KTH ROYAL INSTITUTE
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# Hybrid and Embedded Systems EL2450 - Exercise 8

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## Content

- Transition Systems
- Automaton (Discrete Event System)

*What are they? What's the difference between the two?*

# Exercise 8.2

**Problem**

A vending machine dispenses soda for \$0.45. It accepts only dimes (\$0.10) and quarters (\$0.25). It does not give change in return if your money is not correct. The soda is dispensed only if the exact amount of money is inserted. Model the vending machine using a discrete-event system. Is it possible that the machine does not dispense soda? Prove it formally.

# Exercise 8.3

**Problem**

Consider the automaton describing some discrete-event system shown in Figure 1.

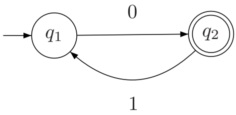



**Figure:** Automaton $A$.

Describe formally the DES. Compute the marked language $L_m$ and the generated language $L$.

## Exercise 8.6

**Problem**

Let the automaton

$$A = (\{q_0, q_1\}, \{0, 1\}, \delta, q_0, \{q_1\})$$

be a nondeterministic automaton where

$$\delta(q_0, 0) = \{q_0, q_1\} \quad \delta(q_0, 1) = \{q_1\} \quad \delta(q_1, 0) = \quad \delta(q_1, 1) = \{q_0, q_1\}.$$

Construct an deterministic automaton $A'$ which accept the same $L_m$.

# Exercise 8.7

**Problem**

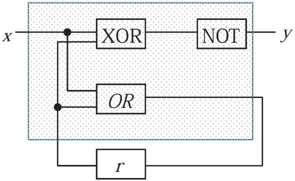
Consider the circuit diagram of the sequential circuit with input variable $x$, output variable $y$, and register $r$, cf. The control function for output variable $y$ is given by

$$\lambda_y = \neg(x \oplus r)$$

where $\oplus$ stands for exclusive (XOR, or parity function). The register evaluation changes according to the circuit function

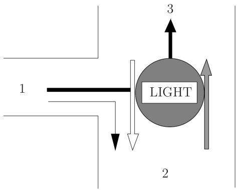
$$\delta_r = x \vee r$$

where $\vee$ stands for disjunction (OR). Initially, the register evaluation is $[r = 0]$. Model the circuit behavior by a finite transition system.



# Exercise 8.9

**Problem**




Consider a simple T-intersection of the traffic system, as shown in the figure. There are four types of vehicles:

- $(1, 2)$-type vehicles coming from point 1 and turning right towards 2;
- $(1, 3)$-type vehicles coming from point 1 and turning left towards 3;
- $(2, 3)$-type vehicles going straight from 2 to 3;
- $(3, 2)$-type vehicles going straight from 3 to 2.

The traffic light is set so that it either turns red for $(1, 2)$ and $(1, 3)$ vehicles (green for $(2, 3)$ and $(3, 2)$ vehicles), or it turns green for $(1, 2)$ and $(1, 3)$ vehicles (red for $(2, 3)$ and $(3, 2)$ vehicles). Model the traffic system as a transition system $\mathcal{T} = (S, Act, \rightarrow, I)$ to describe the changes to the number of the vehicles of the individual types waiting at the intersection over time. Model arrivals of new cars and departures of the waiting cars and changes of the traffic light colors. Assume that initially, there is no vehicle at the intersection.

# Exercise 8.11

**Problem**

Consider the transition system $T = \{S, \Sigma, \rightarrow, S_S\}$, where the cardinality of $S$ is finite. The reachability algorithm is

$$\begin{aligned}
\text{Initialization}: \quad & \mathrm{Reach}_1 = \emptyset; \\
& \mathrm{Reach}_0 = \mathrm{S_S}; \\
& i = 0; \\
\text{Loop}: \quad & \textit{While} \quad \mathrm{Reach_i} \neq \mathrm{Reach_{i-1}} \quad \textit{do} \\
& \quad \mathrm{Reach_{i+1}} = \mathrm{Reach_i} \cup \{\mathrm{s}' \in \mathrm{S} : \exists \; : \mathrm{s} \in \mathrm{Reach_i}, \sigma \in \Sigma, \mathrm{s} \rightarrow^{\sigma} \mathrm{s}' \in \rightarrow\}; \\
& \quad i = i + 1;
\end{aligned}$$

Prove formally that

- the reachability algorithm finishes in a finite number of steps;
- upon exiting the algorithm, $Reach_i = Reach_T(S_S)$.