Math 314: Discrete Mathematics **Spring 2019**
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In Class Practice Problems 2

Solve and discuss the following questions in small groups of 2-4.

Problem 1: Prove that for all $a, b \in \mathbb{Z}$, $n \in \mathbb{N}$ holds $\gcd(a^n, b^n) = \gcd(a, b)^n$.
Hint: First consider the case $\gcd(a, b) = 1$

Problem 2:

i) Use Euclid's extended algorithm to find s, t s.t. $30s + 22t = \gcd(30, 22)$.

ii) Find s', t' such that $0 \leq t' \leq 30$ and $30s' + 22t' = \gcd(30, 22)$.

iii) Is there a multiplicative inverse of $[22]_{30}$ in $\mathbb{Z}_{30}$? If not briefly explain why, otherwise find it.

Problem 3: What is the remainder of 63^{9601} divided by 220.

Problem 4: Find a solution to each of the following congruence relations. Express your solution x by a minimal non-negative integer.

a) $x \equiv 35829 \mod 11$

b) $x \equiv 12 \cdot (17 + 21) \mod 13$

c) $x \equiv 6^{18} \mod 7$

d) $x \equiv 2^{2018} \mod 31$

e) $x \equiv 11^{19} \mod 12$

f) $3 \cdot x \equiv 1 \mod 10$

g) $3^x \equiv 4 \mod 7$

h) $2x + 1 \equiv 0 \mod 5$

Problem 5: Let P be the following recussivly defined set of functions.

1. $\mathrm{Id}_{\mathbb{Z}} \in P$, where $\mathrm{Id}_{\mathbb{Z}} \colon \mathbb{Z} \to \mathbb{Z}$, with $x \mapsto x$
2. for every $k \in \mathbb{Z}$ the constant function c_k is in P, where $c_k \colon \mathbb{Z} \to \mathbb{Z}$, with $x \mapsto k$.

Moreover, for $f, g \in P$ the two functions

3. $(f + g) \colon \mathbb{Z} \to \mathbb{Z}$, with $x \mapsto f(x) + g(x)$, and
4. $(f \cdot g) \colon \mathbb{Z} \to \mathbb{Z}$, with $x \mapsto f(x) \cdot g(x)$ are in P.

Prove that for all $a, b \in \mathbb{Z}$, $n > 1$ and $p \in P$

$$a \equiv b \mod n \implies p(a) \equiv p(b) \mod n \ .$$

Hint: Use structural induction.