Math 314: Discrete Mathematics **Spring 2019**
by Benjamin Schroeter **04/29/2019**

Exercies Sheet 3

Write your name on every sheet that you hand in. Do not use a pencil or a red colored ink. Write down your solution by yourself and *do not copy* it.

Hand in your solution before **Friday May 3 – 8 am**. Have fun!

Exercise 1: For any node, v, in a graph, let $N(v)$ be the set of nodes adjacent to v, that is,

$$N(v) = \{\, u \in V \mid \{u, v\} \text{ is an edge of the graph} \}\,.$$

Suppose ϕ is an isomorphism from graph G to graph H. Carefully prove that $N(\phi(v)) = \{\phi(u) \,|\, u \in N(v)\}$. Conclude that if G and H are isomorphic graphs, then for each $k \in \mathbb{N}_0$, they have the same number of degree k nodes.

Exercise 2: List all the isomorphisms between the two graphs given in Figure 1. Explain why there are no further isomorphisms.

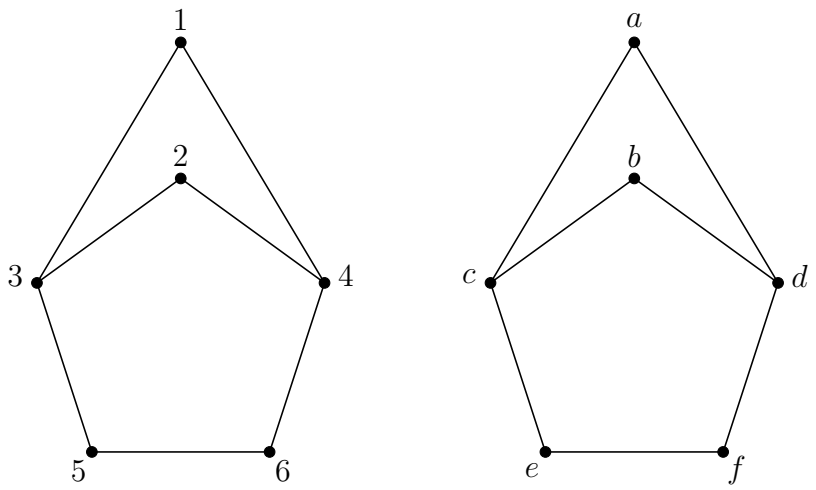

Figure 1: Two isomorphic graphs

Exercise 3: For every $n \in \mathbb{N}$ with $n \geq 3$ give an example of a graph with exactly two vertices of degree 1 and n vertices of degree 2 that is not isomorphic to the line graph L_{n+2}.

Exercise 4: Find the chromatic number of the four graphs in Figure 2. Color the vertices and argue why your coloring uses the minimal number of colors.

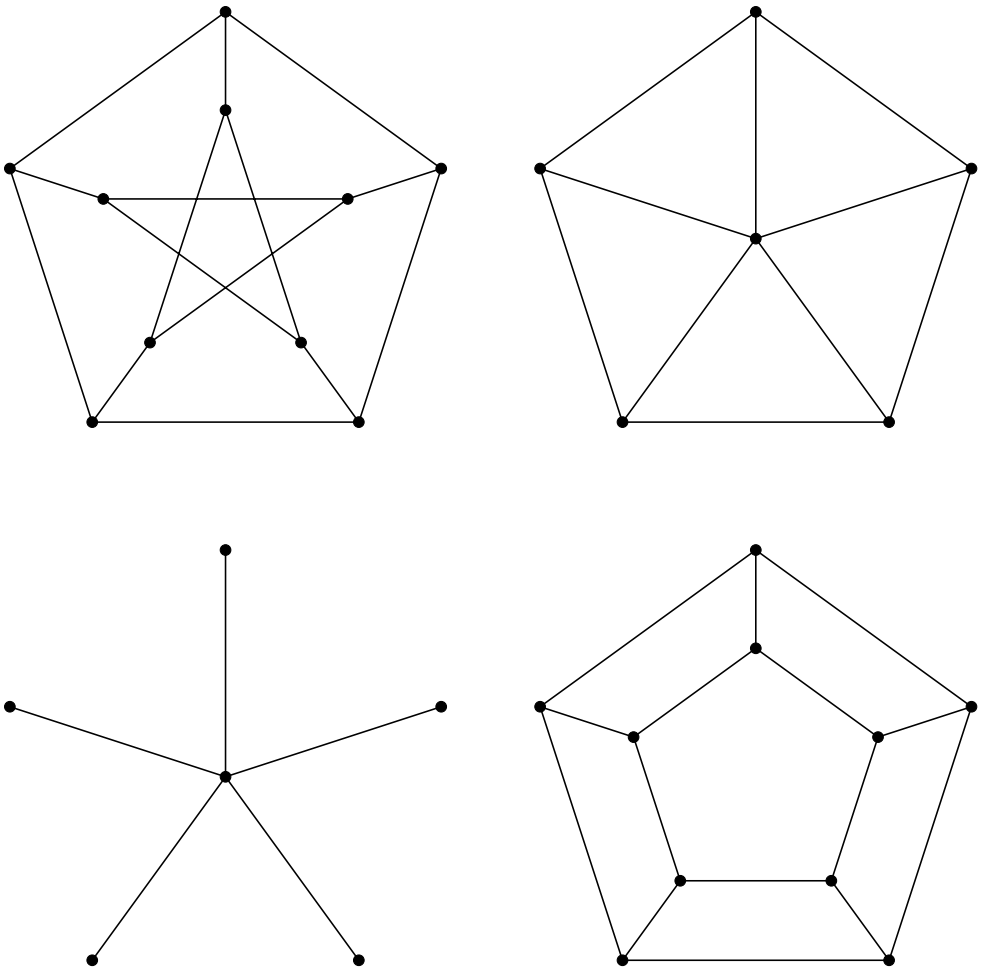

Figure 2: Four connected graphs