Math 314: Discrete Mathematics **Spring 2019**
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In Class Practice Problems 1

Solve and discuss the following questions in small groups (2-4).

Problem 1: Decide which of the following statements are true. Explain your answers.

a) If you pick five cards from a deck of 52 cards, then a least two will be of the same suit.

b) For any five points inside a unit square, two of those points will have a distance of at most $\frac{\sqrt{2}}{2}$.

Problem 2: How likely is it that your opponent in a 2-Players Poker has two pairs, if you have two pairs? (Assume that you only know the cards in your hand.). (You are allowed to use a calculator for this Problem.)

Problem 3: Show the following identity of binomial coefficients.

$$\binom{n-1}{k} - \binom{n-1}{k-1} = \frac{n-2k}{n}\binom{n}{k}$$

Problem 4: Let A, B be sets. Prove $A \subseteq B \iff \text{pow}(A) \subseteq \text{pow}(B)$.

Problem 5: Let R be a relation. Show that $R \circ R^{-1}$ is injective if and only if R is a partial function.

Problem 6: A relation $\sim$ on the set A is called

- *reflexive* if $a \sim a$ for all $a \in A$,
- *symmetric* if $a \sim b$ implies $b \sim a$,
- *transitive* if $a \sim b$ and $b \sim c$ implies $a \sim c$.

Find examples for all combinations of those properties. A relation that fulfills all three properties is called *equivalence relation*.

Problem 7: Prove $n^2 \leq 2^n$ for all $n \geq 4$ with the Well Ordering Principle or by Induction.