

# Liam Solus

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## Academic Positions

**KTH Royal Institute of Technology**, Stockholm, Sweden.

NSF Mathematical Sciences Postdoctoral Research Fellow

September 2016 –

**Massachusetts Institute of Technology**, Cambridge, Massachusetts, USA.

Visiting Scholar: Postdoctoral Researcher with Caroline Uhler

February 2016 – May 2016

**Institute of Science and Technology Austria**, Klosterneuburg, Austria.

Postdoctoral Researcher in Caroline Uhler Group

January 2016 – August 2016

## Education

**University of Kentucky** Lexington, KY, USA.

- Ph.D., Mathematics

December 2015

- *Dissertation:* Polyhedral Problems in Combinatorial Convex Geometry

- *Thesis Advisor:* Benjamin Braun

- M.A., Mathematics

April 2014

**Oberlin College** Oberlin, OH, USA.

- B.A., Mathematics with Honors

May 2011

- *Honors Thesis:* Normal and  $\Delta$ -Normal Configurations in Toric Algebra

- *Thesis Advisor:* Kevin Woods

## Research Grants, Awards, and Honors

**(United States) National Science Foundation**

- NSF Mathematical Sciences Postdoctoral Research Fellowship

2016–

- NSF East Asia and Pacific Summer Institute Fellowship

2014.

- NSF Graduate Research Fellowship Program Honorable Mention

2013.

**University of Kentucky**

- Graduate School of Arts and Sciences Outstanding Teaching Award

2015.

- Summer Research Fellowship (Jointly funded by Dept. of Math./Dept. of Stat.)

2015.

- Max Steckler Fellowship

2012-2014/2015-2016.

**Oberlin College**

- Honors Student in Mathematics

2011.

## Research

**Research Interests:**

- Combinatorics/Discrete Mathematics; Geometric and Algebraic Combinatorics; Discrete Geometry.
- Statistics, Causality, Machine Learning, and Artificial Intelligence.

## Publications:

The authors on papers marked with a  $\star$  are ordered according to contribution (first author is the main contributor). Authors on all other papers are listed alphabetically.

- P. Brändén and L. Solus. *Symmetric decompositions and real-rootedness*. Preprint available on the arXiv at: <https://arxiv.org/abs/1808.04141> (2018).
- L. Solus. *Local  $h^*$ -polynomials of some weighted projective spaces*. To appear in the proceedings of the 2018 Summer Workshop on Lattice Polytopes at Osaka University. Preprint available on the arXiv at: <https://arxiv.org/abs/1807.08223> (2018).
- N. Gustafsson and L. Solus. *Derangements, Ehrhart theory, and local  $h$ -polynomials*. Preprint available on the arXiv at: <https://arxiv.org/abs/1807.05246> (2018).
- F. Liu and L. Solus. *On the relationship between Ehrhart unimodality and Ehrhart positivity*. Preprint available on the arXiv at: <https://arxiv.org/abs/1804.08258> (2018).
- E. Perrone, L. Solus, and C. Uhler. *The Geometry of Discrete Copulas*. Preprint available on the arXiv at: <https://arxiv.org/abs/1802.06969> (2018).
- B. Braun, R. Davis, and L. Solus. *Detecting the integer decomposition property and Ehrhart unimodality in reflexive simplices*. *Advances in Applied Mathematics*. 100 (2018): 122-142.
- B. Braun and L. Solus.  *$r$ -Stable Hypersimplices*. *Journal of Combinatorial Theory, Series A* **157** (2018) 349-388.
- A. Radhakrishnan, L. Solus, and C. Uhler. *Counting Markov equivalence classes for DAG models on trees*. *Discrete Applied Mathematics*. <https://doi.org/10.1016/j.dam.2018.03.015> (2018).
- L. Solus. *Simplices for numeral systems*. *Transactions of the American Mathematical Society*. DOI: <https://doi.org/10.1090/tran/7424> (2017).
- $\star$  Y. Wang, L. Solus, K. Dai Yang, and C. Uhler. *Permutation-based causal inference algorithms with interventions*. *Advances in Neural Information Processing Systems* (2017).
- $\star$  L. Solus, Y. Wang, L. Matejovicova, and C. Uhler. *Consistency guarantees for greedy permutation-based causal inference algorithms*. Preprint available on the arXiv at: <https://arxiv.org/abs/1702.03530> (2017).
- A. Radhakrishnan, L. Solus, and C. Uhler. *Counting Markov equivalence classes by number of immoralities*. *The Proceedings of the 2017 Conference on Uncertainty in Artificial Intelligence and The Proceedings of the 2017 UAI Special Workshop on Causality* (2017).
- $\star$  V. Karwa, D. Pati, S. Petrović, L. Solus, N. Alexeev, M. Raič, D. Wilburne, R. Williams, and B. Yan. *Exact tests for stochastic block models*. Preprint available on the arXiv at: <https://arxiv.org/abs/1612.06040> (2016).
- L. Solus, C. Uhler, and R. Yoshida. *Extremal positive semidefinite matrices whose sparsity pattern is given by graphs without  $K_5$  minors*. *Linear Algebra and its Applications* (2016). DOI 10.1016/j.laa.2016.07.026
- T. Hibi and L. Solus. *The facets of the  $r$ -stable  $(n, k)$ -hypersimplex*. *Annals of Combinatorics* (2016). DOI 10.1007/s00026-016-0325-x
- J. Calcut, J. Metcalf-Burton, T. Richard, L. Solus. *Borromean rays and hyperplanes*. *The Journal of Knot Theory and Its Ramifications* Vol. 23, No. 4 (2014).
- L. Solus. *A topological generalization of partition regularity*. *Involve: A Journal of Mathematics* 3-4 (2010), 421-433. DOI 10.2140/involve.2010.3.421.

