Invariance and symbolic control of cooperative systems for temperature regulation in intelligent buildings

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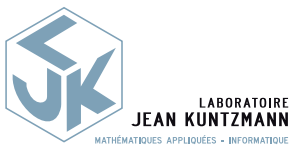
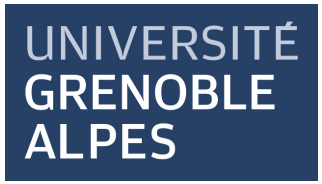


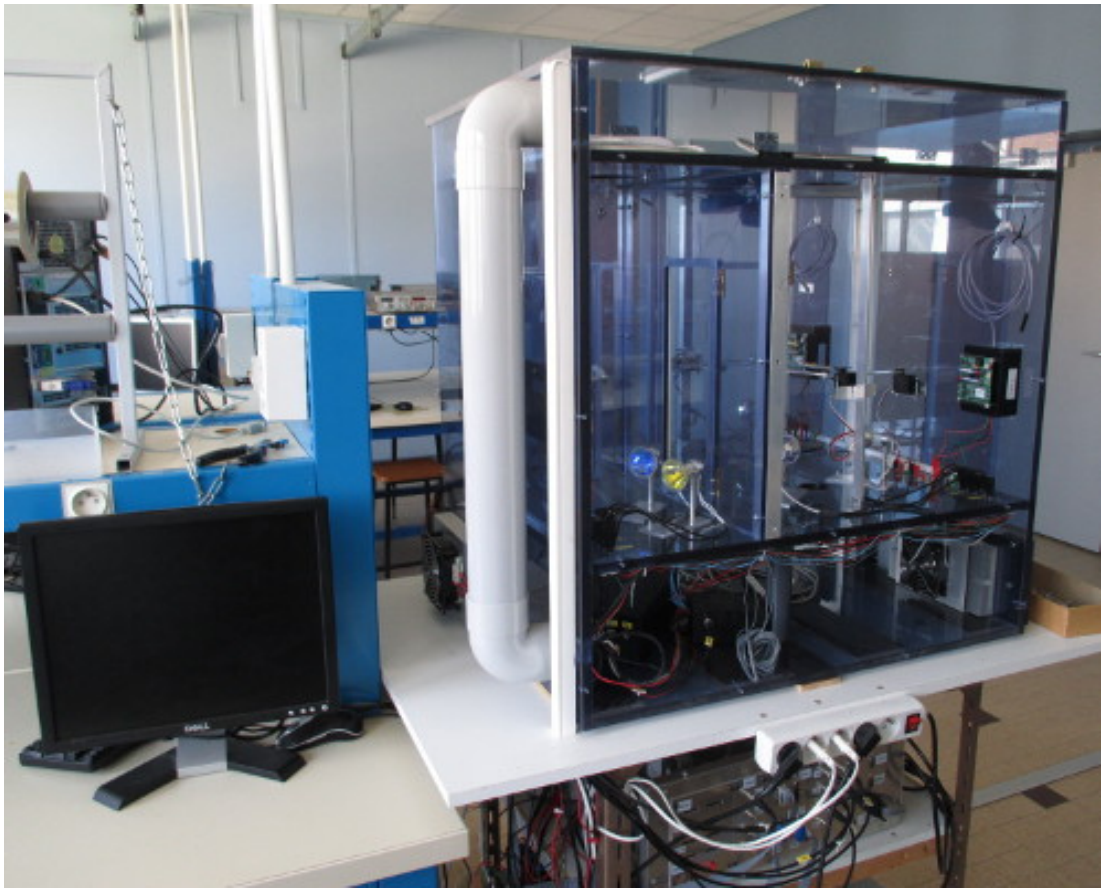


Motivations

RhôneAlpes Région

Develop new control methods for intelligent buildings

Focus on temperature control in a small-scale experimental building



Outline

1. Monotone control system
2. Robust controlled invariance
3. Symbolic control
4. Compositional approach
5. Control in intelligent buildings

System description

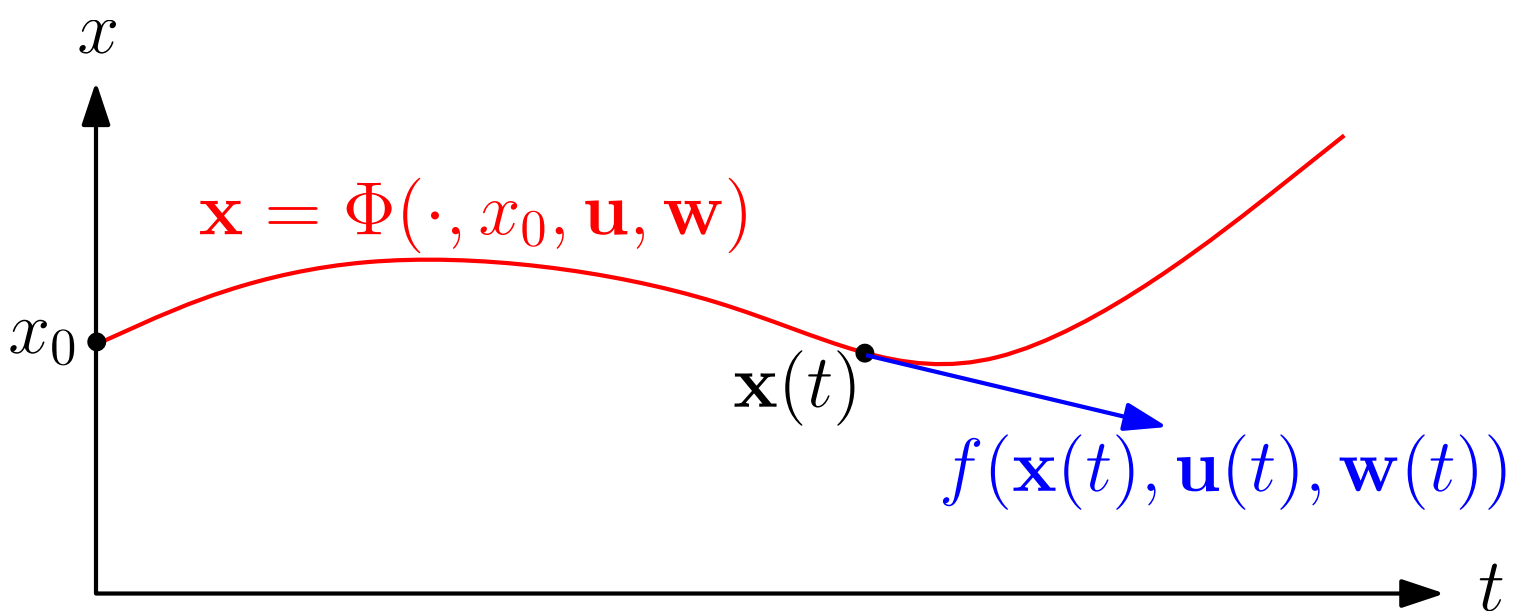
Nonlinear control system:

$$\dot{x} = f(x, u, w)$$

Trajectories:

$$\mathbf{x} = \Phi(\cdot, x_0, \mathbf{u}, \mathbf{w})$$

- x: state
- u: control input
- w: disturbance input
- $\mathbf{x}$, $\mathbf{u}$, $\mathbf{w}$: time functions

