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Title:

An Economic Cost Model for Network Deployment and Spectrum in Wireless Networks

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ABSTRACT

We describe the basic economic theory of cost model for network deployment and spectrum in wireless networks. In particular, we develop a production function for wireless networks. With this production function model, we explore the technical rate of substitution and the elasticity of substitution in the production function for wireless network and find its insight for wireless network. Finally, we compare the engineering value of spectrum and economic value of spectrum.

I. INTRODUCTION

Wireless systems have been evolving to support nonrealtime data service. However, as the data rates for the service increase, the number of users that one basestation can support become smaller. Therefore, the number of basestation per service area is needed to expand, which leads to a high degree of base-station density. The increased base-station density, in turn, results in a high infrastructure cost, which will be directly transferred to a high service cost. Another bottleneck is the radio spectrum available for wireless data service and systems. It is extremely scarce, while demand for this service is growing at a rapid pace. Base-station density and the spectrum are therefore of primary concern in the design of future wireless network deployment.

An initial work [1] of Zander deals with these issues, the cost structure of future wireless access, basestation density and spectrum cost problem. It introduces some simple models to analyze the cost structure of both infrastructure costs and spectrum costs. In this paper, we develop the cost analysis of both infrastructure costs and spectrum costs of [1] using firm theory in microeconomics. By using production function in firm theory, we try to analyze the cost structure of wireless network deployment. The theory of microeconomics is widely used to analyze communication systems but researches have focused on consumer theory such as the utility maximization of each user. In terms of wireless operator's revenue maximization, production function, cost minimization, and the technical rate of substitution in firm theory are much more important and interesting for wireless network operator.

After we formulate the production function of wireless network operator using area spectral efficiency (ASE) [2], we will address the issues of firm theory such as the technical rate of substitution and cost minimization in this paper. Also, by using the technical rate of substitution (TRS), we derive the economic value of spectrum and compare with the engineering value of spectrum.

The rest of this paper is structured as follows. Section II presents a system model for wireless networks and defines the production function for deployment and spectrum in wireless networks. Section III investigates the characteristic of economic cost function. Especially it deals with explore the technical rate of substitution and the elasticity of substitution in the production function for wireless network. Section VI introduces a cost minimization model based on this production function. Section V compares the engineering value of spectrum and economic value of spectrum. Section VI draws some conclusions.

II. SYSTEM MODEL

In this section we introduce the concept of area spectral efficiency (ASE) for fully loaded systems in which the cell's resource (service channels) are fully used and the number of interferers is constant [1]. The ASE of a cell is defined as the sum of the maximum bit rates/Hz/unit area supported by a cell's BS. When the cell radius is R, the area covered by one of this cells is $\pi\left(R\right)^2$. The ASE, $A_e[bits/s/Hz/m^2]$, is therefore approximated by

$$A_e = \frac{\sum_{k=1}^{N_s} C_k}{\pi W_{bs}(R)^2} \qquad (1)$$

where N_s is the total number of active serviced channels per cell, C_K [bits/s] is the maximum data rate of the K th user, and W_{bs} [Hz] is the total allocated bandwidth per cell. We define the maximum rate C_k to be the Shannon capacity of the k th user in the cell, which depends on γ_k , the receives SINR of that user, and W_k , the bandwidth allocated to that user. The Shannon capacity formula assumes that the interference has Gaussian characteristics.

When we assume that there are the coverage service area, A_{svr} and total system bandwidth, W_{sys} , the total system capacity, R_{tot} , is as below by [1],

$$R_{tot} \approx \frac{\sum_{k=1}^{N_s} C_k}{W_{bs}(\pi R^2)} A_{srv} W_{sys}$$

when A_{bs} is the cell coverage of a BS, $\left(\pi R^2\right)$,

$$R_{tot} \approx \frac{\sum_{k=1}^{N_s} C_k}{W_{bs} A_{bs}} A_{srv} W_{sys} = \frac{\sum_{k=1}^{N_s} C_k}{w_{BS} A_{bs}} A_{srv} \cdot W_{sys} = \frac{\sum_{k=1}^{N_s} C_k}{w_{BS}} \cdot \left(\frac{A_{srv}}{A_{bs}}\right) \cdot W_{sys} .$$

