Operator Competition with Asymmetric Strategies in Shared Spectrum

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***Abstract*—As the regulation in wireless communications is moving toward a more flexible and efficient way of managing radio spectrum, it is envisaged that multiple small-sized cellular networks owned by different operators, e.g., facility owners or local operators, will operate in close vicinity on shared spectrum. In this environment, the networks may compete for their own utilities in a selfish manner with giving harmful inter-network interference to competitors. In practice, it is not so unusual that each operator has different fairness criteria or quality of service (QoS) strategies by employing distinct objective functions from competitors. Particularly, we in this paper study power control competition between two networks with the sum of rates (SR) and the minimum rate (MR) as their objective functions, respectively. By exploring Nash equilibria, we identify that the MR network benefits from the objective asymmetry thanks to the adaptability of its competitor, i.e., no constraint in the SR objective. On the other hand, the SR network takes disadvantage due to the fairness requirement reflected in the MR objective of its competitor. However, such asymmetry effects in competition becomes negligible with marginal network separation, e.g., indoor deployment in adjacent buildings. Additionally, we identify cooperation potential with the proper choice of a common objective function although the asymmetric objectives are difficult to be aligned.**

I. Introduction

A migration toward flexible spectrum management becomes eminent to alleviate spectrum scarcity for a rapidly growing mobile broadband access. Accordingly, regulatory bodies recently consider the various ways of spectrum deregulation such as spectrum trading/borrowing or temporal/geographical sharing [1], [2]. From this paradigm shift, various novel network operation models are envisaged [3]. An example is shown in Fig. 1 where small-sized cellular networks managed by different operators each provide services in shared spectrum at adjacent locations. This will create business opportunities to new entrants by lowering the barrier of expensive spectrum cost. However, it induces a new interference environment where each network has interference from its own base stations (BSs) as well as from other BSs in other networks. These networks may compete with others to maximize their own utilities in a selfish manner.

In real business scenarios, it may not be uncommon that operators have different quality of service (QoS) or fairness strategies for differentiated service from their competitors, i.e., competitive advantages [4]. In a network design perspective, this asymmetric strategies may bring the objective function difference between their networks. Unlike the symmetry case where objective functions aim at the same type of QoS or fairness criteria, the objective asymmetry may or may not be beneficial to competing networks since conflicts in spectrum utilization can be alleviated or worsen due to the antithetic optimization purpose. At the same time, the disaccord of objectives may be challenge for operators to agree on a common objective function for cooperation. Thus, it is greatly appealing investigating how the objective asymmetry affects on the network operation in the shared spectrum.

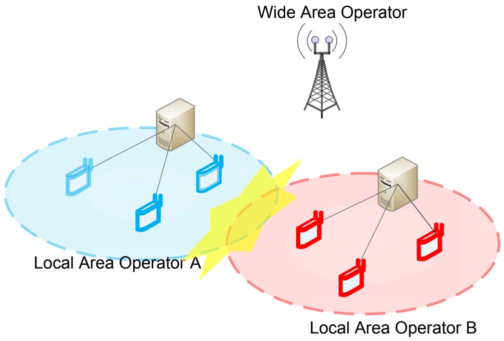


Fig. 1. An example of spectrum sharing scenarios where local area operators A and B serve indoor coverage or capacity on shared spectrum while a conventional macro-cellular operator still provides outdoor seamless coverage in exclusive spectrum.

Most of existing studies on the multi-operator operation have considered price competition for user or spectrum acquisition without any interference between operators [5], [6]. For instance, authors in [5] investigated access competition between heterogeneous networks in non-overlapping channels for maximizing their revenues. [6] considered dynamic spectrum sharing among operators in the form of spectrum resource exchange without harmful interference. Relatively few studies researched an interference problem between competing wireless operators in shared spectrum [7]–[10]. A coverage competition problem has been addressed for attracting freely roaming users [7], [8]. Access probability competition between WLAN networks has been explored in [9]. In our previous work [10], we investigated cooperation and competition in network-wide power control with a symmetric objective function, i.e., the sum of rates, in terms of the network size and deployments. However, those studies implicitly assumed that competing operators provide the similar level of QoS. To our best knowledge, operator competition with asymmetric QoS strategies in shared spectrum has not been investigated yet.