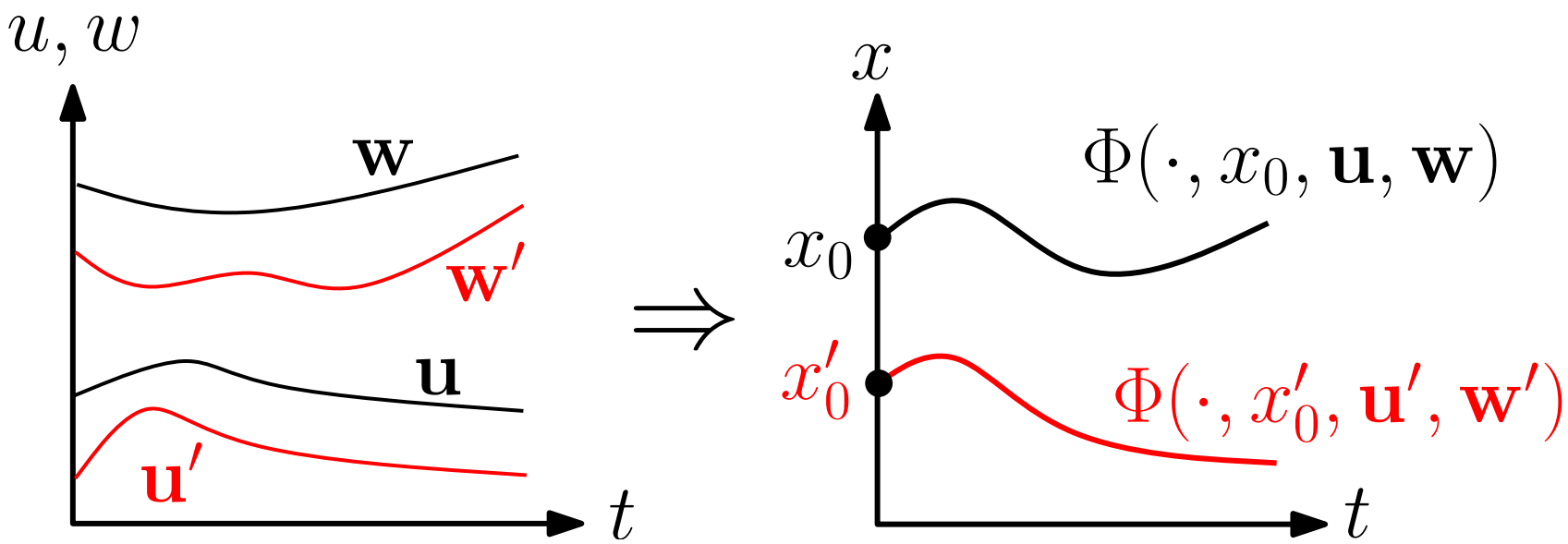


Monotone system

Definition (Monotonicity)

The system is monotone if Φ preserves the componentwise inequality:

$$\mathbf{u} \geq \mathbf{u}',\ \mathbf{w} \geq \mathbf{w}',\ x_0 \geq x_0' \Rightarrow\ \forall t \geq 0,\ \Phi(t, x, \mathbf{u}, \mathbf{w}) \geq \Phi(t, x', \mathbf{u}', \mathbf{w}')$$



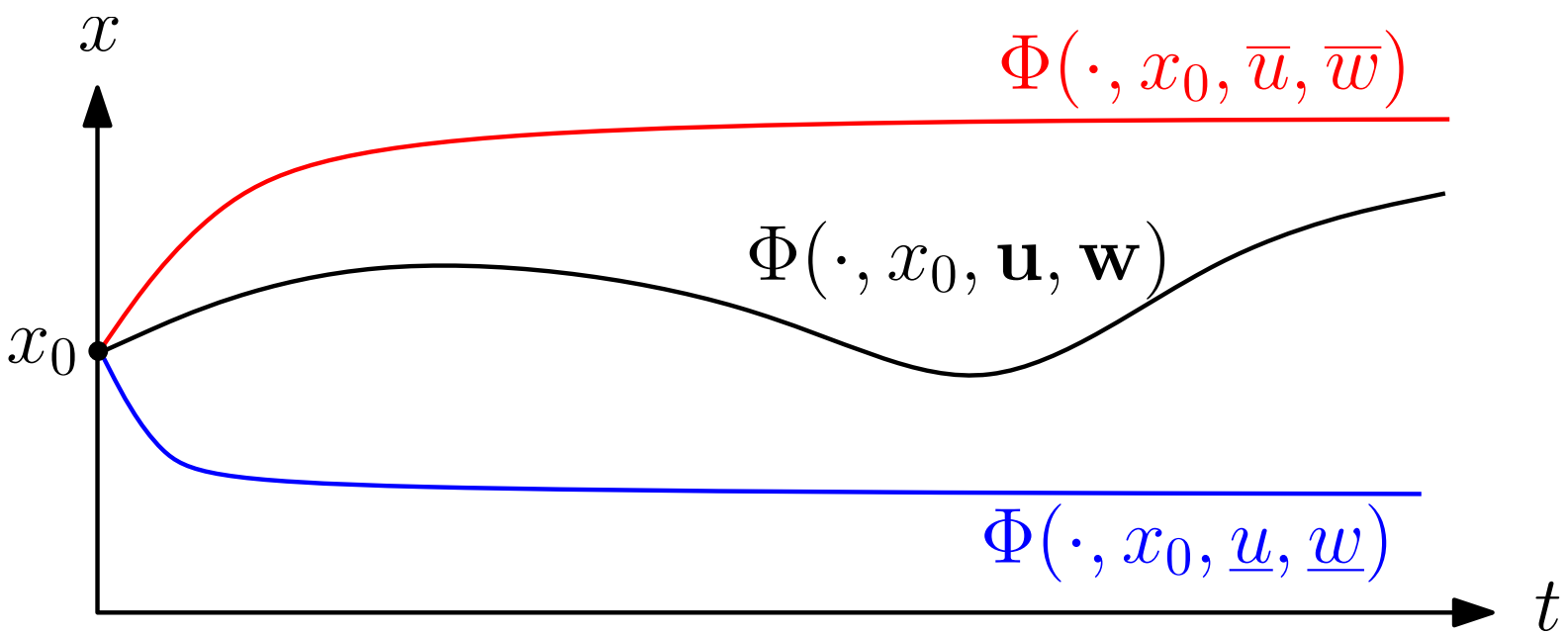
Bounded inputs

Control and disturbance inputs bounded in intervals:

$$\forall t \geq 0,\ \mathbf{u}(t) \in [\underline{u}, \overline{u}],\ \mathbf{w}(t) \in [\underline{w}, \overline{w}]$$

$$\Longrightarrow$$

$$\forall t \geq 0,\ \Phi(t, x_0, \mathbf{u}, \mathbf{w}) \in [\Phi(t, x_0, \underline{u}, \underline{w}), \Phi(t, x_0, \overline{u}, \overline{w})]$$



Characterization

Proposition (Kamke-Müller)

The system $\dot{x} = f(x, u, w)$ is monotone if and only if the following implication holds for all i:

$$u \geq u',\ w \geq w',\ x \geq x',\ x_i = x'_i \Rightarrow f_i(x, u, w) \geq f_i(x', u', w')$$

Proposition (Partial derivatives)

The system $\dot{x} = f(x, u, w)$ with continuously differentiable vector field f is monotone if and only if:

$$\forall i,\ j \neq i,\ k,\ l,\ \frac{\partial f_i}{\partial x_j} \geq 0,\ \frac{\partial f_i}{\partial u_k} \geq 0,\ \frac{\partial f_i}{\partial w_l} \geq 0$$

