Quantum Information Theory
Spring semester, 2017

Assignment 7
Assigned: Friday, May 5, 2017
Due: Friday, May 12, 2017 M. Skoglund

Problem 7.1: Define and explain the concept of *typical subspace* and how it relates to quantum compression.

Problem 7.2: Prove the Holevo bound.

Problem 7.3: Consider the state $\rho = p_1|\psi_1\rangle\langle\psi_1| + p_2|\psi_2\rangle\langle\psi_2|$ with $p_1 = p_2 = 1/2$, $\psi_1 = |0\rangle$ and $\psi_2 = |+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$. What is the best compression rate possible for this state?

Problem 7.4: In the previous problem, consider instead

$$\rho = \frac{1}{2}|0\rangle\langle 0| \otimes |0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| \otimes |+\rangle\langle +|$$

What is the best compression rate for this state? Compare with what you got in Problem 7.3.

Problem 7.5: Consider two pmf's $p(x)$ and $q(x)$ and a collection of states $\{\rho_x\}$. Prove that for $\lambda \in [0, 1]$

$$\lambda\chi(p(x), \rho_x) + (1 - \lambda)\chi(q(x), \rho_x) \leq \chi(r(x), \rho_x)$$

where $r(x) = \lambda p(x) + (1 - \lambda)q(x)$.

Problem 7.6: Consider instead one pmf $p(x)$ and two collections $\{\rho_x\}$ and $\{\sigma_x\}$. Prove that for $\lambda \in [0, 1]$

$$\lambda\chi(p(x), \rho_x) + (1 - \lambda)\chi(p(x), \sigma_x) \geq \chi(p(x), \tau_x)$$

where $\tau_x = \lambda\rho_x + (1 - \lambda)\sigma_x$.

Problem 7.7: For a quantum channel $\mathcal{N}$, and the ensemble $\{p(x), \rho_x\}$, where $\sum p(x) = 1$ and all ρ_x's are valid inputs to $\mathcal{N}$. Let $\rho = \sum p(x)\mathcal{N}(\rho_x)$, and define the classical–quantum state

$$\sigma = \sum_x p(x)|e(x)\rangle\langle e(x)| \otimes \mathcal{N}(\rho_x)$$

The *Holevo information of the channel* $\chi(\mathcal{N})$ is obtained as the maximum over $(p(x), \rho_x)$ in $\chi(p(x), \mathcal{N}(\rho_x)) = H(p) + S(\rho) - S(\sigma)$. Show that if $\mathcal{N}$ is entanglement breaking (c.f., HW problem 5.6), then

$$\chi(\mathcal{N} \otimes \mathcal{N}) = \chi(\mathcal{N}) + \chi(\mathcal{N}) = 2\chi(\mathcal{N})$$