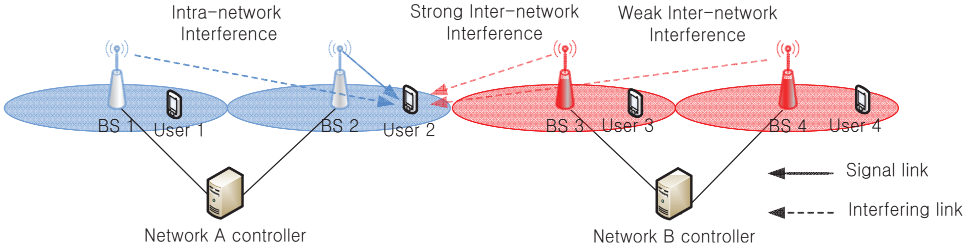


Fig. 2. Example interference environment of two networks (User 2 perspective).

In this paper, we study the operator competition on shared spectrum in terms of downlink power control. For the asymmetric strategies, we consider two networks aiming different objective functions: the sum of rates and the minimum rate. We aim to answer following research questions:

- How do different objective functions affect the competition between networks?
- How does the network performance compare with a symmetric situation?

In order to obtain insights into the basic principles, we investigate the objective asymmetry effects based on a simplified model. This is analyzed according to the practical network separation scenarios. We also examine the possibility of cooperative power control with a common objective function. The rest of the paper is outlined as follows. Section II provides a system model. Section III and Section IV state a simulation methodology and evaluation results, respectively. Finally, Section IV concludes this study with future work.

II. SYSTEM MODEL

A. Topology Model

Since the co-channel operation of networks is presumed, the co-deployments in a fully overlapping geographical region is not realistic as long as operators are rational to avoid excessive interference from other networks. Likewise, it is also a too pessimistic scenario that all BSs in each network are placed along with other BSs in the other network. Instead, we consider that each network is deployed at one vicinity of other networks' service area. As one of practical settings, this can be interpreted as neighboring buildings with BSs installed by different building owners, e.g., hotels, shopping malls, or enterprizes.

Under these assumptions, let us consider two independent networks, denoted by a set $\mathcal{M} = \{A, B\}$, which are individually managed by each operator. Each network i has two BSs which belong to its BS set $\mathcal{B}_i$. Note that BS 1 and 2 belong to network A while BS 3 and 4 are operated by network B. All BSs in two networks are equally spaced along an one-dimensional geometry. As one of practical realizations, this can represent a linear deployment along the corridor in buildings. The closest BSs in two networks are separated at least with inter-BS distance in a given network. Also, BSs in a given network are connected via a network controller so that the transmit powers of BSs are internally coordinated. Here, each controller is presumed to know the complete information of channel gains between its BSs and users based on local measurement reports.

At a given time, one user per BS arrives along one-dimensional geometry within the its cell radius R, following the linear distribution with a probability density function:

$$f(r) = \frac{|r|}{R^2}, \ |r| \leq R, \tag{1}$$

where $\int_{-R}^{R} f(r) = 1$ with r representing a relative location from its serving BS. This can correspond to the projection of two-dimensional uniform distribution into one-dimensional geometry for the analysis simplicity. It also presumes a fully loaded system under equal time-sharing among users in a given BS. For the convenience of notations, we assume that user j associates with BS j. Note that we restrict ourselves to two operators and the linear topology in order to provide an insight into the basic principles of the multi-operator competition.

Let us consider a downlink transmission. Then, each user is exposed in two interference environments as shown in Fig. 2. For instance, User 2 served by BS 2 is affected not only by interference from BS 1 belonging to the same network, i.e., *intra-network interference*, but also in the range of interference from BS 3 and 4 in the other network, which is referred to as *inter-network interference*.

B. Asymmetric Operator Strategies

Let us denote a channel gain between BS j and its user j by g_j. Signal to interference and noise ratio (SINR) received at user j, referred to as γ_j, can be obtained by

$$\gamma_j = \frac{g_j p_j}{I_j^{intra} + I_j^{inter} + N_o}, \tag{2}$$

where p_j and N_o represent the transmit power of BS j and constant noise power, respectively. Note that I_j^{intra} and I_j^{inter} are intra-network interference and aggregate inter-network interference received at user j, respectively. For a given γ_j, we simply compute an achievable rate r_j from Shannon formula given by

$$r_j = \log_2(1 + \gamma_j) \ \text{(bps/Hz)}. \tag{3}$$

At a given user realization, each network i wants to maximize its own objective function U_i. Due to differentiated target services, U_A and U_B can be a different type, i.e., *asymmetry strategies*. As extreme representatives, we consider two types of objective functions: the sum of its users' rates (SR) and the minimum rate among its users (MR). SR objective can correspond to the data-like service without any QoS or fairness constraint while MR objective aims to protect low quality users rather than overall capacity. In particular, we assume that network A and B aim to maximize SR and MR, respectively. Then, the objective functions of two networks can be computed from:

$$U_A = \sum_{j \in \mathcal{B}_A} r_j \text{ and } U_B = \min_{j \in \mathcal{B}_B} \{r_j\},$$

respectively.

