Analysis of a regular multihop system cost model

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Abstract—The coverage and quality of service (QoS) has been a requirement for cell phone networks where minimizing cost is also an important issue. Ongoing researches are trying to find cheaper wireless networks while maintaining the networks performance. In a previous research it has been shown that under some circumstances, infrastructure cost can be reduced by introducing wireless relays while maintaining the same service level [1]. Relays (or routers) have been considered as intermediate equipment between the base station and terminals. This paper verifies the approximations used for the analytical solution in [1] by means of computer simulations. A new relay deployment is also introduced. The results show that deployment has impact on the cost-gain of using relay and system behavior is different than the theoretical prediction.

Index Terms—wireless relay, cost model, multihop.

I. INTRODUCTION

In a traditional or purely cellular network the communication is performed directly between the base station and terminals, which is defined as single-hop cellular network (SCN). Overall cost structure for a new cellular system depends on many factors, for example, radio and transmission equipment, licensing, site rentals, etc. It has been shown in [2] that the total infrastructure cost of a wireless system is linearly proportional to the number of base stations. Considering the Macro base station to be deployed, site cost demands half of the total investment cost [3]. Current economic turmoil forces the wireless operator to rethink if they can cut down the infrastructure cost in a new network. Therefore, deducting the number of Macro base stations by placing comparatively cheaper wireless relays could be one solution to get ‘cost gain’.

In heterogeneous three-layer multihop network, communication is carried out between base station, relays and terminals also known as multihop cellular networks (MCN) [4]. The concept of using relays in multihop networks has been studied for different purposes, for instance, coverage extension, power reduction and cost effectiveness in [1] and [5]. In [1] it has been shown that wireless relays are useful for providing more coverage when the user density is low, however the density of base stations cannot be made arbitrarily low, even if the wireless relays have zero cost, as a consequence of increased transport traffic and a limited channel capacity.

In [1] an analytical solution to a dimensioning problem in a system using fixed wireless relays as an ad hoc extension of a cellular system is presented. Analytical cost and relay deployment models were introduced. A hexagonal deployment is used, which was needed for the analytical modeling (Fig.1). The cost model was proved to be useful to investigate the cost effectiveness of such systems. An interesting question is whether an analytical solution like the one in [1] can be obtained in the case of other relay deployment models.

Fig. 1. Hexagonal relay deployment. Small dots represent mobiles, relays are represented by circles. The base station is located at the centre of the cell [1]

II. PROBLEM STATEMENT

In [1] models and approximations were used to solve a dimensioning problem. As a result an analytical solution is presented.

This work is aimed to investigate if a solution to the dimensioning problem obtained with other deployment model and by excluding some of the approximations used previously is similar to the analytical solution obtained.

For this purpose most of the models (cost, propagation model) will be maintained with the only exception of the deployment model. The same applies to the approximations, only those regarding interference are not supported. Instead calculations (through computer simulations) are carried out using the same propagation model. In the following section approximations, assumption and models are described.

III. MODELS, APPROXIMATIONS AND ASSUMPTIONS

In this section a brief description of the models in [1] is made as they are also being used here. Also a new relay deployment model is presented.
A. Cost model

The cost equation is defined by the following linear relationship.

\[ \prod = \pi_b \lambda_b + \pi_r \lambda_r + \pi_{\text{const}} \]  \hspace{1cm} (1)

where

- \( \prod \) = total cost of the system
- \( \pi_b \) = cost per base station
- \( \lambda_b \) = base station density
- \( \pi_r \) = cost per relay
- \( \lambda_r \) = wireless relay density
- \( \pi_{\text{const}} \) = other costs

When relays are not considered, set \( \lambda_r = 0 \).

B. New Relay deployment Model

By using the tier-based deployment it was not possible to generate relay densities in a more continuous fashion when considering a fixed cell area, as tiers provided the number of relays as a multiple of six. It is relevant in the process of obtaining the so called iso-quant curves described later.

Thus a new deployment is considered where the relays do not form tiers though they are still placed in a regular fashion. This was needed for the routing algorithm to work properly. The resultant placement scheme for relays can be observed in Fig.2.

![Regular deployment of relays with density 18 [relay/km^2]](image)

C. Assumptions

1) **Base stations:** Omni directional antennas are assumed to be used in every base station. Power Control is used so that signal to interference ratio (\( \gamma \)) is the same for all users [6]. Furthermore, the same average power is assumed for all cells, if a base station contains relays then the power of this base station is scaled down.

2) **Wireless relay:** Wireless relays have also omni directional antennas and use power control. Links are allocated in TDMA fashion (no STDMA). The wireless relays use regenerative relaying. After receiving a packet, a wireless relay delivers it to the terminal, or routes it to the closest relay in the terminal direction.