### Contributions to The Online Encyclopedia of Integer Sequences (OEIS):

- A318408: sequence authored. Status: approved, Aug 26 2018.
- A318407: sequence authored. Status: approved, Aug 26 2018.
- A318406: sequence authored. Status: approved, Aug 26 2018.
- A318405: sequence authored. Status: approved, Aug 26 2018.
- A318404: sequence authored. Status: approved, Aug 26 2018.
- A318376: sequence authored. Status: approved, Aug 24 2018.
- A000045: comment and link added, Aug 23 2018.
- A011973: comment and link added, Aug 23 2018.
- A046739: comment and link added, Aug 23 2018.
- A130477: comment and link added, Aug 23 2018.
- A002301: comment and link added, Aug 23 2018.
- A034807: comment and link added, Aug 23 2018.
- A001610: comment and link added, Aug 23 2018.

### Refereeing Responsibilities:

- Journal of Combinatorial Theory, Series A,
- Discrete and Computational Geometry,
- European Journal of Combinatorics,
- Journal of Algebraic Combinatorics,
- Annals of Combinatorics,
- ARS Mathematica Contemporanea,

### Invited Presentations

#### Osaka University, Osaka, Japan.

- *Thinking  $\gamma$ -positively about Lattice Polytopes*. Summer Workshop on Lattice Polytopes. (August 2018)
- *Real-rootedness, Unimodality, and Symmetric Decompositions of Polynomials*. Summer Workshop on Lattice Polytopes. (August 2018)

#### Aalto University, Helsinki, Finland.

- *Geometry, Combinatorics, and the Shape of a Distribution*. (April 2018)

#### Mathematisches Forschungsinstitut Oberwolfach, Oberwolfach, Germany.

- *Simplices for Numeral Systems*. Miniworkshop on Lattice Polytopes: Methods, Advances, and Applications. (September 2017)

#### MSRI Semester on Geometric and Topological Combinatorics, MSRI, Berkeley, CA, USA.

- *Ehrhart Unimodality*. Video available at: <https://www.msri.org/workshops/813>. Introductory Workshop. (September 2017)

#### Georgia Institute of Technology, Atlanta, Georgia, USA.

- *Combinatorial geometry of Bayesian networks*. SIAM Conference on Applied Algebraic Geometry, Minisymposium on Algebra and Combinatorics of Graphical Models. (July 2017)

#### Freie Universität, Berlin, Germany.

- *Simplices for Numeral Systems*. Methods for Discrete Structures Seminar. (April 2017)

#### Mathematisches Forschungsinstitut Oberwolfach, Oberwolfach, Germany.

- *Learning bayesian networks via edgewalks on DAG associahedra*. Workshop on Algebraic Statistics. (April 2017)

#### Osaka University, Osaka, Japan.

- *Learning bayesian networks via edgewalks on DAG associahedra*. Workshop on Convex Polytopes. (January 2017)

**Joint Mathematics Meetings**, Atlanta, Georgia, USA.

- *On consistency guarantees in DAG model selection*. Special Session on Algebraic Statistics I (a Mathematics Research Communities Session). (Upcoming January 2017)

**Joint Mathematics Meetings**, Atlanta, Georgia, USA.

- *Combinatorial optimization in Markove equivalence of DAG models*. Special Session on Gaussian Graphical Models and Combinatorial Algebraic Geometry II. (Upcoming January 2017)

