distribution to approximate the pdf of I_a^T as in (13).

$$f_{I_a^T}(z) = \frac{1}{z\sqrt{2\pi\sigma_{I_a^T}^2}} \exp\left(\frac{\ln(z) - \mu_{I_a^T}}{2\sigma_{I_a^T}^2}\right). \quad (13)$$

The parameters $\mu_{I_a^T}$ and $\sigma_{I_a^T}^2$ of the PDF can be obtained from the first and second cumulant computations as:

$$k_{I_a^T}(1) = E[I_a^T] = \exp[\mu_{I_a^T} + \sigma_{I_a^T}^2/2], \quad (14)$$

$$k_{I_a^T}(2) = Var[I_a^T] = (\exp[\sigma_{I_a^T}^2] - 1)\exp[2\mu_{I_a^T} + \sigma_{I_a^T}^2]. \quad (15)$$

IV. NUMERICAL RESULTS

In this section, we present the results of the numerical experiments. The primary and secondary systems are modeled with the parameter values summarized in Table I. The basic propagation loss model used in this study is the C1-Suburban WINNER model which is proposed for 5GHz band by WINNER project in [13]. The parameters of the propagation model in dB scale are:

$$L(d) = 41.2036 + 3.5225\log_{10}(d[\text{meter}]). \quad (16)$$

Notice that in section III, we introduced a general mathematic framework to calculate the aggregate interference, nevertheless when it comes to the experimental part we need to define specific primary receiver and secondary users. Thus, we have set a meteorological radar as a primary receiver and WLANs as secondary transmitters. However, we assume that the primary receiver or the radar has an omnidirectional antenna. This is because we do not compute the aggregate interference by snapshots (when the radar is facing a portion of the total area), but the contrary. This assumption leads to obtain conservative results.

The CDF of the total aggregate interference I_a^T with heterogeneous secondary users density is presented in Fig. 3. We can notice a good match with the Monte Carlo simulation. Two different I_{thr} are presented, one with lower requirement of interference protection I_{thr}=-100dBm which create higher aggregate interference because 99.97% of the secondary users are allowed to transmit. The second with higher requirement of interference protection I_{thr}=-160dBm which induce less aggregate interference to the primary receiver because only 52.61% of the secondary users can transmit.

The next two experiment presented in Fig. 4 and Fig. 5 have the purpose to show the impact of the hot zone's parameters $(r_H,\ and\ \Delta_{r_H})$ on the aggregate interference. In Fig. 4 we vary the distance between the primary receiver and the center of the hot zone r_H, for the two mentioned interference protection requirement. It is observed that there is an intersection point or a value for r_H from where considering the distribution of secondary users as homogeneous (uniform) or heterogeneous lead to the same result in term of aggregate interference. This value of r_H depends on the I_{thr} requirements. Also, it is found that at some point of r_H and high I_{thr} considering homogeneous distribution of secondary transmitters overestimates the aggregate interference.

In Fig. 5 we experiment with different values for the depth of the hot zone but keeping r_H fixed. This shows how the distribution of dispersion in the hot zone, $r_H - \Delta_{r_H} < y \leq r_H + \Delta_{r_H}$, affects the aggregate interference when different I_{thr} are required. If the center of the hot zone is 80 Km away from the primary receiver, a variation in Δ_{r_H} is not affecting the aggregate interference. The contrary case happens when the center of the hot zone is only 30Km away from the primary user. Then, the aggregate interference is increasing or decreasing (according I_{thr}) with the increment of Δ_{r_H}. This suggests that the impact on the aggregate interference will depend on the given I_{thr}, r_H and Δ_{r_H}.

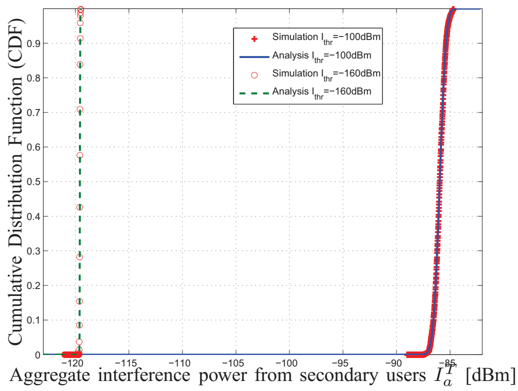

Fig. 3. CDF of I_a^T with heterogeneous users density, with one hot zone with parameters $I_{thr} = -80$dBm and -140dBm, r_H=15Km, Δ_{r_H}=5Km, θ_H=10°, ρ_B=1/Km^2, and ρ_H=20/Km^2.

V. CONCLUSION

In this paper we proposed a mathematical model to consider the heterogeneous distribution of secondary users and investigated the impact of this heterogeneity on the aggregate

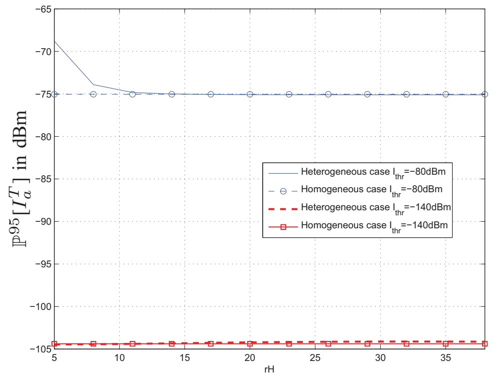

Fig. 4. Impact of the distance between the center of the hot zone and the primary user r_H in I_a^T, with the hot zone parameters Δ_{r_H}=5Km, $Area_H$=78.5Km^2, ρ_B=1/Km^2, and ρ_H=20/Km^2. The operator $\mathbb{P}^{95}[I_a^T]$ denotes the 95th percentile of I_a^T.