Therefore, N_{bs} is $\frac{A_{svr}}{A_{bs}}$,

$$R_{tot} \approx \frac{\sum_{k=1}^{N_s} C_k}{w_{BS}} \cdot N_{bs} \cdot W_{sys} .$$

When we normalize R_{tot} with the coverage service area, A_{srv},

$$\frac{R_{tot}}{A_{srv}} \approx \frac{\sum_{k=1}^{N_s} C_k}{W_{bs}} \cdot \left(\frac{1}{A_{bs}}\right) \cdot W_{sys} = \frac{\sum_{k=1}^{N_s} C_k}{W_{bs}} \cdot \left(\frac{N_{bs}}{A_{srv}}\right) \cdot W_{sys} .$$

Here, the basestation density $K\ [BSs/m^2]$,

$$K = \frac{\text{number of basestation}}{\text{coverage are}} = \frac{N_{bs}}{A_{srv}} = \frac{1}{A_{bs}} = \frac{1}{\text{a cell coverage}} .$$

Therefore, we define the system capacity function of basestation density and spectrum,

$$\frac{R_{tot}}{A_{srv}} \approx \eta \cdot \left(\frac{N_{bs}}{A_{srv}}\right) \cdot W_{sys}$$

where η is the sum of maximum average data rates per unit bandwidth by a cell's basestation, $(\sum_{k=1}^{N_s} C_k)/(W_{bs}),\ [b/s]/[Hz \cdot BS]$. Simply, R_{tot} is the increasing function of the number of basestaion, N_{bs}, and amount of spectrum, W_{sys}, as below [1],

$$R_{tot} \approx \eta \cdot N_{bs} \cdot W_{sys} \qquad (2)$$

$$R_{tot} \approx \frac{C_{sys}}{c_{BS} A} \eta W_{sys} = \frac{\eta}{A} N_{BS} W_{sys} = \frac{\eta}{A} NW .$$

III. ECONOMIC MODEL FOR PRODUCTION FUNCTION AND COST

The simplest and most common way to describe the technology of a firm is the production function. A firm produces output from various combination of input. In wireless operators, these inputs are power, spectrum, and number of basestations, deployment to increase capacity. In order to research firm choices, we need a convenient way to summarize the production possibility of the firm, i.e., which is the combination of inputs and outputs are technologically feasible [3].

Based on (2), we can define the production function of wireless systems. We can say inputs as amount of spectrum, W_{sys}, and number of basestations, N_{bs}, and an output as capacity, R_{tot} as below,

$$R_{\ tot} = R(w_{sys}, N_{bs}) = R(w, n) ,$$

where, w is the amount of spectrum and n is the number of the basestations. With the above equation, we can define the marginal product (MP) of production function for wireless systems.

$$\frac{\partial R}{\partial w} = \frac{dR(w,n)}{dw} = MP_w = p_w ,$$

$$\frac{\partial R}{\partial n} = \frac{dR(w,n)}{dn} = MP_n = p_n,$$

where MP_w is the marginal product of spectrum, price of spectrum (p_w) and MP_n is the marginal product of a basestation, price of a basestation (p_n).

A. The technical rate of substitution (TRS)

An analogue to the marginal rate of substitution in consumer theory is the technical rate of substitution (TRS) in production theory [3].This measures the rate at which one input can be substituted for another without changing the amount of output produced. Assume that we have some technology summarized by a smooth production function and that we are producing at a particular point $R_{tot}^* = R(w^*, n^*)$. Suppose that we want to increase the amount of spectrum and decrease the number of basestations so as to maintain a constant level of output. Let $n(w)$ be the (implicit) function that tells us how much of n it takes to produce R if we are using w units of the other input. Then by definition, the function $n(w)$ has to be satisfy the identity,

$R(w, n(w)) \equiv R^*$

We are after an expression on $\frac{\partial n(w^*)}{\partial w}$. Differentiating the above identity, we find:

$$\frac{\partial R(w,n)}{\partial w} + \frac{\partial R(w,n)}{\partial n}\frac{\partial n(w^*)}{\partial w} = 0,$$

$$\frac{\partial n(w^*)}{\partial w} = -\frac{\frac{\partial R(w,n)}{\partial w}}{\frac{\partial R(w,n)}{\partial n}} = -\frac{MP_w}{MP_n} = TRS.$$

This gives an explicit expression for the technical rate of substitution.

