# On Achievability of Linear Source Coding over Finite Rings

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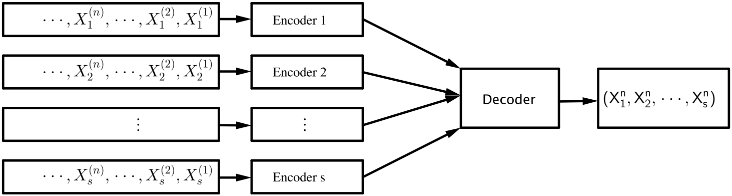
# Outline

# Linear Source Coding over Finite Field / Ring (I)

Consider the Slepian–Wolf Source Network:



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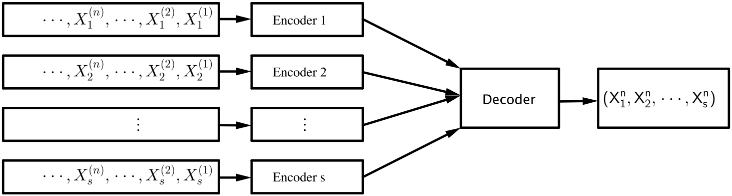


1. [Elias(1955), Csiszár(1982)] propose to use linear mappings (over finite field) as encoders for Slepian–Wolf data compression;
2. Linear coding over finite field (LCoF) is optimal, i.e. achieves the Slepian–Wolf region [Slepian and Wolf(1973)].

# Linear Source Coding over Finite Field / Ring (II)

How about linear coding over finite ring (LCoR)?

## Definition 1

The tuple $[\mathfrak{R}, +, \cdot]$ is called a *ring* if the following criteria are met:

1. $[\mathfrak{R}, +]$ is an *Abelian group*;
2. There exists a *multiplicative identity* $1 \in \mathfrak{R}$, namely, $1 \cdot a = a \cdot 1 = a$, $\forall\, a \in \mathfrak{R}$;
3. $\forall\, a, b, c \in \mathfrak{R}$, $a \cdot b \in \mathfrak{R}$ and $(a \cdot b) \cdot c = a \cdot (b \cdot c)$;
4. $\forall\, a, b, c \in \mathfrak{R}$, $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$ and $(b + c) \cdot a = (b \cdot a) + (c \cdot a)$.

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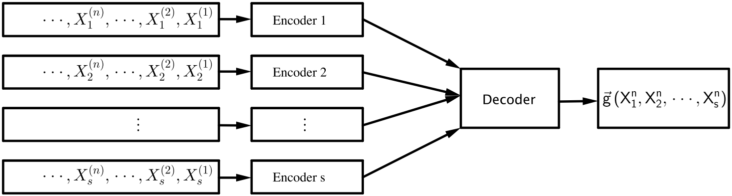
- † Why ring in particular?
- † Will LCoR be optimal as LCoF for Slepian–Wolf coding?
- † What is the benefit?

# Motivation: Source Coding for Computing (I)

Source Coding for Computing $g$ (a discrete function):

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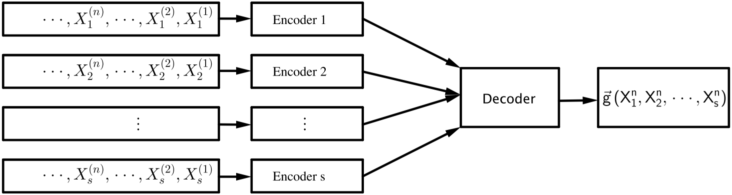


First considered by [Körner and Marton(1979), Ahlswede and Han(1983)] for $g$ being the modulo-two sum.

One trick: Let $Z^{(n)} = g\left(X_1^{(n)}, X_2^{(n)}, \cdots, X_s^{(n)}\right)$ and $\phi$ be a linear encoder (over some field / ring) such that

$$\begin{aligned} \epsilon &> \Pr\{\psi(\phi(Z^n)) \neq Z^n\} \\ &\overset{(a)}{=} \Pr\{\psi(\vec{g}(\phi(X_1^n), \phi(X_2^n), \cdots, \phi(X_s^n))) \neq Z^n\}, \end{aligned}$$

where $(a)$ holds when $g$ is a linear function over some field / ring.