3) **Routing algorithm and Resource allocation:** Terminals are assumed to get connected to the nearest relay or base station since the system model is only distance dependent. Time share allocated to the links is proportional to the amount of traffic handled by that link, which implies that the link closer to the base station will get more time share compare to the links away from it.

4) **The end user, service level and propagation:** Users are assumed to be uniformly distributed with a density of \( \lambda_u [1/km^2] \). The same bit rate \( R \) [kbps] is assigned to each user. So the service level in an area \( A \) is defined as providing a bit rate \( \lambda_u RA \).

5) **Propagation and receiver models:** Shadow fading is not considered where a Cost 231-Hata propagation model is used [7]. Ideal receiver is assumed this means that in order to receive a bit rate \( R \) using a bandwidth \( W \), SIR should be:

\[ \gamma = 2 \frac{R}{W} - 1 \]  \hspace{1cm} (2)

The value \( \gamma \) is set same for all the receivers in a cell.

D. Approximations

- Users directly connected to a base station or relays are uniformly distributed within a circle.
- The power transmitted by a cell (i.e., transmitted by a base station and its associated relays) is approximated by the average value, \( P_0 \).
- The interference generated by a base station and its associated relays is approximated by the interference generated by a base station transmitting with \( P_0 \).
- The interference experienced at a distance \( r \) from the base station is approximated to a constant value. This value is approximated to be:

\[ I = c_1 \cdot I_{\text{closest interferer}} \]  \hspace{1cm} (3)

Where \( c_1 = 8.5 \) and \( I \) closest interferer is the interference generated by the closest co-channel base station, transmitting with \( P_0 \), experienced at the base station (\( r = 0 \)).

IV. VERIFICATION PROCEDURE

A simulator program has been written which incorporates assumptions, approximations and all the characteristics of the models described as in section III. Simulations were carried out for several loads, user densities and system sizes.

**Iso-quant curves:** The locus of pairs \( (\lambda_r, \lambda_b) \) satisfying the constrained average power \( (P_0) \) for a determined service level are termed iso-quant curves. The average cell power depends on several parameters as expressed in the following relationship:

\[ P_0 = P_0(\rho_b, \lambda_r; K; W_{\text{tot}}, R, \lambda_u). \]
Where $\lambda_r$, $\lambda_b$ and $\lambda_u$ are defined as before, $A_c$ is the cell area, $W_{tot}$ is the total available bandwidth [MHz] assigned to a service provider, $\lambda_b$ is the cell radius, $K$ is the reuse factor and $R$ is the bit rate [Kbps].

Due to the nonlinear dependence of the average cell power on the cell radius, the base station density can be obtained through numerical calculations:

$$\lambda_b = \frac{1}{A_0} = 2/\left(3\sqrt{3}\rho_b^2\right) = f(\lambda_r, P_0, K; W_{tot}, R, \lambda_u | P_{cell} \equiv P_0)$$

(4)

Obtaining iso-quant curves:

Fig.4 shows cell radius verses Average cell power curves for different relay densities. It can be observed that by deploying more relays, cell radius and consequently cell area (in other words, coverage area) can be increased. For a certain relay density by constraining the average cell power ($P_0$), for example, 20W (as in the figure) a certain cell radius can be found. The equation 4 shows that base station density is inversely proportional to the cell radius. Therefore, for a predefined relay density corresponding base station density can be computed, where the average cell power needs to be constrained. Because of new relay deployment model it has been possible to obtain any relay density. Iso-quant curve is the locus of $(\lambda_r, \lambda_b)$ as in the Fig.5.

It will be shown later that the shape of the iso-quant curve is important to verify if the system model is cost effective or not. It is also noteworthy to mention that budget-constrain (straight line having the slope as the ratio of a relay price and a base station price) is also important factor to verify a system model.

V. RESULTS AND DISCUSSION

Two study cases are chosen to demonstrate our results. Table I identifies the parameter values that were used during simulations, where constants $\alpha, C_0$ describe propagation conditions.

**Study case I**

Service level in this case is $\lambda_u R = 100$ [Kbps/km$^2$] with a system size of 9 clusters. Resulting iso-quant curves are shown in Fig.5. Theoretical (curve obtained using the analytical models) and simulated iso-quant curves can be compared. It can be observed that for any value of relay density a higher value of base station density is obtained for the simulated case.

**Study case II**

Service level is raised to $\lambda_u R = 200$ [Kbps/km$^2$]. Again 9 clusters were used in the simulations (Fig.6). For higher service level more infrastructures is required.

