**Math 314: Discrete Mathematics** **Spring 2019**
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# Homework 1

Write your name on every sheet that you hand in. Do not use red colored ink. Write down your solution by yourself and do not copy it. Your solutions should be structured, understandable, and readable. **Hand in your solution before Friday February 15 – 8 am.** Have fun!

**Problem 1:** Let $A$ and $B$ be sets.

a) Show that $\operatorname{pow}(A \cap B) = \operatorname{pow}(A) \cap \operatorname{pow}(B)$.

b) Find an example such that $\operatorname{pow}(A \cup B) \neq \operatorname{pow}(A) \cup \operatorname{pow}(B)$.

**Problem 2:** Show that the relation $R\colon D \to C$ is a total injection if and only if $R^{-1} \circ R$ is the identity $\mathrm{Id}_D$ on $D$, i.e., $x \,\mathrm{Id}_D\, y$ if and only if $x = y$.

**Problem 3:** Let $f : A \to B$ and $g : B \to C$ be partial functions, and $h = g \circ f$ their composition. Prove the following statements.

a) If $f$ and $g$ are surjective, then $h$ is a surjection.

b) The function $f$ is a bijection if and only if $f^{-1}$ is bijective.

Bonus: If $h$ is injective and $g$ is total, then $f$ must be injective. Provide a counterexample showing how this claim could fail if $g$ was not total.

**Problem 4:** How many total functions are there from $D$ to $C$ if $|D| = 4$ and $|C| = 8$? Prove the correctness of your result.