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1. Monotone control system
2. Robust controlled invariance
3. Symbolic control
4. Compositional approach
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Definitions

Definition (Robust Controlled Invariance)

A set $\mathcal{S}$ is a *robust controlled invariant* if there exists a controller such that the closed-loop system stays in $\mathcal{S}$ for any initial state and disturbance:

$$\exists \mathrm{u} : \mathcal{S} \to [\underline{u}, \overline{u}] \mid \forall x_0 \in \mathcal{S},\ \forall \mathbf{w} \in [\underline{w}, \overline{w}],\ \forall t \geq 0,\ \Phi_{\mathrm{u}}(t, x_0, \mathbf{w}) \in \mathcal{S}$$

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Definition (Local control)

Each control input affects a single state variable:

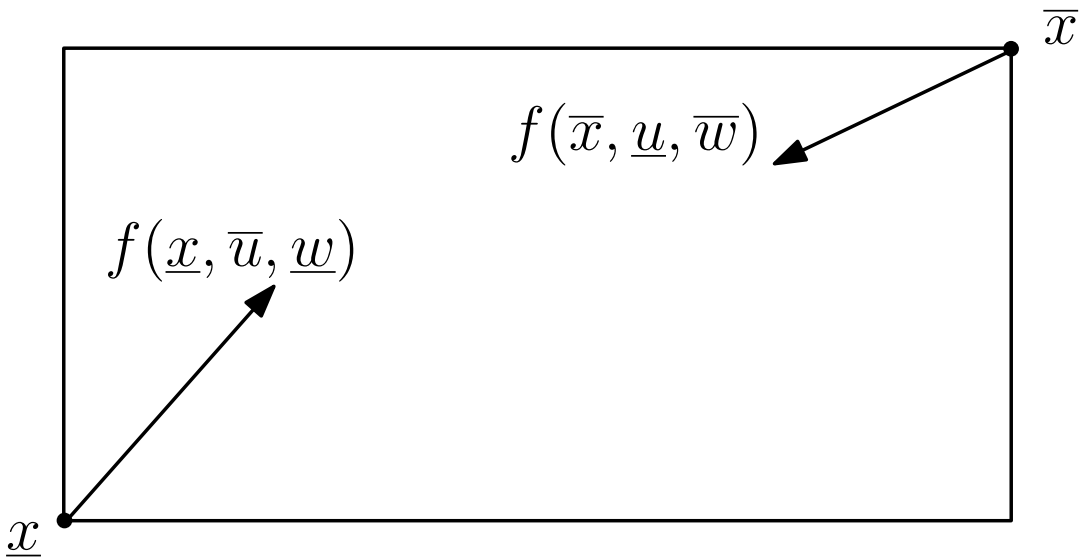
$$\forall k,\ \exists! i \mid \frac{\partial f_i}{\partial u_k} \neq 0$$

Robust controlled invariance

Theorem (Meyer, Girard, Witrant, CDC 2013)

With the monotonicity and local control properties, the interval $[\underline{x}, \overline{x}] \subseteq \mathbb{R}^n$ *is robust controlled invariant if and only if*

$$\begin{cases} f(\overline{x}, \underline{u}, \overline{w}) \leq 0 \\ f(\underline{x}, \overline{u}, \underline{w}) \geq 0 \end{cases}$$



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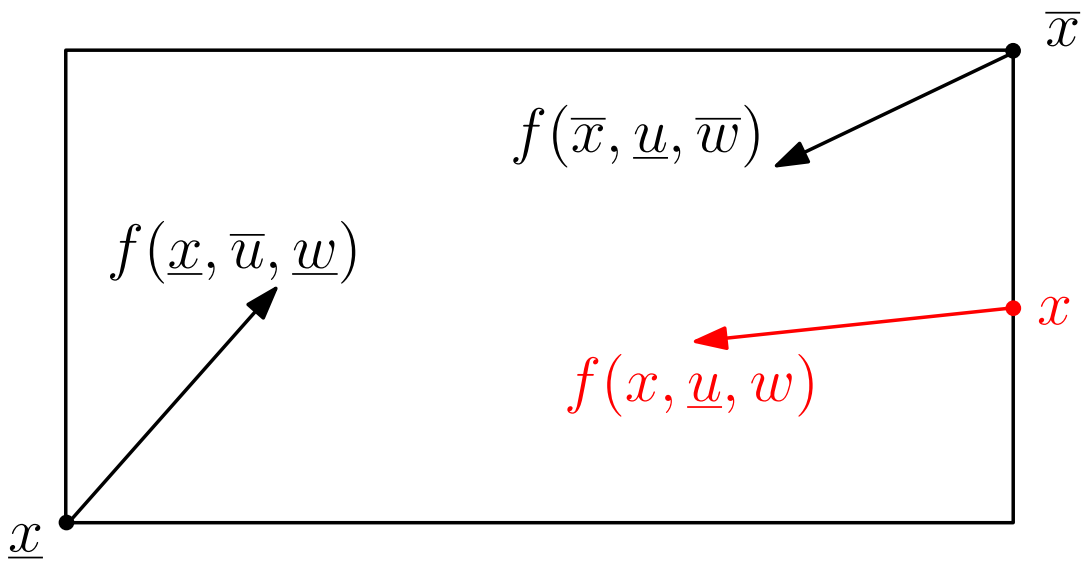

Sides of $[\underline{x}, \overline{x}]$

- $x \leq \overline{x}$, $x_1 = \overline{x}_1$
- $w \leq \overline{w}$

Kamke-Müller condition:

$$f_1(x, \underline{u}, w) \leq f_1(\overline{x}, \underline{u}, \overline{w}) \leq 0$$