Here is another way to derive the technical rate of substitution. Think of a vector of small change in the input levels which we write $dx = (dw, dn)$. The associated change in the output is approximated by,

$$dy = \frac{\partial f}{\partial w}dw + \frac{\partial f}{\partial n}dn \ .$$

This expression is known as the total differential of the function $f(w,n)$. Consider a particular change in which only spectrum and basestation change, and the change is such that output remains constant. That is, dw , and dn adjust "along an isoquant."
Since the output remains constant, we have,

$$0 = \frac{\partial f}{\partial w}dw + \frac{\partial f}{\partial n}dn,$$

which can be solve for

$$\frac{dn}{dw} = -\frac{\partial f / \partial w}{\partial f / \partial n} \ .$$

Either the implicit function method or the total differential method may be used to calculate the technical rate of substitution. The implicit function method is a bit rigorous, but the total difference method is perhaps more intuitive.

B. The elasticity of substitution

The technical rate of substitution measures the slope of an isoquant. The elasticity of substitution measures the curvature of an isoquant. More specifically, the elasticity of substitution measures the percentage change in the factor ratio divided by the percentage changes in TRS, with output being held fixed. If we let $\Delta(n/w)$ be the change in the factor ratio and ΔTRS be the change in the technical rate of substitution, we can be expressed this as,

$$\theta = \frac{\dfrac{\Delta(n/w)}{(n/w)}}{\dfrac{\Delta TRS}{TRS}} .$$

This is a relatively natural measure of curvature: it asks how the ratio of factor inputs changes as the slope of the isoquant changes. This quantity measures the extent to which operators can substitute basestations for spectrum as the relative productivity or relative cost of two factors changes. When θ is large, it means that the operator can easily substitute between spectrum and basestations. If a small change in slope gives us a large change in the factor input ratio, the isoquant is relatively flat which means the elasticity of substitution is large.

In practice, we think of the percent change as being very small and take the limit of this expression as Δ goes to zero. Hence the expression for θ becomes,

$$\theta = \frac{TRS}{(n/w)} \frac{d(n/w)}{dTRS} \ .$$

It is often convenient to calculate θ using the logarithmic derivative. In general, if $y = g(x)$, the elasticity of y with respect to x refers to the percentage in y induced by a small percentage change in x. That is,

$$\varepsilon = \frac{\frac{dy}{y}}{\frac{dx}{x}} = \frac{dy}{dx}\frac{x}{y} \ .$$

Provided that x and y are positive, this derivative can be written as,

$$\varepsilon = \frac{d \ln y}{d \ln x} \ .$$

Applying this to the elasticity of substitution, we can write,

$$\theta = \frac{d \ln(n / w)}{d \ln TRS} \ .$$

In general, the closer θ is to zero, the model L-shaped the isoquants are and the more 'difficult' substitution between inputs; the larger θ is, the flatter the isoquants and 'easier' substitution between them.

IV COST MINIMIZATION

In this Section, we will investigate the behavior of a cost-minimizing firm. This is of interest for two reasons: Firs it gives us another way to look at the supply behavior of a firm facing competitive output markets, and the second, the cost function allows us to model the production behavior of firms that do not face competitive output markets. In addition, the analysis of cost minimization gives us a taste of the analytic methods used in constrained optimization problems.

A. Calculus analysis of cost minimization

Here we introduce cost minimization model as below,

$$\text{minimize} \quad \mathbf{pr}$$
$$\text{subject to}$$
$$R(\mathbf{r}) = R_{sys}$$

where $\mathbf{p}$ is price vector and $\mathbf{r}$ is resource vector, and R^*_{sys} is the required system capacity.

