Quantum Information Theory
Spring semester, 2017

Assignment 3
Assigned: Friday, March 31, 2017
Due: Friday, April 7, 2017 M. Skoglund

Problem 3.1: For linear spaces, define and explain the concepts of *tensor*, *tensor product space* and *rank* (of a tensor). For Hilbert spaces, also describe how to form inner products and linear operations for the product space in terms of the component spaces.

Problem 3.2: For finite-dimensional spaces, relate the abstract concepts of tensor and tensor product to Kronecker and outer product for matrices.

Problem 3.3: For Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$, consider a linear operator T on $\mathcal{H}_1 \otimes \mathcal{H}_2$. Define and describe how to compute the *partial trace* $\text{Tr}_{\mathcal{H}_2} T$. Show how the general definition leads to the formula

$$\text{Tr}_{\mathcal{H}_2} T = \sum_{ij} a_{ij} \text{Tr}(T_i^{(2)}) T_j^{(1)}$$

in terms of operators $\{T_i^{(1)}\}$ and $\{T_j^{(2)}\}$ on $\mathcal{H}_1$ and $\mathcal{H}_2$.

Problem 3.4: Define and explain the concept of *entangled state*. In what sense does entanglement correspond to dependence/correlation between subsystems?

Problem 3.5: Let $\mathcal{H}$ be a 2-dimensional/qubit space, and let $\hat{\mathcal{H}}$ be the space of compact self adjoint operators $O : \mathcal{H} \to \mathcal{H}$ with inner product $\langle A, B \rangle = 2^{-1}\text{Tr}(A^* B)$, for $A, B \in \hat{\mathcal{H}}$. Show that the operators represented by the four Pauli matrices are a basis for $\hat{\mathcal{H}}$, i.e. any compact self-adjoint operator on qubits can be written as a linear combination (with real coefficients) of Pauli operators.

Problem 3.6: For $\mathcal{H}$ a qubit space, as above, construct a basis for compact self-adjoint operators $T : \mathcal{H} \otimes \mathcal{H} \to \mathcal{H} \otimes \mathcal{H}$ using the Pauli matrices. What can you say about extending your construction to the n-fold product space?