Probability and Random Processes
2026

Assignment 2
Assigned: Thursday, January 29, 2026
Due: Thursday, February 5, 2026 M. Skoglund

Problem 2.1: Define 'simple function' and prove that if f is a nonnegative Lebesgue measurable function, then there is a nondecreasing sequence of simple nonnegative functions that converges pointwise to f. Also prove the converse, i.e., that the pointwise limit of a sequence of simple nonnegative functions is Lebesgue measurable.

Problem 2.2: Define the Lebesgue integral of a nonnegative Lebesgue measurable function. Use the result in Problem 2.1 to illustrate graphically how the value of the integral is obtained as a nondecreasing sequence of approximations to the final value (the limit).

Problem 2.3: Prove the MCT.

Problem 2.4: Prove Fatou's lemma (you can use the MCT without proving it).

Problem 2.5: Prove the DCT (you can use Fatou's lemma without proving it).