Aggregate Interference in Secondary Access with Interference Protection

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***Abstract*—This paper presents a derivation of the probability distribution function (pdf) of the aggregate interference in a secondary access network where multiple secondary users cause interference to a single primary user. The derivation considers a practical interference protection mechanism that the transmission of each secondary user is regulated by an interference threshold. Analytic pdf of the interference from a secondary user is obtained. Then, the distribution of the aggregate interference is approximated based on its cumulants. The derived pdf shows a good agreement with Monte Carlo simulation.**

***Index Terms*—Aggregate interference, secondary spectrum access, interference protection.**

I. Introduction

Radio spectrum has become a scarce resource with the increasing demand for wireless services. On the other hand, actual measurement results suggest that the spectrum is poorly utilized under the current regime of static spectrum allocation. This brought a paradigm of secondary spectrum access which allows secondary users to share the radio resource with primary users provided that the secondary users do not cause intrusive interference to the primary users. The protection of the primary users is a key requirement of the secondary access. In a situation where multiple secondary users interfere with a primary user at the same time, the accurate characterization of the aggregate interference is of crucial importance for the implementation of the secondary access.

The probability distribution of the aggregate interference by multiple secondary users has recently been investigated [1]–[3]. The most popular approach is to obtain the characteristic function of the aggregate interference in a Poisson field of independent interferers. Cumulants (or moments) are employed to approximate the probability density function (pdf) of the aggregate interference. To avoid detrimental interference, an exclusion (no-talk) region is introduced which is a disk of a fixed radius where secondary users are prevented from transmitting.

In this paper, we derive the pdf of the aggregate interference when propagation loss to the primary receiver is known to secondary users. We employ a distributed interference protection mechanism which resembles dynamic frequency selection (DFS) specified in IEEE 802.11h standard [4]. A secondary user is prohibited from transmission if its individual interference to the primary user exceeds a certain threshold. Our model differs from existing work in the sense that the no-talk region has an irregular shape in the presence of the fading. We obtain the pdf of interference from an arbitrary secondary user. Then, we exploit the cumulants of the pdf to approximate the distribution of the aggregate interference. The derived pdf is compared with Monte Carlo simulation.

The paper is organized as follows: In Section II, the system model is explained. The pdf of the aggregate interference is derived in Section III. The comparison with the simulation results are provided in Section IV. Finally, conclusions are drawn in Section V.

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II. System Model

A circle of radius R is assumed where a primary receiver is located at its origin. We consider a moment when N uniformly distributed secondary users in the circle desire to transmit. We assume that each secondary transmitter can accurately estimate the propagation loss to the primary receiver. This is a reasonable assumption when a radar is the primary user. The assumption also holds when the secondary users are assisted by a beacon signal from the primary receiver.

We consider a distributed interference protection mechanism: an interference threshold I_{thr} is applied to each secondary user so that its transmission is prohibited if it will generate interference higher than I_{thr}. The DFS scheme employed by WLAN devices in 5GHz radar spectrum [4] can be regarded as a practical implementation of our model.

Let us consider an arbitrary secondary user j whose distance from the primary user is denoted by a random variable (RV) r_j with the following pdf:

$$f_{r_j}(y) = \frac{2y}{R^2},\ 0 < y \le R. \tag{1}$$

We define ξ_j as the interference that the primary user would receive from the user j if it were to transmit. Then, ξ_j is given by

$$\xi_j = GP_tL(r_j)X_j, \tag{2}$$

where P_t denotes the transmit power of the secondary user, X_j is a RV modeling fading effect, and $L(r_j)$ is the distance-dependent path loss modeled as $L(r_j) = Cr_j{}^{-\alpha}$ where C is a constant and α is an exponent. The other gains and losses are accounted for by G.

Let I_j be the interference from the user j with the interference protection. The interference I_j is regulated such that

$$I_j = \begin{cases} \xi_j, & \xi_j \le I_{thr}, \\ 0, & \text{otherwise}. \end{cases} \tag{3}$$

Let $\mathbb{R}^2(O)$ denote the area that the user j is not allowed to transmit. In the presence of the fading, $\mathbb{R}^2(O)$ usually forms an irregular shape. On the other hand, the exclusion region model [1]–[3] assumes $\mathbb{R}^2(O)$ to be a disk of the radius r_o centered at the origin. The interference under the exclusion region model, $I_j(r_o)$, is given as

$$I_j(r_o) = \begin{cases} \xi_j, & r_j > r_o, \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

The exclusion region model represents a situation that the secondary users know the distance from the primary receiver, but are ignorant of the propagation loss, e.g. the use of geo-location database. Overall, it can be viewed that the models based on I_{thr} and r_o portray different levels of knowledge about the primary receiver.