Robust controlled invariance

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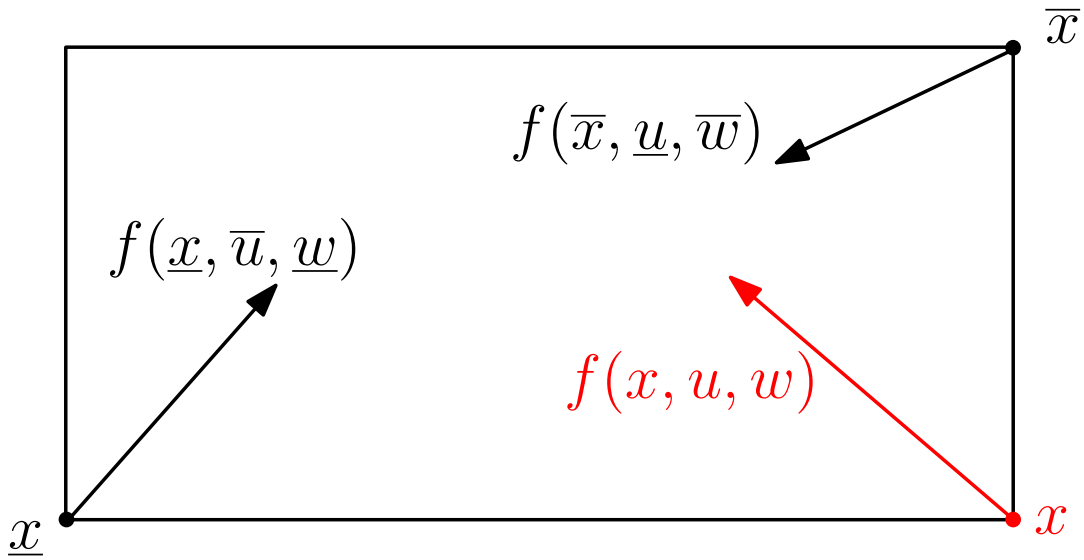
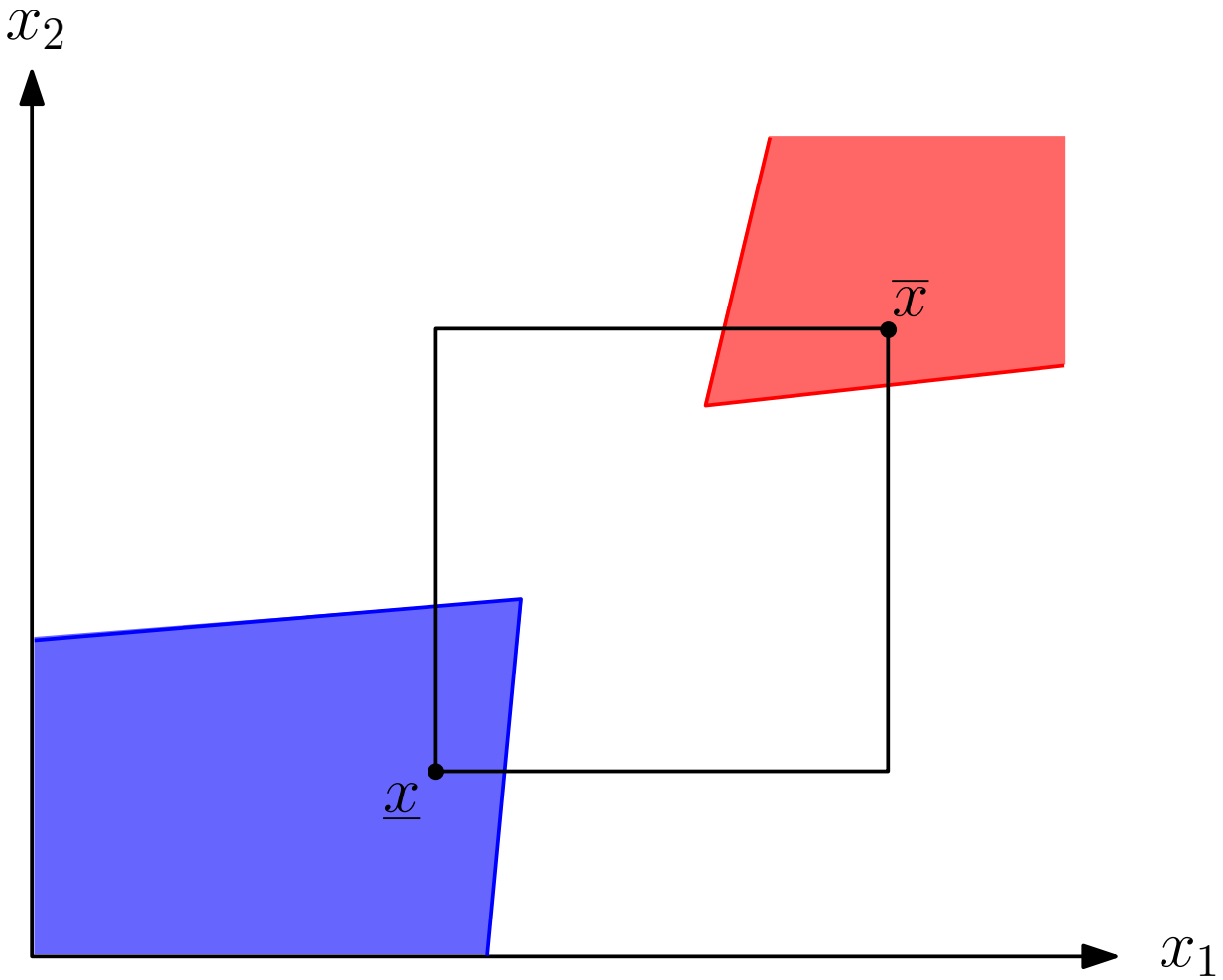


Other vertices of $[\underline{x}, \overline{x}]$

$$f_1(x, \underline{u}, w) \leq f_1(\overline{x}, \underline{u}, \overline{w}) \leq 0$$
$$f_2(x, \overline{u}, w) \geq f_2(\underline{x}, \overline{u}, \underline{w}) \geq 0$$

- Choice of u ?
- Use local control property

Choice of the interval



$$x \in \mathbb{R}^2$$

2 conditions on $\overline{x}$

- $f_1(\overline{x}, \underline{u}, \overline{w}) \leq 0$
- $f_2(\overline{x}, \underline{u}, \overline{w}) \leq 0$

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- $f_1(\underline{x}, \overline{u}, \underline{w}) \geq 0$
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Robust set stabilization

Definition (Stabilizing controller)

A controller $\mathrm{u} : \mathcal{S}_0 \to [\underline{u}, \overline{u}]$ is a stabilizing controller from $\mathcal{S}_0$ to $\mathcal{S}$ if

$$\forall x_0 \in \mathcal{S}_0,\ \forall \mathbf{w} \in [\underline{w}, \overline{w}],\ \exists T \geq 0 \mid \forall t \geq T,\ \ \Phi_{\mathrm{u}}(t, x_0, \mathbf{w}) \in \mathcal{S}$$

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Let $\mathcal{S}_0 = [\underline{x_0}, \overline{x_0}]$ and $\mathcal{S} = [\underline{x}, \overline{x}] \subseteq [\underline{x_0}, \overline{x_0}]$

Assumption

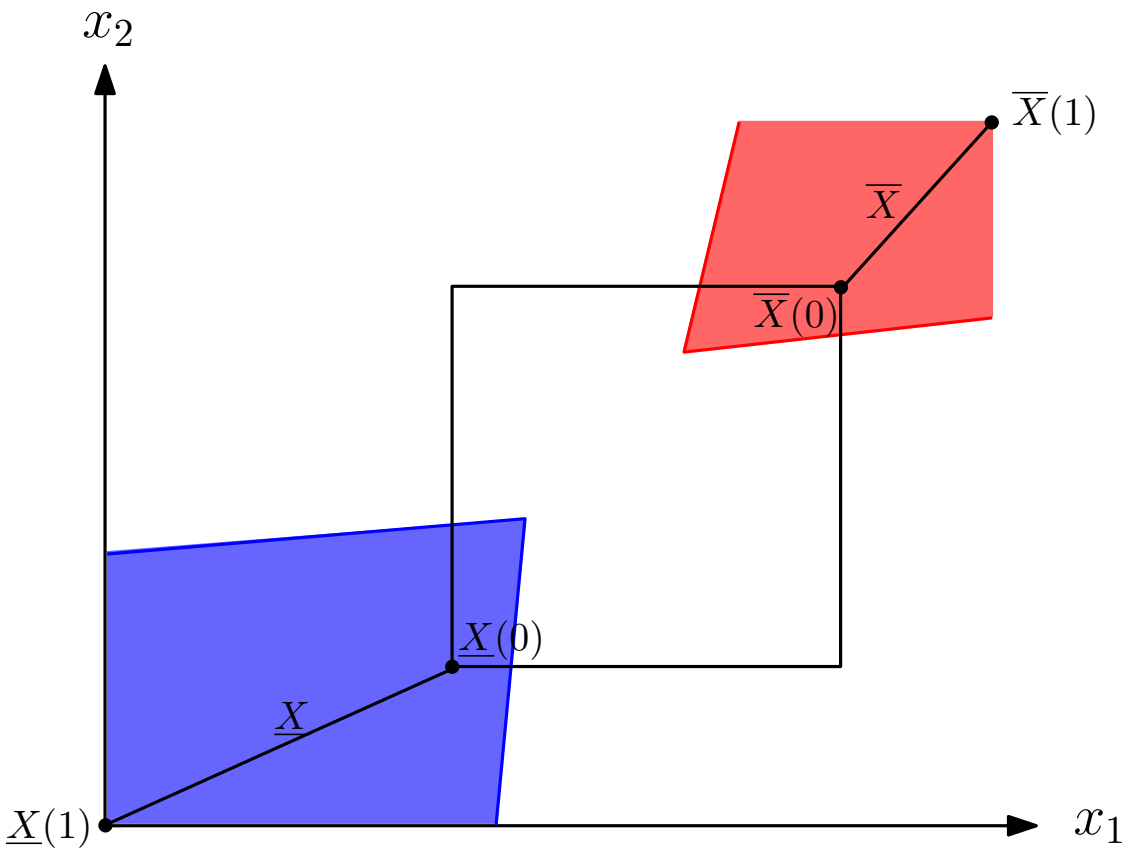
$\exists\ \underline{X},\ \overline{X} : [0,1] \to \mathbb{R}^n$, *respectively* **strictly decreasing and increasing**, *such that* $[\underline{X}(1), \overline{X}(1)] = [\underline{x_0}, \overline{x_0}]$, $[\underline{X}(0), \overline{X}(0)] = [\underline{x}, \overline{x}]$ *and satisfying*