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Facts:

1. Very discrete function over a finite domain is equivalent to a *restriction* of some *polynomial function* over a finite field / ring [Huang and Skoglund(2013a), conclusion of Fermat's little theorem or Galios theory];

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2. The *characteristic* of a non-field ring is not necessary a prime: linear coding over ring strictly outperforms its field (any finite field) counterpart in terms of achieving larger achievable region for computing (infinite) many $g$'s [Huang and Skoglund(Submitted)];

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3. Some non-field rings contain *zero divisors*: functions are not classified (the classification results of [Han and Kobayashi(1987)]), e.g. polynomial function over ring $\mathfrak{R} = \mathbb{Z}_6 \times \mathbb{Z}_6$

$$(X_1 + X_2)X_3,$$

where $X_1 \in \{(0,2),(2,0)\}$, $X_2 \in \{(0,0),(2,2)\}$ and $X_3 \in \{(1,3),(3,1)\}$.

# Random Linear Mapping over Ring

## Definition 2

A (left) linear mapping $\phi : \mathfrak{R}^n \to \mathfrak{R}^k$ is defined as

$$\phi : \mathbf{x} \mapsto \mathbf{A}\mathbf{x}, \forall\, \mathbf{x} \in \mathfrak{R}^n.$$

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## Lemma 3 ([Huang and Skoglund(2013b)])

*Let $\mathfrak{R}$ be a finite ring and choose uniformly at random a linear mapping*

$$\phi : \mathfrak{R}^n \to \mathfrak{R}^k.$$

*Given* $\mathbf{x}, \mathbf{y} \in \mathfrak{R}^n$ *with* $\mathbf{y} - \mathbf{x} = [a_1, a_2, \cdots, a_n]^t$, *we have*

$$\Pr\left\{\phi(\mathbf{y}) = \phi(\mathbf{x})\right\} = |\mathfrak{I}|^{-k},$$

*where* $\mathfrak{I} = \langle a_1, a_2, \cdots, a_n \rangle_l = \left\{ \sum_{i=1}^{n} r_i a_i \middle| r_i \in \mathfrak{R} \right\}.$

# Proof of Lemma 3 for $k = 1$

Define linear function $f : \mathfrak{R}^n \to \mathfrak{R}$ by

$$f : \phi \mapsto \phi(\mathbf{y} - \mathbf{x}), \forall\, \phi \in \mathfrak{R}^n.$$

It is obvious that the image

$$f(\mathfrak{R}^n) = \mathfrak{I}$$

by definition. Moreover, $\forall\, r_1 \neq r_2 \in \mathfrak{I}$, the pre-images

$$f^{-1}(r_1) \cap f^{-1}(r_2) = \emptyset$$

and

$$\left|f^{-1}(r_1)\right| = \left|f^{-1}(r_2)\right| = |f^{-1}(0)|.$$

Therefore, $|\mathfrak{I}| \left|f^{-1}(0)\right| = |\mathfrak{R}|^n$, i.e.

$$\frac{\left|f^{-1}(0)\right|}{|\mathfrak{R}|^n} = \frac{1}{|\mathfrak{I}|}.$$

# A Generalized Conditional Typicality Lemma (I)

## Definition 4

Let $X \sim p_X$ be a discrete random variable with sample space $\mathscr{X}$. The set $\mathcal{T}_\epsilon(n, X)$ of *strongly $\epsilon$-typical sequences* of length $n$ with respect to $X$ is defined to be

$$\left\{ \mathbf{x} \in \mathscr{X}^n \,\middle|\, \left| \frac{N(x; \mathbf{x})}{n} - p_X(x) \right| \leq \epsilon, \forall\, x \in \mathscr{X} \right\},$$

where $N(x; \mathbf{x})$ is the number of occurrences of $x$ in the sequence $\mathbf{x}$.

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## Definition 5

Given a finite ring $\mathfrak{R}$ and one of its left ideal $\mathfrak{I}$, the *coset* $\mathfrak{R}/\mathfrak{I}$ is the set

$$\{r_1 + \mathfrak{I}, r_2 + \mathfrak{I}, \cdots, r_m + \mathfrak{I}\},$$

where $m = \frac{|\mathfrak{R}|}{|\mathfrak{I}|}$, $r_i \in \mathfrak{R}$ for all feasible $i$ and $r_i + \mathfrak{I} \cap r_j + \mathfrak{I} = \emptyset \Leftrightarrow i \neq j$.