C. Network Power Control Model

With the asymmetry strategies in two networks, they may compete selfishly or cooperate for mutual benefits. In this subsection, we model the competition and cooperation in the downlink power control perspective.

1) Competitive Network Power Control: Without any regulation constraints or operators agreement, each network may compete for maximizing only its objective regardless of how much interference it harms to the other network. In a practical system, network i may adapt its transmit power vector $\mathbf{p}_i$ only according to monitored interference resulting from other network. Note that each network is still capable of internally coordinating the transmit powers of two BSs so as to maximize its objective. Then, the other network reconsiders its power vector since the network i changes $\mathbf{p}_i$. Such interactive adaptation process between two networks will be continued until they reach into the equilibrium or the monitoring phase ends. This can be analyzed by using a game model [11]. Thus, we formulate this as a strategic game denoted by $\mathcal{G}$. Let us define the feasible set of $\mathbf{p}_i$ as $\Omega_i = \prod_{j \in \mathcal{B}_i} \mathcal{P}_j$ where $\mathcal{P}_j = \{p_j \mid 0 \leq p_j \leq p_{max}\}$ and Π stands for Cartesian product. For practicability, we here assume that the transmit power p_j in all BSs is limited to the maximum allowed power p_{max}. Since $\mathcal{G}$ is composed of triplets, the competitive power control game of two networks is described as follows:

- Player: $\mathcal{M}$
- Action space: $\mathbf{p}_i \in \Omega_i$ for $i \in \mathcal{M}$
- Payoff function: $U_i(\mathbf{p}_i, \mathbf{p}_{-i})$ for $i \in \mathcal{M}$.

Note that $\mathbf{p}_{-i}$ represents the transmit power vector of the other network aside from network i. Since network A and B employ asymmetry strategies, we refer to this situation as *asymmetry competition*. For comparison purpose, *symmetry competition* is also regarded as a reference case where two networks attempt to maximize the same type of objective functions. In this case, we have two reference cases, i.e., SR or MR symmetry competition.

2) Cooperative Network Power Control: Two networks may want to cooperate by agreeing a common objective function U^{coop} as long as mutual benefits is identified. However, it is so difficult to find a proper U^{coop} which can improve both U_A and U_B since they fundamentally have distinct criteria. Instead, we examine the potential of using a weighted linear sum of U_A and U_B:

$$U^{coop} = w\frac{U_A}{N_A} + (1-w)U_B,$$

where $w \in [0,1]$ and $N_A = |\mathcal{B}_A|$. Note that we normalize U_A with N_A to make it comparable with U_B since SR objective aggregates multiple users' rates.

In order to maximize U^{coop}, two networks may adjust transmit powers in a decentralized manner by employing an penalty function due to architectural simplicity or considerable extra cost for an additional inter-network coordinator [12]. In this case, they follow pre-agreed protocols or behave based on a statistical information of the other network. On the other hand, they cooperate in a centralized manner via the explicit inter-network coordinator. As the upper bound performance of the cooperation, we consider the centralized case since the local information given in the distributed approach is more limited than the information available to the central controller. Then, this can be seen as a conventional centralized wireless system owned by single operator with respect to implementation and network behavior. A difference from the single operator case is that each operator is still interested in its individual performance, i.e., U_A or U_B, even though the unified network behaves to maximize U^{coop}. Transmit power vector $\mathbf{p}$ of all four BSs can be defined in a feasible power vector space $\Omega = \prod_{j \in \mathcal{B}_A \cup \mathcal{B}_B} \mathcal{P}_j$. Then, we formulate the corresponding cooperative power control problem as follows:

$$\begin{aligned} &\text{maximize} \quad U^{coop}(\mathbf{p}), \\ &\text{subject to} \quad \mathbf{p} \in \Omega. \end{aligned}$$

D. Performance Metric

In order to investigate how the asymmetry strategies in two networks affect on their performances, we measure U_i as each operator's utility according to different situations. Let us differentiate U_i in the situations of asymmetry (symmetry) competition and cooperation by marking the superscript $asy(sym)$ and $coop$, respectively. To evaluate how the performance in the competition is different from the symmetry case, we also define the average performance difference of network i as

$$\Delta_i^{\text{diff}} = \frac{E[U_i^{asy}] - E[U_i^{sym}]}{E[U_i^{sym}]} \times 100\ (\%).$$