**KTH Royal Institute of Technology**, Stockholm, Sweden.

- *Ehrhart  $h^*$ -polynomials and regular unimodular triangulations*. Combinatorics Seminar. (October 2016)

**RIMS**, Kyoto, Japan.

- *On reflexive IDP lattice simplices and unimodality of  $h^*$ -polynomials*. RIMS Workshop on Computational Commutative Algebra and Convex Polytopes. (August 2016)

**SIAM Annual Meeting**, Boston, MA, USA.

- *Recovery of covariance matrices for Gaussian graphical models*. SIAM Annual Meeting, Algebraic Statistics Mini-Symposium . (July 2016)

**Georgia Institute of Technology**, Atlanta, GA, USA.

- *Extremal positive semidefinite matrices for graphs without  $K_5$  minors*. Algebra Seminar. (November 2015)

**Loyola University**, Chicago, IL, USA.

- *Extremal positive semidefinite matrices for certain graphs*. Special Session on Geometric, Algebraic, and Enumerative Combinatorics, AMS Sectional Meeting. (October 2015)

**Institute of Science and Technology Austria**, Klosterneuburg, Austria.

- *Extremal positive semidefinite matrices for certain graphs*. Research seminar. (July 2015)

**Rikkyo University**, Tokyo, Japan.

- *Gorenstein  $r$ -stable  $(n, 2)$ -hypersimplices*, Combinatorics Seminar. (Summer 2014)

**Kobe University**, Kobe, Japan.

- *A shelling of the odd second hypersimplex*, Workshop on Applications of Algebraic Geometry and Algebraic Analysis. (Summer 2014)

**Osaka University**, Osaka, Japan.

- *Facets of the  $r$ -stable second hypersimplices*, Conference on Gröbner Bases. (Summer 2014)

#### Contributed Presentations

**Conference on Uncertainty in Artificial Intelligence**. Sydney, Australia.

- *Counting Markov equivalence classes by number of immoralities*. Special Workshop on Causality. (August 2017)

**SIAM Conference on Applied Algebraic Geometry**, National Institute of Mathematical Sciences, Daejeon, South Korea.

- *Extremal positive semidefinite matrices for certain graphs*. Contributed Session III. (August 2015)

**Osaka University**, Osaka, Japan.

- *Facets of the  $r$ -stable  $n, 2$ -hypersimplex*. Combinatorics Seminar. (Summer 2014)
- *Unimodality of  $h^*$ -polynomials of the  $n, k$ -hypersimplex*. Combinatorics Seminar. (Summer 2014)

**University of Minnesota**, Minneapolis, MN, USA.

- *r-stable hypersimplices*, Poster Session, IMA Workshop on Geometric and Enumerative Combinatorics. (Fall 2014)

**University of Kentucky**, Lexington, KY, USA.

- *Identifying extreme rays with cuts*, Combinatorics Seminar. (Spring 2015)
- *Connecting two problems with a spectrahedron*, Algebra Seminar. (Spring 2015)
- *r-stable hypersimplices*, Qualifying Exam. Combinatorics Seminar. (Fall 2014)
- *Not all simplicial polytopes are weakly vertex decomposable*, Master's Exam. Combinatorics Seminar. (Spring 2014)
- *Semigroup algebras of the Veronese type*, Algebra Seminar. (Fall 2013)

**The Ohio State University**, Columbus, OH, USA.

- *Metric and topological spaces for studying partition regularity*, The Young Mathematicians Conference. (Summer 2010)

**Mathfest 2010**, Pittsburgh, PA, USA.

- *Partition regularity for the jordan canonical form and nilpotent linear operators*, Mathfest. (Summer 2010)

## Teaching Experience

**PhD Students:**

- Petter Restadh
  - **Thesis topic:** Combinatorics in Causality.
  - August 2018 –
  - **Date:** Co-advised by Svante Linusson.

**Masters Students:**

- Nils Gustafsson
  - **Thesis topic:** Box Polynomials of Lattice Simplices.
  - **Date:** January 2018 – June 2018
- Jonas Bederoff Eriksson
  - **Thesis topic:** Graph Properties of DAG Associahedra and Related Polytopes.
  - **Date:** January 2018 – June 2018

**Classes Taught as Primary Instructor:** Duties include writing lectures, homework, exams, and quizzes, grading homework, exams, and quizzes, and leading lectures and group discussions.

- *Combinatorial Models in Statistics and Causality*. PhD Course at the AI-MATH Graduate School. Wallenberg AI, Autonomous Systems, and Software Program (WASP), Sweden (Proposed date of course: Spring 2019).
- *MA 114, Calculus II*, University of Kentucky (Spring 2015).
- *MA 123, Elementary Calculus and its Applications*, University of Kentucky (Summer 2013).