III. Probability Distribution of Aggregate Interference

A. Interference from an arbitrary secondary user

We begin with the pdf of ξ_j. Note that ξ_j is a function of two RVs, r_j and X_j. Let us introduce a RV U_j denoting $GP_tL(r_j)$ so that we have $\xi_j = U_jX_j$. From (1), the pdf of U_j is given by

$$f_{U_j}(u) = \Psi u^{\frac{-2}{\alpha}-1},\ Q \leq u < \infty, \tag{5}$$

where

$$\Psi = \frac{2}{R^2\alpha}\left(\frac{1}{GP_tC}\right)^{\frac{-2}{\alpha}}, \tag{6}$$

$$Q = GP_tL(R). \tag{7}$$

As for the fading effect, we consider shadow fading such that X_j follows a log-normal distribution. By denoting the standard deviation of the shadowing by $\sigma_{X_j}^{dB}$ in dB scale, we have

$$f_{X_j}(x) = \frac{1}{x\sqrt{2\pi\sigma_{X_j}^2}}\exp\left[\frac{-(\ln x)^2}{2\sigma_{X_j}^2}\right],\ 0 < x < \infty, \tag{8}$$

where $\sigma_{X_j} = \sigma_{X_j}^{dB}\ln(10)/10$.

We assume the shadow fading does not depend on the location of the secondary user, i.e. X_j and U_j are independent of each other. Then, the pdf of ξ_j can be expressed by the following formula [5]:

$$f_{\xi_j}(z) = \int \frac{1}{|x|} f_{X_j}(x) f_{U_j}\left(\frac{z}{x}\right) dx. \tag{9}$$

The range of x is obtained from (5). Thus, we have

$$f_{\xi_j}(z) = \int_0^{z/Q} \frac{\Psi}{x^2\sqrt{2\pi\sigma_{X_j}^2}}\exp\left[\frac{-(\ln x)^2}{2\sigma_{X_j}^2}\right]\left(\frac{z}{x}\right)^{\frac{-2}{\alpha}-1} dx. \tag{10}$$

With a few mathematical manipulations, (10) can be simplified by using the Gaussian error function:

$$f_{\xi_j}(z) = \Omega z^{\frac{-2}{\alpha}-1}\left[1 + \text{erf}\left(\frac{\ln(z/Q) - 2\sigma_{X_j}^2/\alpha}{\sqrt{2\sigma_{X_j}^2}}\right)\right], \tag{11}$$

where

$$\Omega = \frac{1}{2}\Psi\exp\left[2\sigma_{X_j}^2/\alpha^2\right]. \tag{12}$$

When I_{thr} is applied to the user j, the transmission is not allowed if $\xi_j > I_{thr}$. This means there will be a portion of secondary users who have the *zero* transmission power. That portion is given by $1 - F_{\xi_j}(I_{thr})$, where $F_Y(\cdot)$ denotes the cumulative distribution function (CDF) of a RV Y. Thus, the pdf of I_j is as follows:

$$f_{I_j}(z) = \begin{cases} 1 - F_{\xi_j}(I_{thr}), & z = 0, \\ f_{\xi_j}(z), & 0 < z \leq I_{thr}, \\ 0, & \text{otherwise.} \end{cases} \tag{13}$$

B. Aggregate interference

The aggregate interference from N secondary users is denoted by I_a, and given by

$$I_a = \sum_{j=1}^{N} I_j. \tag{14}$$

We employ a cumulant-based approach to approximate the pdf of I_a. Cumulants have an attractive property that the i^{th} cumulant of the sum of independent RVs is equal to the sum of the individual i^{th} cumulants [3]. While Edgeworth expansion and shifted log-normal distribution have been proposed for the approximation in the exclusion region model [1], we found that a simple log-normal distribution well describes I_a under the considered interference protection.

Let $\kappa_a(i)$ be the i^{th} cumulant of I_a. Then,

$$\kappa_a(i) = \sum_{j=1}^{N} \kappa_j(i), \tag{15}$$

where $\kappa_j(i)$ is the i^{th} cumulant of I_j which can be easily computed from (13). By using the first two cumulants of I_a, the pdf of I_a is approximated by the following log-normal distribution:

$$f_{I_a}(z) = \frac{1}{z\sqrt{2\pi\sigma_{I_a}^2}}\exp\left[\frac{-(\ln z - \mu_{I_a})^2}{2\sigma_{I_a}^2}\right], \tag{16}$$

where the parameters μ_{I_a} and $\sigma_{I_a}^2$ are obtained from the following equations:

$$\kappa_a(1) = \exp\left[\mu_{I_a} + \sigma_{I_a}^2/2\right], \tag{17}$$

$$\kappa_a(2) = \left(\exp\left[\sigma_{I_a}^2\right] - 1\right)\exp\left[2\mu_{I_a} + \sigma_{I_a}^2\right]. \tag{18}$$

If there are large number of secondary users, central limit theorem (CLT) can also be applied such that I_a is approximated by a Gaussian distribution with the mean and variance of $\kappa_a(1)$ and $\kappa_a(2)$, respectively.