In the cost model, the cost $\Pi$ depends on the values of relay and base station densities. For a given service level, the optimum base station and relay densities, are those point for what the budget line is tangent to the iso-quant curve, as shown in Fig.7 by an arrow. Also the cost gain is defined as in the figure.

As it is observed from the figure simulated curves are shifted upwards, which results in a larger optimum base station density than in theoretical case. The optimum point is obtained at a higher value of $\lambda_r$ in the simulated case. For a determined budget constraint cost gain are smaller in theoretical case.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$W$ [MHz]</th>
<th>$R$ [Kbps]</th>
<th>$\mu$ [1/km$^2$]</th>
<th>$P_0$ [20W]</th>
<th>$K$</th>
<th>$\alpha$</th>
<th>$C_0$ [-38.8 dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
<td>10</td>
<td>10, 20</td>
<td></td>
<td>4</td>
<td>3.5</td>
<td>-38.8</td>
</tr>
</tbody>
</table>

TABLE II

**COMPARATIVE COST GAIN**

Table II shows a comparative study of cost gain for simu-
lated and analytical cases. It is evident from the Fig.7 that an absolute cost increase of about 50% from the hexagonal (analytical) to the square (simulated) case, but a cost gain of about 10% in both cases (0.012/0.08 in the hexagonal case, 0.018/0.13 in the square case).

In our simulation we have constrained the average cell power, $P_0$, for fair comparison with the analytical model. By average cell power, we mean the average total power transmitted by a base station and its associated relays in a cell. Perhaps it would be more realistic to constrain peak cell power, $P_{peak}$ instead.

In appendix, analytical expression has been developed to investigate the reason of shifting the simulated curve from the theoretical one. For doing that we considered square tiers. It has been shown that for square tiers hop distance is smaller than the hexagonal tiers. In other word, to serve a certain bit rate to the users it needs more transport traffic to be carried for square tiers deployment. As a consequence, for a certain area and predefined relay density it needs more base station density for square tiers case.

VI. CONCLUSION

In this project work we have verified, through computer simulations, the validity of the approximations used in [1] for obtaining analytical expressions for the multihop network cost model. Numerical results obtained from those simulations show that the system behavior is different than the theoretical prediction. The deployment has been proved to have impact on cost gains and also on the initial infrastructure investment.

VII. FUTURE WORK

Multihop network has got reasonable attention in recent days because of its reliable and efficient operation while increasing overall network throughput. Other main advantage of multihop network is that it conserves transmit energy resources, which also becomes enormously important to keep the interference level lower as a whole. At the emergence of 3G where mobile communication has titled not only a media of voice communication, this field has got substantial research interest. It would be interesting to investigate if this simple model is also suitable for the scenario when shadow fading is considered. In reality some times it is not possible to deploy relays with a regular fashion with the same hop distance because of difficult topographical features. Therefore a random deployment of relays where hop distance varies from one hop to another could be another dimension of investigation.

REFERENCES

yogy, Stockholm, Sweden.
VIII. APPENDIX

Minimum hop distance calculations

A. Hexagonal

The relay density is $\lambda_r = N_r/A$ defined as, for the hexagonal case we have that the number of relays per tier is $6T$, so we can express, the relay density:

$$\lambda_{r,h} = \sum_{i=1}^{6i} \frac{6i}{3\sqrt{3}b^2} = \frac{3(T_h^2 + T_h)}{3\sqrt{3}b^2} \quad (5)$$

$$\lambda_{r,h} = \frac{3(T_h^2 + T_h)}{3\sqrt{3}T_h^2 \Delta_h^2} = \frac{2\sqrt{3}}{3\Delta_h^2} (1 + 1/T_h) \quad (6)$$

By introducing the additional relationship, $\rho_h = T_h \cdot \Delta_h$ as depicted in Fig.8 we obtain,

B. Square

Following the same procedure outlined before, we obtain:

$$\lambda_{r,s} = \sum_{i=1}^{4i} \frac{4i}{2T_s \cdot \Delta_s^2} = \frac{2(T_s^2 + T_s)}{2T_s \cdot \Delta_s^2} = \frac{1}{\Delta_s^2} (1 + 1/T_s) \quad (7)$$

Equating both expressions the relationship between the minimum hop distances,

$$\Delta_s = \sqrt{\frac{3(1 + 1/T_s)}{2\sqrt{3}(1 + 1/T_s)}} \Delta_h \quad (8)$$

If the same number of tiers is considered $(T_s = T_h)$ then, the last expression simplifies to

$$\Delta_s = \sqrt{\frac{\sqrt{3}}{2} \cdot \Delta_h} \quad (9)$$