**Classes Taught as Recitation Leader:** Duties include leading recitation discussions and group work sessions, grading homework, quizzes, and written projects, and writing quizzes.

- *MA 113, Calculus I*, University of Kentucky (Fall 2015).
- *MA 114, Calculus II*, University of Kentucky (Spring 2015).
- *MA 138, Calculus II for the Life Sciences*, University of Kentucky (Spring 2014).
- *MA 137, Calculus I for the Life Sciences*, University of Kentucky (Fall 2014, Fall 2013).
- *MA 123, Elementary Calculus and its Applications*, University of Kentucky (Spring 2013, Fall 2012).

**Additional Experience:** Grading for graduate classes.

- *MA 561, Algebra I*. University of Kentucky (Fall 2014).

## Outreach

**KTH PRAO session leader:** (December 2017) The PRAO program brings small groups of 8–9 students to the mathematics department at KTH for one week, during which time they discuss and work with various researchers on problems aimed at engaging them in higher level mathematics. In this two hour PRAO session, I guided the students through learning counting methods and visualization techniques in discrete geometry. The students computed the face vectors for a variety of polyhedra, from which they discovered Euler’s theorem. The students then worked on problems aimed at guiding them through a proof of Euler’s theorem and an application to combinatorial game.

**IST Austria Sommercampus session leader; “What is Science?”:** (August 2016) IST Austria Sommercampus is a week long Summer camp for children of primary school age in the greater Vienna area. The “What is Science?” session begins the sommercampus with a group discussion with the children lead by two different-bodied researchers studying in two different areas. Together we lead a discussion introducing the children to the various aspects of science and the research areas they will encounter throughout the week. We then lead special hands-on sessions where the children experiment with an engaging problem from each of our fields. My problem had the children constructing their own polyhedral complexes with toys, and then searching for Hamiltonian paths on the edge graphs of their creations. The children also worked together to build origami icosohedra encoding the solution with colors.

**University of Kentucky Graduate Student Mentor:** (Fall 2015) Duties include offering individual mentorship and guidance to a first-year graduate student from an underrepresented group who is transitioning in a PhD program in mathematics.

**High School Math Circle Writer/Leader:** Duties include writing and leading inquiry-based learning sessions for high school students, and managing graduate and undergraduate assistants. High School Math Circles bring 9 – 12<sup>th</sup> grade students from the Bluegrass region together to participate in mathematical activities with the goal of exposing young students to ideas in higher mathematics not typically seen until college. Activities are designed to stimulate a deeper interest in mathematics and promote mathematical discussion amongst high school students.

- *Graph Laplacians and the Matrix-Tree Theorem* (September 2015).
- *Symmetries of Polygons: Introducing Abstract Algebra* (November 2014).
- *The Rubik’s Cube: Introducing Abstract Algebra* (November 2014).

**High School Math Circle Leader:** Duties include leading inquiry-based learning sessions for high school students, and managing graduate and undergraduate assistants.

- *Adjacency Matrices of Graphs* (September 2015).
- *Euler’s formula* (Fall 2013).

**High School Math Circle Assistant:** Duties include assisting with inquiry-based learning sessions for high school students by encouraging group discussions and mathematical experimentation.

- *Exploring addition and multiplication of big numbers.* University of Kentucky (Spring 2013).

**Elementary School Math Circle Assistant:** Duties include assisting with inquiry-based learning sessions for  $K - 5^{th}$  grade students by encouraging group discussions and mathematical experimentation.

- *Polygons, Rigidity, and Platonic Solids* (Spring 2015).

## Seminars and Workshops Organized

**Minisymposium on Combinatorics and Algorithms in Decision and Reason.** Organized jointly with Svante Linusson, SIAM Conference on Applied Algebraic Geometry, Bern, Switzerland (Upcoming July 2019).

**Session on Combinatorial Commutative Algebra and Polytopes,** Organized jointly with Robert Davis, Joint Mathematics Meetings, San Deigo, California, USA (January 2018).

**KTH Mathematics Postdoc Day**, Organized jointly with Svante Linusson and Barbara Gris, KTH Royal Institute of Technology, Stockholm, Sweden (September 2017).

**Minisymposium on the Algebra and Combinatorics of Graphical Models**, Organized jointly with Caroline Uhler, SIAM Conference on Applied Algebraic Geometry, Georgia Institute of Technology, Atlanta, Georgia, USA (July-August 2017).