$$f(\underline{X}(\lambda), \overline{u}, \underline{w}) > 0,\ \ f(\overline{X}(\lambda), \underline{u}, \overline{w}) < 0,\ \ \forall \lambda \in [0,1]$$

Robust set stabilization

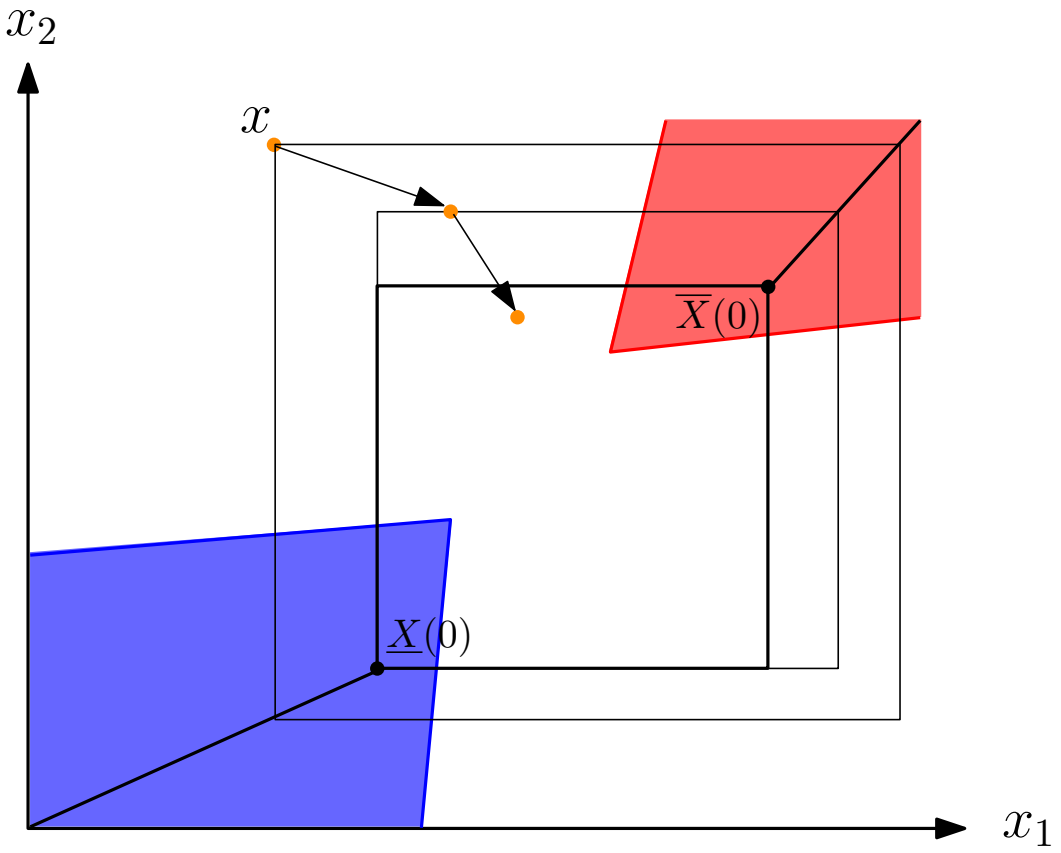
$$f(\underline{X}(\lambda), \overline{u}, \underline{w}) > 0, \;\; f(\overline{X}(\lambda), \underline{u}, \overline{w}) < 0, \;\; \forall \lambda \in [0,1]$$

$\forall \lambda, \lambda' \in [0,1]$, $[\underline{X}(\lambda), \overline{X}(\lambda')]$ is a robust controlled invariant interval



Robust set stabilization

Take the smallest interval of this family containing the current state x
Apply any invariance controller in this interval



Robust set stabilization

$$\begin{cases} \overline{\lambda}(x) = \min\{\lambda \in [0,1] \mid \overline{X}(\lambda) \geq x\} \\ \underline{\lambda}(x) = \min\{\lambda \in [0,1] \mid \underline{X}(\lambda) \leq x\} \end{cases}$$

$[\underline{X}(\underline{\lambda}(x)), \overline{X}(\overline{\lambda}(x))]$ is the smallest interval containing the current state x

Robust set stabilization

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$[\underline{X}(\underline{\lambda}(x)), \overline{X}(\overline{\lambda}(x))]$ is the smallest interval containing the current state x

Invariance controller	Candidate stabilizing controller	
$u_i(x) = \underline{u}_i + (\overline{u}_i - \underline{u}_i)\frac{\overline{x}_i - x_i}{\overline{x}_i - \underline{x}_i}$	$u_i(x) = \underline{u}_i + (\overline{u}_i - \underline{u}_i)\frac{\overline{X}_i(\overline{\lambda}(x)) - x_i}{\overline{X}_i(\overline{\lambda}(x)) - \underline{X}_i(\underline{\lambda}(x))}$	(1)

Theorem (Meyer, Girard, Witrant, prov. accepted in Automatica)

(1) *is a stabilizing controller from* $[\underline{X}(1), \overline{X}(1)]$ *to* $[\underline{X}(0), \overline{X}(0)]$.