We analyze this constrained minimization problem using the method of Lagrange multipliers. Begin by writing the Lagrangian,

$$L(\lambda, \mathbf{r}) = \mathbf{pr} - \lambda(R(\mathbf{r}) - R^*_{sys})$$

and differentiate it with respect to each of the choice variables, r_i and the Lagrange multiplier, λ. The first-order conditions characterizing and interior solution $\mathbf{r}^*$ are,

$$p_i - \lambda \frac{\partial R(\mathbf{r}^*)}{\partial r_i} = 0 \quad \text{for } i = 1, \ldots n$$
$$\partial R(\mathbf{r}^*) = R_{sys}.$$

These conditions can also be written in vector notation. Letting $\mathbf{\Delta R(r)}$ be the gradient vector, the vector of partial derivative of $R(\mathbf{r})$, we can write the derivative condition as

$$\mathbf{p} = \lambda \mathbf{\Delta R(r^*)}.$$

We can interpret the first order conditions by dividing the i th condition by the j th condition to get,

$$\frac{p_i}{p_j} = \frac{\frac{\partial R(\mathbf{r}^*)}{\partial r_i}}{\frac{\partial R(\mathbf{r}^*)}{\partial r_i}} \quad i, j = 1, \ldots n \quad (3)$$

The right-hand side of this expression is the technical rate of substitution, the rate at which factor j can be substitute for factor i while maintaining a constant level of output. The left-hand side of this expression is the economic rate of substitution at what rate factor j can be substituted for factor i while maintaining a constant cost. The conditions given above require that the technical rate of substitution can be equal to the economic rate of substitution. If this were not so, there would be some kind of adjustment that would result in a lower cost way of producing the same output.

For example, suppose

$$\frac{p_i}{p_j} = \frac{2}{1} \neq \frac{1}{1} = \frac{\frac{\partial R(\mathbf{r}^*)}{\partial r_i}}{\frac{\partial R(\mathbf{r}^*)}{\partial r_i}} \quad .$$

Then if we use one unit less of factor i and one unit more of factor j, output remains essentially unchanged but cost have gone down. For we have saved two dollars by hiring one unit of loss of factor i and incurred an additional cost of only one dollar by hiring more of factor j.

This first order condition can also be represented graphically. In Figure 1, the curved lines represent isoquants and the straight lines represent constant cost curves. When R is fixed, the problem of the firm is to find a cost-minimizing point on a given isoquant. The equation of a constant cost curve, $C = p_w w + p_n n$, can be written as $n = C / p_n - (p_w / p_n)w$. For fixed p_w and p_n, the firm wants to find a point on a given isoquant where the associated constant cost curve has minimal vertical intercept. It is clear that such a point will be characterized by the tangency condition that the slope of the constant cost curve must be equal to the slope of the isoquant. Substituting the algebraic expressions for these two slopes gives us equation (3).

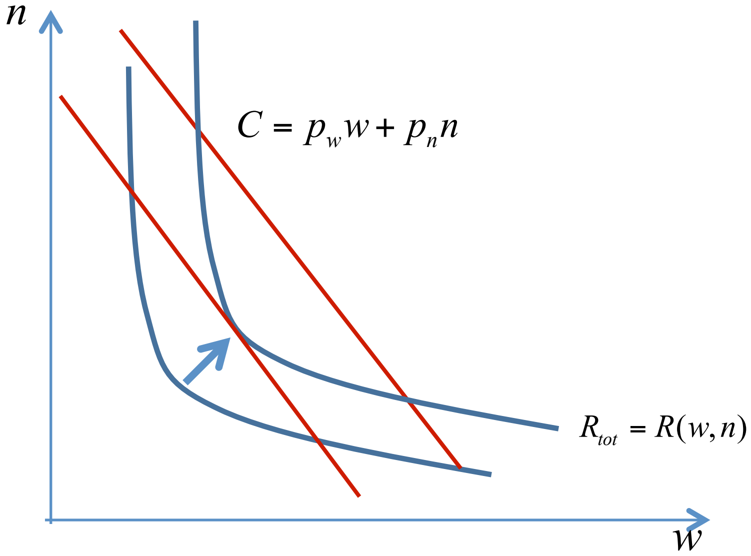

Figure 1. **Cost minimization.** At a point that minimize costs, the isoquant must be tangent to the constant line