Note that network A and B target to maximize MR and SR, respectively. Accordingly, U_A^{sym} means the SR of network A in the SR symmetry competition, and U_B^{sym} represents the MR of network B in the MR symmetry competition. Similarly, we can measure the cooperation gain or loss in an average sense by comparing the performance in the asymmetry competition. For this, we define average cooperation gain of network i at a given w as

$$\Delta_i^{\text{coop}} = \frac{E[U_i^{coop}] - E[U_i^{asy}]}{E[U_i^{asy}]} \times 100\ (\%).$$

III. SIMULATION METHODOLOGY

A. *Nash Equilibria in Competition*

In the case of the competitive power control, we analyze pure strategy Nash equilibria (NEs) as a solution concept. By definition, it is the action profile that no player can yield a better payoff from unilateral deviation. The NEs can be determined by finding the intersections of the best response curves of two players [11]. Also, the closed form solution for the best response function is generally unknown when the payoff function is non-convex for a given inter-network interference. For brevity, our approach is adopting the numerical approximation by quantizing transmit power instead of analytically finding NEs. In a practical system, power control algorithms may generate different outcomes depending on implementations. While remaining this issue out of the scope, we apply an exhaustive search to obtain all NEs in the quantized action space for a given user realization. It is also noteworthy that the pure NE does not necessarily exist in the quantized action space and there might be more than one NE depending on the payoff matrix. Accordingly, we randomly select $\mathbf{p}_i$ to give non-zero payoff if no NE exists and randomly choose one NE if multiple NEs are identified[1]. This reflects the finite iteration and the random initialization of transmit powers in a real system.

B. *Social Optimum in Cooperation*

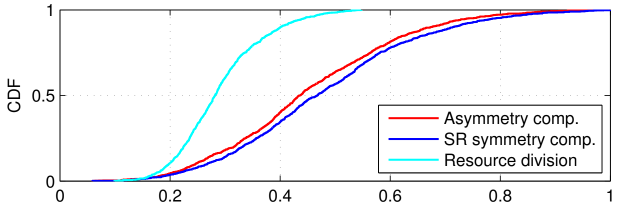
The cooperative network power control aims to obtain the globally optimal solutions. However, solving the cooperative network power control problem is also inherently challenging due to the lack of convexity in U^{coop} for a given w [13] even though an centralized algorithm is applied. Thus, we again approximate the optimal power allocation by solving a discrete optimization problem based on discrete power levels. Then, we find the optimal solution of cooperative networks by an exhaustive search as an upper bound of performance.

C. *Simulation Parameters*

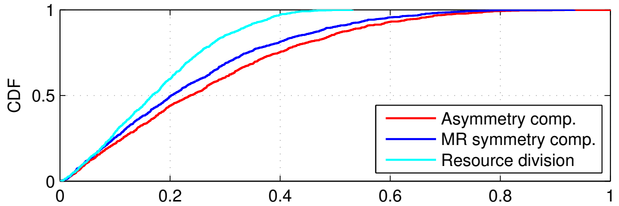
We perform Monte-Carlo simulation with 1000 randomly generated user locations and channel realizations. All relevant simulation parameters are followed. The cell radius is set as 100 m so that distance between the closest BSs in the neighboring networks are set as 200 m. Path loss is described as $PL = 127 + 30\log_{10}(d)$ (dB) where d accounts for distance (km) from a transmitter [14]. Additionally, log-normal shadow fading with standard deviation $\sigma = 6$ dB is included. $\mathcal{P}$ is quantized into a finite set $\{-\infty, -3, 8.5, 20\}$ dBm. The noise power N_o is assumed to be -95 dBm.

IV. NUMERICAL RESULTS

A. *Effects of Asymmetry Competition*



(a) Normalized U_A w/ SR



(b) Normalized U_B w/ MR

Fig. 3. Performance difference of the asymmetry competition from the symmetry case.

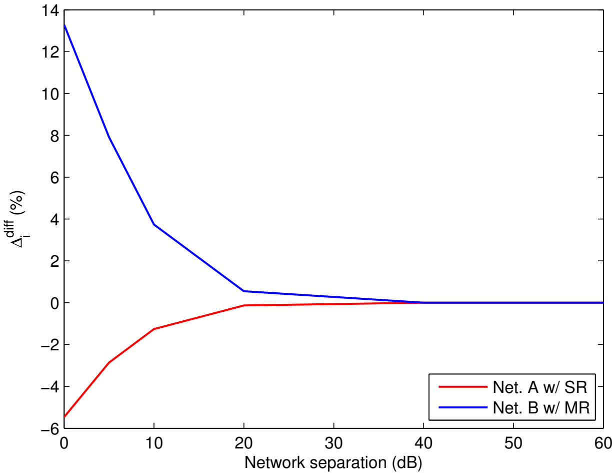


Fig. 4. Average performance difference according to network separation.