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Abstraction-based synthesis

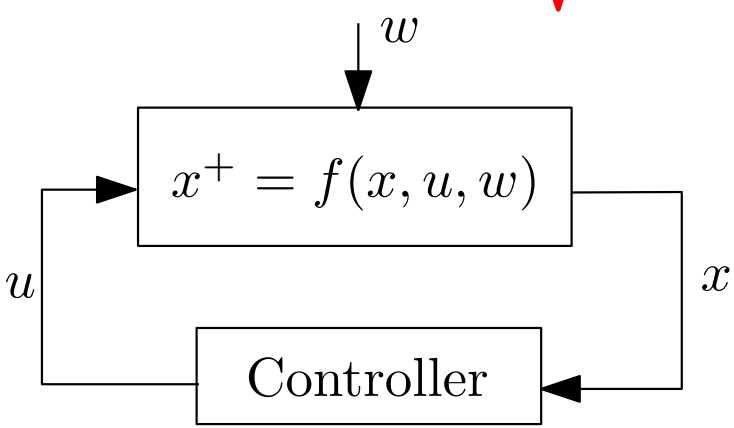
Continuous state

Uncontrolled

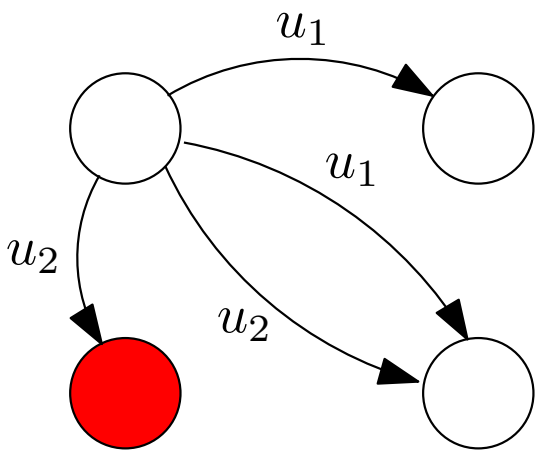
$x^+ = f(x, u, w)$

Synthesis

Controlled



Abstraction-based synthesis

	Continuous state	Discrete state
Uncontrolled	$x^+ = f(x, u, w)$	
Controlled		

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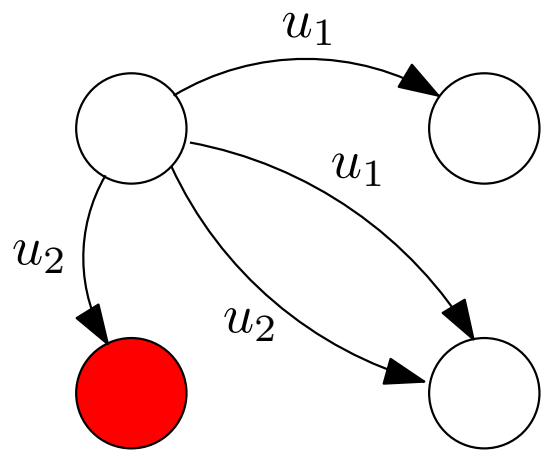
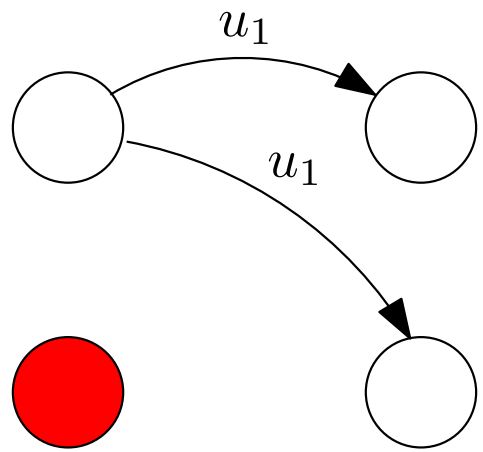
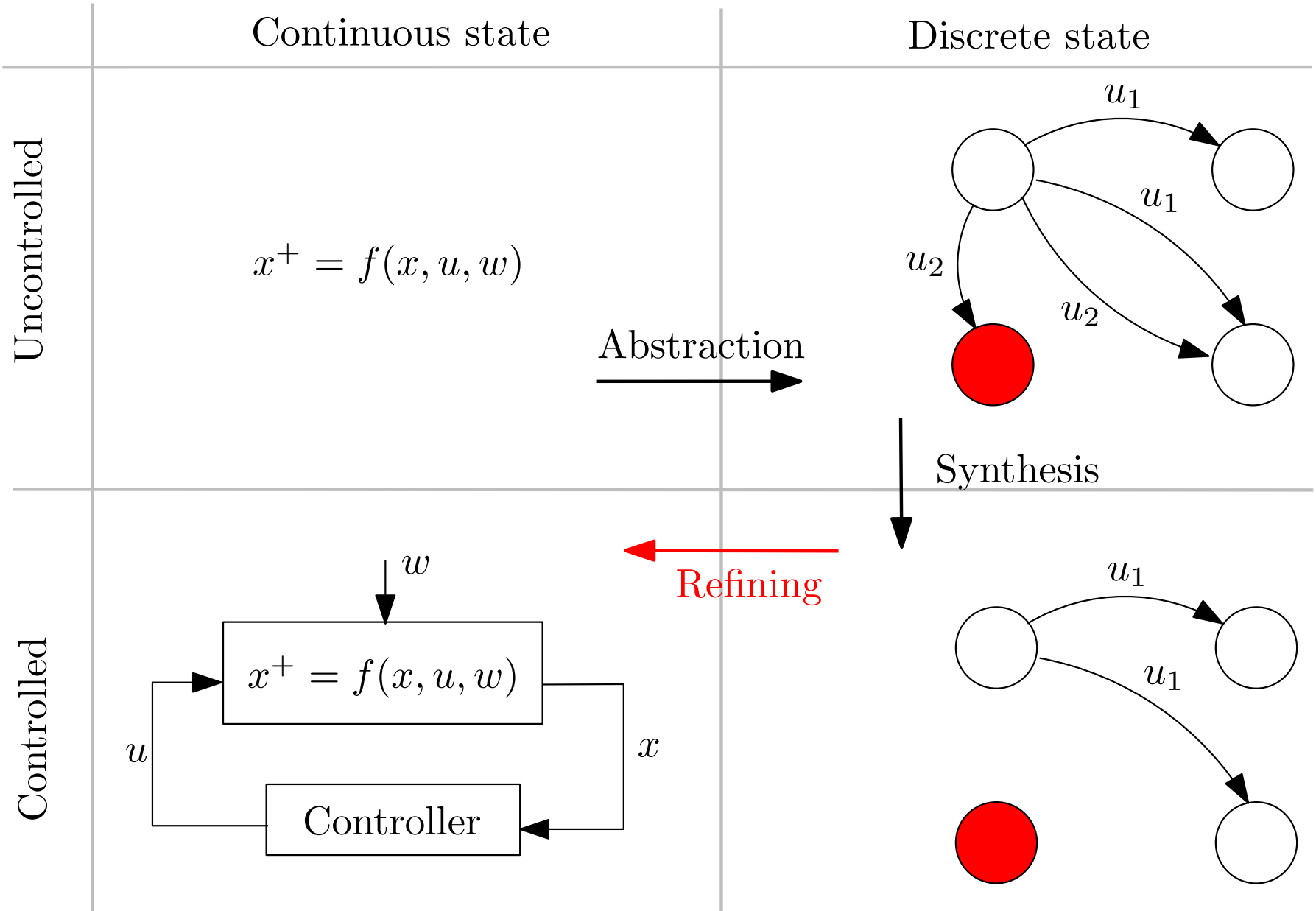
Uncontrolled	$x^+ = f(x, u, w)$	
Controlled		