EXAMPLE 1. Based on the above production function concept, we finally formulate and solve the below cost minimization problem using firm theory in microeconomic and deal with the revenue maximization problem.

$$\min_{w,n\geq 0} \quad p_w w + p_n n$$
$$\text{subject to}$$
$$R(w,n) \geq R_{sys}^*$$

Where p_w and p_n are the price of spectrum and a basestation, and R_{sys}^* is the required system capacity.

V. ENGINEERING AND ECONOMIC VALUES OF SPECTRUM

By [3], we can calculate the engineering value of spectrum as below,
Providing a certain average data rate $R_{tot}(bits / Hz / Km^2)$ a certain area of size A in a highly loaded (interference limited) cellular data system requires the following number of basestations N_{BS} and spectrum W_{sys} as we mentioned [3].

$$R_{tot} \approx \frac{\eta}{A} N_{BS} W_{sys}$$

The constant η can be interpreted as spectral efficiency of the system (in bit/s/Hz) and is a property of the design of the radio transmission technology used. Now assuming an operator providing the data rate R_{tot} and interested in increasing his data rate by ΔR,

$$R_{tot} + \Delta R \approx \frac{\eta}{A} N_{BS} W_{sys} + \frac{\eta}{A} \Delta N W_{sys} + \frac{\eta}{A} N_{BS} \Delta W$$

The second term corresponds to increasing the number of base station by ΔN , where as the last term represents increasing the spectrum allocation by ΔW . Using this, the additional cost ΔC as below,

$$C_{sys} + \Delta C \approx C_{sys} + c_{BS} \Delta N + \left(\Delta c_{BS} N_{BS} + c_{sp} \right) \Delta W,$$

where c_{BS} is the cost of an (additional) basestation and Δc_{BS} is the cost of refitting the existing basestation with equipment for the new spectrum and c_{sp} is the cost(per Hz) for the new spectrum. So we can compare the next equation to minimize cost,

$$\min \Delta C = \min \left(c_{BS} \frac{\Delta R}{\eta W_{SYS}} A, \left(\Delta c_{BS} N_{BS} + c_{sp} \right) \frac{\Delta R}{\eta N_{BS}} A \right).$$

We can get the next equations

$$c_{BS} \frac{1}{W_{SYS}} = \left(\Delta c_{BS} N_{BS} + c_{sp} \right) \frac{1}{N_{BS}}$$

$$c_{BS} N_{BS} = \left(\Delta c_{BS} N_{BS} + c_{sp} \right) W_{SYS}$$

$$c_{sp} = \frac{c_{BS} N_{BS}}{W_{sys}} - \Delta c_{BS} N_{BS}.$$

The above equation is called the engineering value of spectrum.
Using the technical rate of substitution (TRS), also we can calculate the economic value of spectrum as below,

$$\frac{\partial R(w,n)}{\partial w}\Delta W + \frac{\partial R(w,n)}{\partial n}\Delta N = 0$$

$$\frac{\partial R(w,n)}{\partial w} = -\frac{\partial R(w,n)}{\partial n}\frac{\Delta N}{\Delta W}$$

$$p_w = -p_n \frac{\Delta N}{\Delta W}.$$

In the engineering value of spectrum, if we ignore the refitting cost Δc_{BS}, or set $\Delta c_{BS} = 0$, the engineering value of spectrum equals the economic value of spectrum.

VI. CONCLUSION

We deal with the basic economic theory of cost model for network deployment and spectrum in wireless networks. In particular, we develop a production function for wireless networks. With this production function model, we explore the technical rate of substitution and the elasticity of substitution in the production function for wireless network and find its insight for wireless network. Finally, we compare the engineering value of spectrum and economic value of spectrum.

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