Fig. 3-(a) and (b) provide the cumulative distribution function (CDF) of normalized U_A and U_B as the result of asymmetry competition, respectively. Note that resource division represents that two networks equally divide the spectrum so that the rate of each user is simply reduced by half. This case sketches a conventional static spectrum allocation between networks or an orthogonal spectrum usage agreement between networks [2]. Two sub-figures illustrate how different the average utilities of two operators is from the symmetry case. Interestingly, Fig. 3-(a) shows that the performance of network A with SR objective worsens than SR symmetry competition. In terms of network B with MR objective, this result becomes conversed. As shown in Fig. 3-(b), network B has better performance than MR symmetry competition.

This can be interpreted from the nature of SR and MR objective. Since MR objective function maximizes the minimum rate out of two users, it cannot abandon up any users. On the other hand, SR objective may give up either one of two users for reducing intra-interference as long as it benefits the sum of two users' rates. In our topology model, network B

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[1] With any channel realization, the existence and the uniqueness of NE are not guaranteed due to the lack of quasi-concavity in our payoff function. By intensive experiments, we identify the considered system model mainly yields the unique NE, and thus the impact of randomly selected outcome is negligible to the performance.

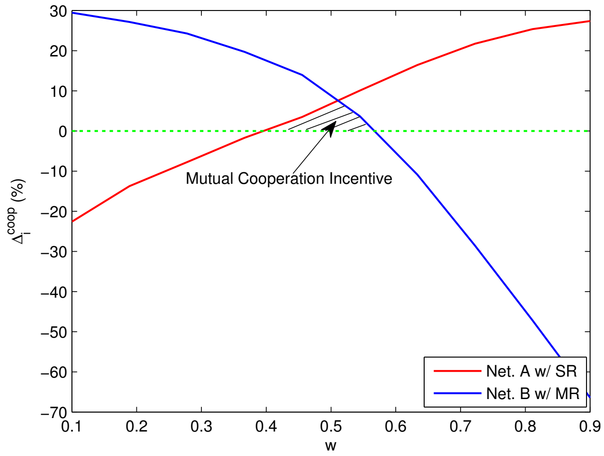

Fig. 5. Cooperation possibility for mutual benefits with proper weight selection.

attempts to protect user 3 due to the tight fairness condition with causing stronger inter-network interference to network A. Conversely, network A is more likely to deactivate BS 2 since user 2 experiences strong inter-network interference from network B thanks to the flexibility of SR objective, i.e., no fairness constraint. In return, this creates less inter-network interference to network B.

One of realistic assumptions in multi-operator deployments is that each network somehow is separated, e.g., geographically or with high wall penetration. By simply adding constant path loss in channel gains across two networks, i.e., *network separation*, we can evaluate this effect. Fig 4 plots Δ_i^{diff} according to the network separation. As the network separation increases, the interference coupling effect between neighboring networks is drastically reduced to make the effects of the objective asymmetry become marginal.

B. Possibility of Cooperation

In order for operators to decide whether or not to cooperate, they first need to inspect its potential possibility. Fig. 5 plots Δ_i^{coop} by varying w. We can identify that the considered cooperative power control does not always give positive gain to both operators at the same time. In most of the range of w, either one of operators has negative cooperation gain. However, we recognize that there at least exists a proper w which can simultaneously improve utilities of both operators. Although the asymmetry strategies is difficult to be agreed, this implies that two operators can still enhance their utilities with adequate cooperation.

V. CONCLUSION

When small-sized cellular networks owned by different local operators are deployed in interference range on shared spectrum, we addressed a competitive power control problem between operators with asymmetry strategies. The asymmetry strategies was modeled as different objective functions in competing networks. As representative cases, we considered two competing networks: one aiming to maximize the sum of rates (SR) of its users, and the other one maximizing the minimum rate (MR) of its users. We modeled this competition as a strategic game. By analyzing the Nash equilibria of the game, we identified that the MR network benefits thanks to the flexible nature of SR objective in its competitor. On the other hand, the SR network experienced a disadvantage due to the protective strategy of its competitor. We also observed that the performance difference from a symmetry case quickly vanishes as networks are reasonably separated, e.g., deployment in neighboring buildings. Furthermore, we found that two operators can have the mutual performance improvement with the proper level of cooperation. The results were obtained using a specific network topology and equal maximum output power constraints in two networks. Thus, a more general multiple networks scenario needs to be studied as a future work.

ACKNOWLEDGMENT

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