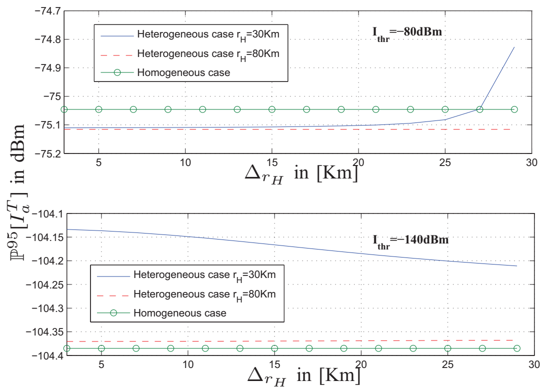

Fig. 5. Impact of the variation of Δ_{r_H} on I_a^T, when N_H=1,500 users, and ρ_B=1/Km^2. The operator $\mathbb{P}^{95}[I_a^T]$ denotes the 95th percentile of I_a^T.

interference to the primary user. We assume a large area which is a mixture of cities, towns, and rural areas. The regions with higher secondary user densities are termed hot zone, and modeled as annulus sectors on a circle of background user density. In order to obtain the pdf of the aggregate interference, first we derived the pdf of an arbitrary secondary user in either hot zone or background. Then, the method of moments is employed to approximate the pdf of the aggregate interference as a log-normal distribution. Comparison of the analytic pdf with Monte Carlo simulation shows a good agreement. This suggests that the derived pdf can obviate the need for complicated and time-consuming simulation. Our hot zone model also has an advantage that it can have a various shapes within the frame of the annulus sector. The impact of shaping parameters was evaluated.

Based on the numerical experiments, our findings are as follows: the aggregate interference is affected considerably by the heterogeneous density or concentration of secondary users when the hot zone is close to the primary receiver. On the other hand, if the hot zone is far away, we can simplify the model by not considering secondary user heterogeneity. The depth of hot zone, i.e. the vertical length of the annulus sector, and the interference threshold imposed on the secondary users are key parameters to determine the behavior of the aggregate interference.

Several important issues remain as further research. First, the hot zone model should be examined in practical geographical regions to know how well it reflects the real environment. Second, the impact of multiple hot zones should be investigated. Third, more practical primary and secondary system characteristics should be considered such as the directional antenna.

ACKNOWLEDGMENT

The research leading to these results has received partial funding from the European Union's Seventh Framework Programme FP7/2007-2013 under grant agreement n° 248303 (QUASAR). The authors also would like to acknowledge the VINNOVA project MODyS for providing partial funding.

REFERENCES

[1] D. Cabric, I. O'Donnell, M.-W. Chen, and R. Brodersen, "Spectrum Sharing Radios," *IEEE Circuits and Systems Magazine*, vol. 6, no. 2, pp. 30–45, 2006.

[2] J. Mitola III and G. Q. Maguire Jr, "Cognitive Radio: Making Software Radios More Personal," *IEEE Personal Communications*, vol. 6, no. 4, pp. 13–18, Aug. 1999.

[3] A. Ghasemi and E. S. Sousa, "Interference Aggregation in Spectrum-Sensing Cognitive Wireless Networks," *IEEE Journal of Selected Topics in Signal Processing*, vol. 2, no. 1, pp. 41–56, Feb. 2008.

[4] A. Ghasemi, "Interference Characteristics in Power-Controlled Cognitive Radio Networks," in *Proc. 5th International Conference on Cognitive Radio Oriented Wireless Networks and Communications (CrownCom)*, Cannes, Jun. 9-11 2010.

[5] X. Hong, C.-X. Wang, and J. Thompson, "Interference Modeling of Cognitive Radio Networks," in *Proc. 67th IEEE Vehicular Technology Conference (VTC)*, Singapore, May 11-14 2008, pp. 1851–1855.

[6] M. Aljuaid and H. Yanikomeroglu, "A Cumulant-Based Characterization of the Aggregate Interference Power in Wireless Networks," in *Proc. 71st IEEE Vehicular Technology Conference (VTC)*, Taipei, May 16-19 2010.

[7] K. W. Sung, M. Tercero, and J. Zander, "Aggregate Interference in Secondary Access with Interference Protection," *IEEE Communications Letters*, 2011, accepted for publication, temporariliy available at http://dl.dropbox.com/u/18150297/sung.pdf.

[8] M. Tercero, K. W. Sung, and J. Zander, "Impact of Aggregate Interference on Meteorological Radar from Secondary Users," in *Proc. IEEE WCNC*, Cancun, Mar. 28–31 2011.

[9] K. W. Sung, E. Obregon, and J. Zander, "On the Requirements of Secondary Access to 960-1215 MHz Aeronautical Spectrum," in *Proc. IEEE DySPAN*, Aachen, May 3–6 2011.

[10] IEEE Std 802.11h-2003, "Spectrum and Transmit Power Management Extensions in the 5 GHz band in Europe," Oct. 2003.

[11] A. Rabbachin, T. Quek, M. Win, and H. Shin, "Cognitive Network Interference," *IEEE Journal on Selected Areas in Communications*, vol. 29, no. 2, pp. 480–493, 2011.

[12] M. Aljuaid and H. Yanikomeroglu, "Impact of Secondary Users Field Size on Spectrum Sharing Opportunities," in *proc. IEEE WCNC*, Sydney, Apr. 18–21 2010.

[13] IST-4-027756 WINNER II, "D1.1.2 v1.2 WINNER II Channel Models," [Online]. Available: https://www.ist-winner.org/WINNER2-Deliverables/.