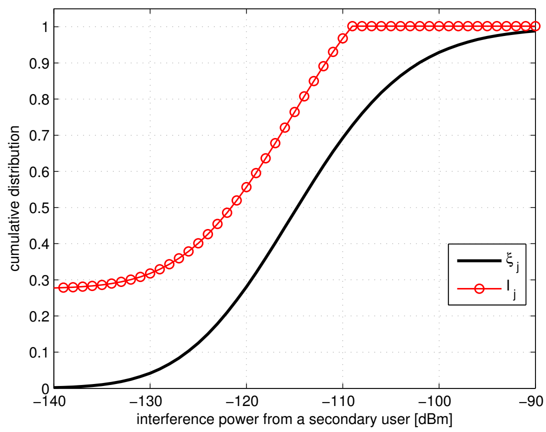

Fig. 1. CDF of ξ_j and I_j

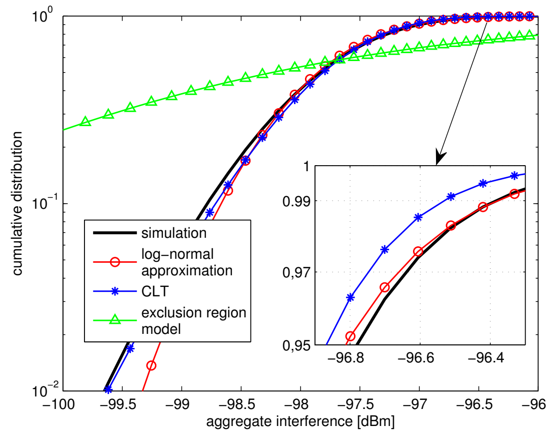

Fig. 2. CDF of I_a with $N = 79$ (0.01 user/km^2); $r_o = 40.9$ km

IV. Numerical Result

We consider a reference scenario that mobile devices share radar spectrum in 5.6 GHz. The following parameters are chosen from [6]: $R = 50$ km, $P_t = 20$ dBm, $\sigma_{X_j}^{dB} = 8$ dB, and $I_{thr} = -109$ dBm. We also employ the radar antenna gain of 40 dBi and on-tune rejection of 7 dB for G. WINNER D1 model [7] is used for $L(r_j)$.

Fig. 1 shows the CDF of ξ_j and I_j. It is observed that about 27% of secondary users are not allowed to transmit due to the I_{thr} threshold. Thus, I_j has a truncated distribution of ξ_j. Also, note that $F_{I_j}(I_{thr}) = 1$.

In Fig. 2, the CDF of I_a is compared with the Monte Carlo simulation when the secondary users are sparsely distributed. Both the log-normal and Gaussian approximations give good matches with the simulation result, while the log-normal approximation provides better performance in the tail part of the CDF. The accuracy in the tail regime is more important since the probability of harmful interference is usually the main concern of the secondary access. Fig. 3 depicts the CDF of I_a with more secondary users. The gap between the log-normal approximation and the CLT decreases as the number of secondary users increases.

The interference with the exclusion region is also presented in Fig. 2 and Fig. 3 where r_o is chosen to minimize the gap (mean squared error in CDF) with the result by I_{thr}. Significantly higher deviation is observed in both figures when the circular exclusion region is applied. This suggests that the knowledge of primary receiver plays an important role in regulating the aggregate interference.

V. Conclusion

We derived the pdf of aggregate interference in a secondary access when the propagation loss to the primary receiver is known to the secondary users. A distributed interference protection mechanism was considered such that the transmission of each secondary user is regulated by an interference threshold. It represents practical schemes such as DFS in IEEE 802.11h specification. We obtained an analytic pdf of the interference coming from an arbitrary secondary user. Then, the aggregate interference is approximated as a log-normal distribution. The approximation shows a good agreement with the simulation result particularly in the tail region of the distribution. Thus, the pdf derived in this paper can obviate time-consuming simulations in estimating the probability of harmful interference generated by multiple secondary users. Impacts of fast fading and non-uniform distribution of secondary users remain as interesting further studies.

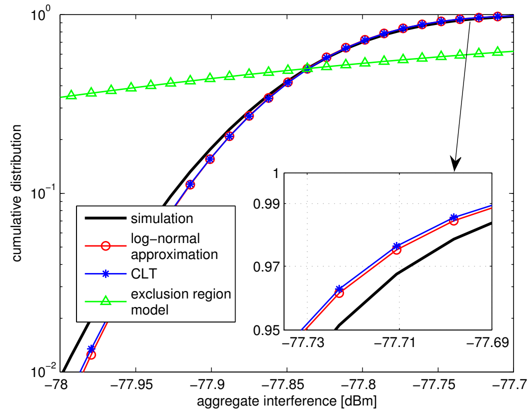

Fig. 3. CDF of I_a with $N = 7854$ (1 user/km^2); $r_o = 42.1$ km

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