Abstraction

Synthesis

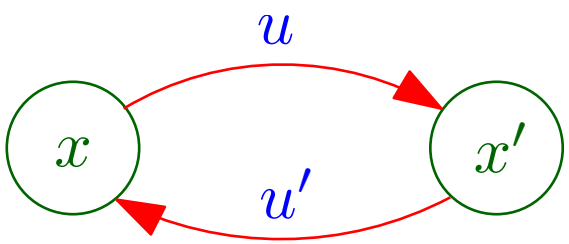
Abstraction-based synthesis



Transition systems

$S = (X, U, \longrightarrow)$

- Set of states X
- Set of inputs U
- Transition relation $\longrightarrow$
- Trajectories: $x_1 \xrightarrow{u_1} x_2 \xrightarrow{u_2} x_3 \xrightarrow{u_3} \dots$



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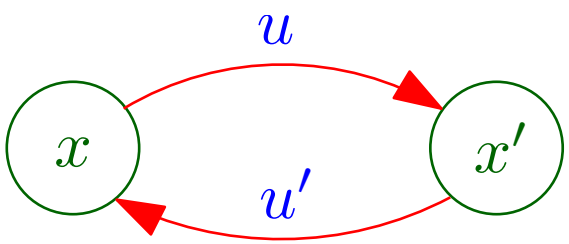
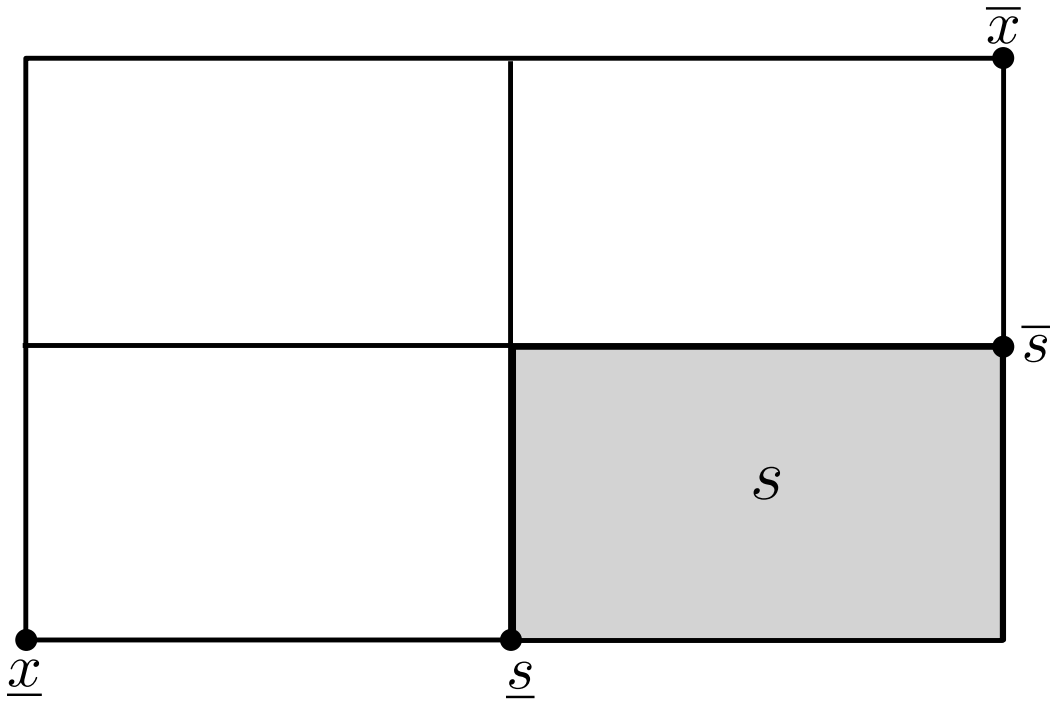


Sampled dynamics (sampling τ)

- $X = \mathbb{R}^n$
- $U = [\underline{u}, \overline{u}]$
- $x \xrightarrow{u} x' \iff \exists \mathbf{w} : [0, \tau] \to [\underline{w}, \overline{w}] \mid x' = \Phi(\tau, x, u, \mathbf{w})$
- Safety specification in $[\underline{x}, \overline{x}] \subseteq \mathbb{R}^n$

Abstraction

- Discretization of the control space $[\underline{u}, \overline{u}]$
- Partition $\mathcal{P}$ of the interval $[\underline{x}, \overline{x}]$ into **symbols**



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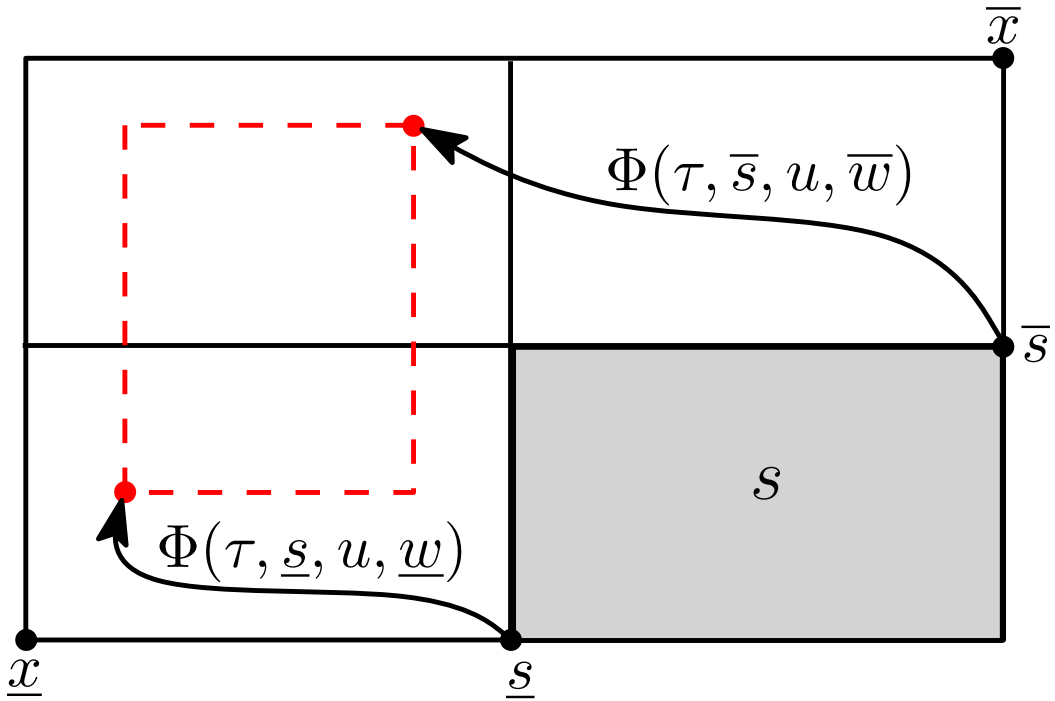
- Over-approximation of the reachable set (monotonicity)



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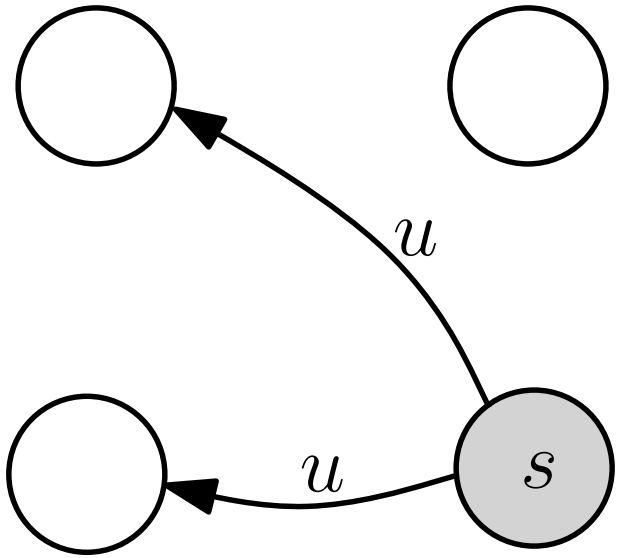
- Partition $\mathcal{P}$ of the interval $[\underline{x}, \overline{x}]$ into **symbols**
- Over-approximation of the reachable set (monotonicity)
- Intersection with the partition