**KTH Young Combinatorists Seminar**, KTH Royal Institute of Technology, Stockholm, Sweden (Summer 2017).

#### Conferences and Workshops Attended

**SIAM Conference on Applied Algebraic Geometry**, Bern, Switzerland (Upcoming July 2019).

**Einstein Workshop on Algebraic Combinatorics**, ZUSE Institute, Berlin, Germany (Upcoming November 2018).

**Einstein Workshop on Geometric and Topological Combinatorics**, ZUSE Institute, Berlin, Germany (Upcoming October 2018).

**Summer Workshop on Lattice Polytopes**, Osaka University, Osaka, Japan (July-August 2018).

**Joint Mathematics Meetings**, San Diego, California, USA (January 2018).

**Oberwolfach Mini-Workshop on Lattice Polytopes: Methods, Advances, Applications**, Mathematisches Forschungsinstitut Oberwolfach, Oberwolfach, Germany (September 2017).

**Interactions with Lattice Polytopes**, Universität Magdeburg, Magdeburg, Germany (September 2017).

**MSRI Introductory Workshop: Geometric and Topological Combinatorics**, MSRI, Berkeley, California, USA (September 2017).

**Conference on Uncertainty in Artificial Intelligence**, Sydney, Australia (Upcoming August 2017).

**SIAM Conference on Applied Algebraic Geometry**, Georgia Institute of Technology, Atlanta, Georgia, USA (Upcoming July-August 2017).

**Interactions Between Algebra and the Sciences**, Max Planck Institute for Mathematics in the Sciences and University of Leipzig, Leipzig, Germany (May 2017).

**The Chow Lectures**, Max Planck Institute for Mathematics in the Sciences and University of Leipzig, Leipzig, Germany (May 2017).

**Oberwolfach Workshop on Algebraic Statistics**, Mathematisches Forschungsinstitut Oberwolfach, Oberwolfach, Germany (April 2017).

**Workshop on Convex Polytopes**, Osaka University, Osaka, Japan (January 2017).

**Joint Mathematics Meetings**, Atlanta, GA, USA (January 2017).

**MRC Collaboration Workshop on Linear Structural Equation Models**, University of Washington, Seattle, WA, USA (December 2016).

**MRC Collaboration Workshop on Network Models**, Illinois Institute of Technology, Chicago, IL, USA (November 2016).

**RIMS Workshop on Computational Commutative Algebra and Convex Polytopes**, RIMS, Kyoto, Japan (August 2016).

**SIAM Annual Meeting**, Boston, MA, USA (July 2016).

**Summer School on Algebra, Statistics, and Combinatorics**, Helsinki, Finland (June 2016).

**MRC (Mathematical Research Community) in Algebraic Statistics**, Snowbird, UT, USA (June 2016).

**AMS Sectional Meetings**, Loyola University, Chicago, IL, USA (October 2015).

**SIAM Conference on Applied Algebraic Geometry**, National Institute of Mathematical Sciences, Daejeon, South Korea (Summer 2015).

**BMS School on Discrete/Convex Geometry**, Berlin Mathematical School, Berlin, Germany (Summer 2015).

**Midwest Combinatorics Conference**, University of Minnesota, Minneapolis, MN, USA (Summer 2015).

**Graduate Student Combinatorics Conference**, University of Kentucky, Lexington, KY, USA (upcoming, April 2015).

**IMA Workshop on Geometric and Enumerative Combinatorics**, University of Minnesota, Minneapolis, MN, USA (November 2014).

**Workshop on Applications of Algebraic Geometry and Algebraic Analysis**, Kobe University, Kobe, Japan (July 2014).

**Conference on Gröbner Bases**, Osaka University, Osaka, Japan (July 2014).

**Triangle Lectures in Combinatorics**

- High Point University, High Point, NC, USA (October 2014).
- North Carolina State University, Raleigh, NC, USA (September 2013).

**The Young Mathematicians Conference**, The Ohio State University, Columbus, OH, USA (August 2010).

**MAA Mathfest**, Pittsburgh, PA, USA (August 2010).

Professional Memberships

<b>Society for Industrial and Applied Mathematics</b> , <i>Member</i>	2017–.
<b>American Mathematical Society</b> , <i>Member</i>	2016–.
<b>American Mathematical Society</b> , <i>Graduate Student Member</i>	2012–2015.
<b>Association for Women and Mathematics</b> , <i>Graduate Student Member</i>	2012–2015.
<b>U of Kentucky Mathematics Graduate Student Council</b> , <i>Treasurer</i>	2012–2013.
<b>U of Kentucky Mathematics Graduate Student Council</b> , <i>First-Year Representative</i>	2012.