# A Generalized Conditional Typicality Lemma (II)

**Lemma 6 ([Huang and Skoglund(2013b)])**

*Let $(X_1, X_2) \sim p$ be a jointly random variable whose sample space is a finite ring $\mathfrak{R} = \mathfrak{R}_1 \times \mathfrak{R}_2$. For any $\eta > 0$, there exists $\epsilon > 0$, such that, $\forall\ (\mathbf{x}_1, \mathbf{x}_2)^t \in \mathcal{T}_\epsilon(n, (X_1, X_2))$ and for any left ideal $\mathfrak{I}$ of $\mathfrak{R}_1$,*

$$|D_\epsilon(\mathbf{x}_1, \mathfrak{I}|\mathbf{x}_2)| < 2^{n\left[H(X_1|X_2) - H(Y_{\mathfrak{R}_1/\mathfrak{I}}|X_2) + \eta\right]}, \tag{1}$$

*where*

$$D_\epsilon(\mathbf{x}_1, \mathfrak{I}|\mathbf{x}_2) = \left\{ (\mathbf{y}, \mathbf{x}_2)^t \in \mathcal{T}_\epsilon(n, (X_1, X_2)) \middle| \, \mathbf{y} - \mathbf{x}_1 \in \mathfrak{I}^n \right\}$$

*and $Y_{\mathfrak{R}_1/\mathfrak{I}} = X_1 + \mathfrak{I}$ is a random variable with sample space $\mathfrak{R}_1/\mathfrak{I}$.*

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## Remark 1

If $\mathfrak{I} = \mathfrak{R}_1$, then $D_\epsilon(\mathbf{x}_1, \mathfrak{I}|\mathbf{x}_2)$ is the set of all $\epsilon$-typical sequences $(\mathbf{y}, \mathbf{x}_2) \in \mathcal{T}_\epsilon(n, (X_1, X_2))$. Obviously, $|D_\epsilon(\mathbf{x}_1, \mathfrak{I}|\mathbf{x}_2)| < 2^{n[H(X_1|X_2)+\eta]}$.

# A Generalized Conditional Typicality Lemma (III)

## Example 7 (Single Source)

Let $\mathfrak{R} = \{0, 1, a, b\}$ and $\mathfrak{I} = \{0, a\}$ be a left ideal of $\mathfrak{R}$. Then

$$\mathfrak{R}/\mathfrak{I} = \{\mathfrak{I}, \{1, b\}\}$$
$$\mathfrak{R}/\mathfrak{R} = \{\mathfrak{R}\}.$$

For strongly $\epsilon$-typical sequences

$$\mathbf{x}_1 : 1 - 0 - b - a - a - 0 - 1 - a - b - 1 - 0 - 1$$
$$\mathbf{y}' : b - a - 1 - a - 0 - a - b - 0 - 1 - 1 - 0 - b$$
$$\mathbf{y}'' : 0 - a - 1 - b - 0 - a - b - 0 - 1 - 1 - 0 - a.$$

We have $\mathbf{y}' \in D_\epsilon(\mathbf{x}_1, \mathfrak{I})$, when $\mathbf{y}'' \notin D_\epsilon(\mathbf{x}_1, \mathfrak{I})$ but $\mathbf{y}', \mathbf{y}'' \in D_\epsilon(\mathbf{x}_1, \mathfrak{R})$.

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## Remark 2

The above typicality lemma is a special case of the typicality lemma of *Supremus typicality sequences* [Huang and Skoglund(2013a)].

# Achievability Theorem

Assume that the sample space of $X_i$ $(1 \leq i \leq s)$ is a finite ring $\mathfrak{R}_i$, and let $X_T = \prod_{i \in T} X_i$ and $\mathfrak{R}_T = \prod_{i \in T} \mathfrak{R}_i$ for $\emptyset \neq T \subseteq \{1, 2, \cdots, s\}$.

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## Theorem 8 ([Huang and Skoglund(2013b)])

*The coding rate* $(R_1, R_2, \cdots, R_s) \in \mathbb{R}^s$ *satisfying*

$$\sum_{i \in T} \frac{R_i \log |\mathfrak{I}_i|}{\log |\mathfrak{R}_i|} > H(X_T | X_{T^c}) - H(Y_{\mathfrak{R}_T/\mathfrak{I}_T} | X_{T^c}),$$

$$\forall\, \emptyset \neq T \subseteq \{1, 2, \cdots, s\} \text{ and for all left ideal } \mathfrak{I}_T \text{ of } \mathfrak{R}_T,$$

*where* $Y_{\mathfrak{R}_T/\mathfrak{I}_T} = X_T + \mathfrak{I}_T$*, is achievable with linear coding over* $\mathfrak{R}_1, \mathfrak{R}_2, \cdots, \mathfrak{R}_s$.

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Example: linear coding over $\mathbb{Z}_6$ is optimal for the scenario that $\mathfrak{R}_1 = \mathbb{Z}_6$ and $X_1 \sim p$ satisfying

| $X_1$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $p(X_1)$ | 0.05 | 0.1 | 0.15 | 0.2 | 0.2 | 0.3 |

# Analysis of Error Probability

For simplicity, assume single source scenario and $\mathbf{X} \in \mathcal{T}_\epsilon(n, X_1)$ is the encoded data. Let $L$ be the set of all non-trivial left ideals of $\mathfrak{R}_1$.

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$$\begin{aligned}
\Pr\{Error\} &\leq \sum_{\mathbf{x} \in \mathcal{T}_\epsilon(n, X_1) \setminus \{\mathbf{X}\}} \Pr\{\phi(\mathbf{x}) = \phi(\mathbf{X})\} + \delta \\
&= \sum_{\mathfrak{I}_1 \in L} \sum_{\mathbf{x} \in D_\epsilon(\mathbf{X}, \mathfrak{I}_1)} \Pr\{\phi(\mathbf{x}) = \phi(\mathbf{X})\} + \delta \\
&= \sum_{\mathfrak{I}_1 \in L} \sum_{\mathbf{x} \in D_\epsilon(\mathbf{X}, \mathfrak{I}_1)} |\mathfrak{I}_1|^{-k} + \delta \\
&< \sum_{\mathfrak{I}_1 \in L} 2^{n\left[H(X_1) - H(Y_{\mathfrak{R}_1/\mathfrak{I}_1}) + \eta\right]} |\mathfrak{I}_1|^{-k} + \delta.
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\end{aligned}$$

If $R_1 = \frac{k \log |\mathfrak{R}_1|}{n} > \frac{\log |\mathfrak{R}_1|}{\log |\mathfrak{I}_1|} \left[H(X_1) - H(Y_{\mathfrak{R}_1/\mathfrak{I}_1})\right]$, then

$$\Pr\{Error\} \to 0 \text{ as } n \to \infty.$$

# Non-field Ring vs Field

| | field | non-field ring | properties |
|---|---|---|---|
| Slepian–Wolf coding (side information) | $\surd$ | exist optimal encoders for all scenarios | inverse & typicality lemma |
| Slepian–Wolf coding (memory) | $\surd$ | not yet proved, optimal shown by examples | inverse & typicality lemma |
| Implementation Complexity | | $\surd$ | polynomial long division algorithm |
| Alphabet sizes of encoders | | $\surd$ | prime subfield |
| Coding for Computing | | $\surd$ | characteristic & zero divisor |
| Coding for Computing (memory) | | $\surd$ | characteristic & zero divisor |

# Other Algebraic Structures

- † "A ring is a group" – exact words from many math textbooks.
- † Is LCoR a subclass of "group coding"? NO! Because our arguments actually involve properties, e.g. inverse, characteristic, zero divisor and etc, which are defined based on the multiplicative operation.
- † The mathematicians mean that consider the base set $\mathfrak{R}$ of a ring $[\mathfrak{R}, +, \cdot]$ with its operation $+$, some conclusion follows.
- † It is out of context to draw a conclusion based on the sentence

  "a ring is a group".
- † An algebraic structure needs to be understood based on its associated operation(s).

$\mathbb{Z}_2$ can be either a set of two symbols, a semi-group, a group, the binary field, a ring, a vector space, a module or an algebra over a field.

# Thanks

♫ ♬

## Thanks!

S. Huang and M. Skoglund, *On Existence of Optimal Linear Encoders over Non-field Rings for Data Compression with Application to Computing*, in Proc. IEEE ITW September 2013. Available: `http://www.ee.kth.se/~sheng11` or `http://people.kth.se/~sheng11`

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