Obtain a finite abstraction $S_a = (X_a, U_a, \underset{a}{\longrightarrow})$

Alternating simulation

Definition (Alternating simulation relation)

$H : X \to X_a$ is an alternating simulation relation from S_a to S if:

$$\forall u_a \in U_a,\ \exists u \in U \mid x \xrightarrow{u} x' \text{ in } S \Longrightarrow H(x) \xrightarrow[a]{u_a} H(x') \text{ in } S_a$$

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Proposition

The map $H : X \to X_a$ defined by

$$H(x) = s \Longleftrightarrow x \in s$$

is an alternating simulation relation from S_a to S:

$$\forall u_a \in U_a \subseteq U \mid x \xrightarrow{u_a} x' \text{ in } S \Longrightarrow H(x) \xrightarrow[a]{u_a} H(x') \text{ in } S_a$$

Safety synthesis

Specification: safety of S_a in $\mathcal{P}$ (the partition of the interval $[\underline{x}, \overline{x}]$)

$$F_{\mathcal{P}}(Z) = \{s \in Z \cap \mathcal{P} \mid \exists\ u,\ \forall\ s \xrightarrow[a]{u} s',\ s' \in Z\}$$

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Fixed-point Z_a of $F_{\mathcal{P}}$ reached in **finite time**
Z_a is the **maximal safe set** for S_a, associated with the safety controller:

$$C_a(s) = \{u \mid \forall\ s \xrightarrow[a]{u} s',\ s' \in Z_a\}$$

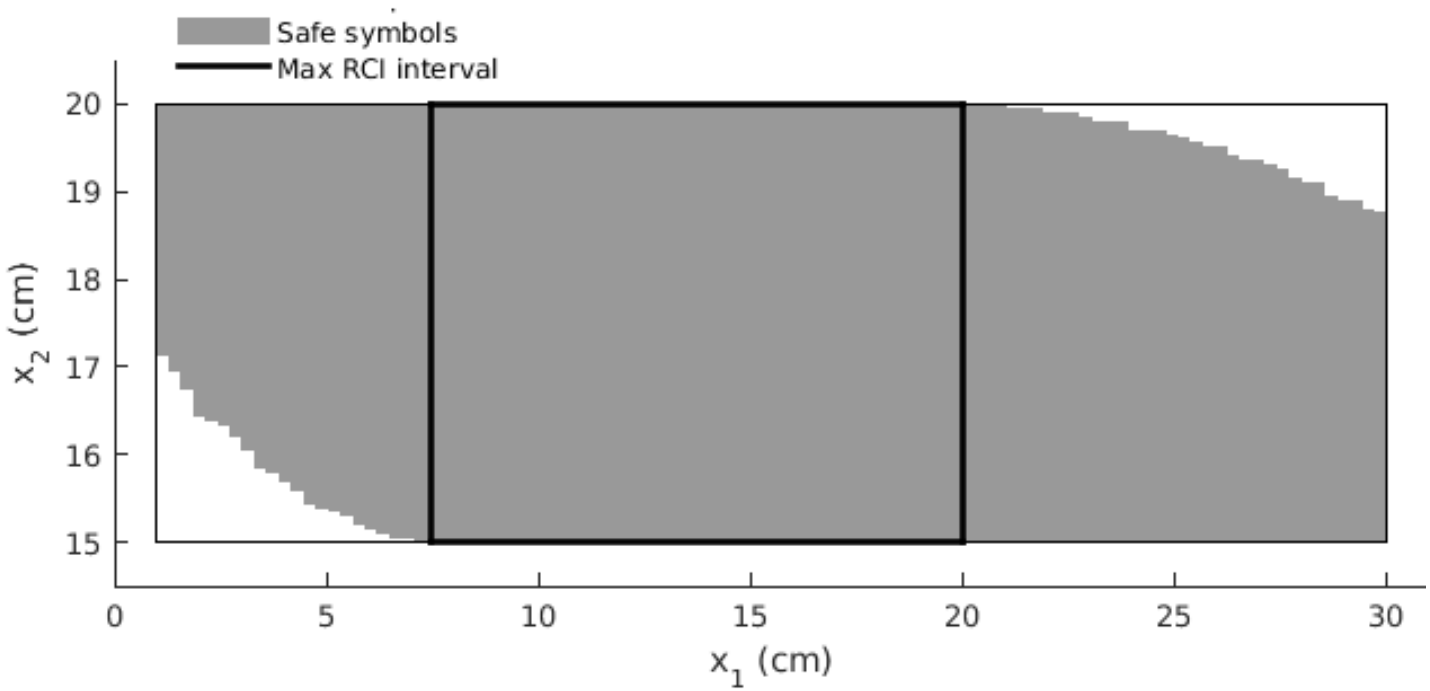
Theorem

C_a is a safety controller for S in Z_a.

Safety vs invariance

$2D$ example with a partition of 100×100 symbols

- Chosen interval is **not** robust controlled invariant
- Compare the safe set Z_a with the largest robust controlled invariant sub-interval



Performance criterion

Minimize on a trajectory $(x^0, u^0, x^1, u^1, \dots)$ of S:

$$\sum_{k=0}^{+\infty} \lambda^k g(x^k, u^k)$$

with a cost function g and a **discount factor** $\lambda \in (0, 1)$

Performance criterion

Minimize on a trajectory $(x^0, u^0, x^1, u^1, \dots)$ of S:

$$\sum_{k=0}^{+\infty} \lambda^k g(x^k, u^k)$$

with a cost function g and a **discount factor** $\lambda \in (0, 1)$

Cost function on S_a: $g_a(s, u) = \max_{x \in s} g(x, u)$

Focus the optimization on a **finite horizon** of N sampling periods

Accurate approximation if $\lambda^{N+1} \ll 1$

Optimization

Dynamic programming algorithm:

$$J_a^N(s) = \min_{u \in C_a(s)} g_a(s,u)$$

$$J_a^k(s) = \min_{u \in C_a(s)} \left(g_a(s,u) + \lambda \max_{s \xrightarrow[a]{u} s'} J_a^{k+1}(s') \right), \ \forall k < N$$

$J_a^0(s)$ is the **worst-case minimization** of $\sum_{k=0}^{N} \lambda^k g_a(s^k, u^k)$

Optimization

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$J_a^0(s)$ is the **worst-case minimization** of $\sum_{k=0}^{N} \lambda^k g_a(s^k, u^k)$

Receding horizon controller:

$$C_a^*(s) = \arg\min_{u \in C_a(s)} \left(g_a(s,u) + \lambda \max_{s \xrightarrow[a]{u} s'} J_a^1(s') \right)$$

Performance guarantees

Theorem (Meyer, Girard, Witrant, HSCC 2015)

Let $(x^0, u^0, x^1, u^1, \dots)$ *be a trajectory of* S *controlled with* C_a^*.
Let $s^0, s^1, \dots$ *such that* $x^k \in s^k$, *for all* $k \in \mathbb{N}$. *Then,*

$$\sum_{k=0}^{+\infty} \lambda^k g(x^k, u^k) \leq$$

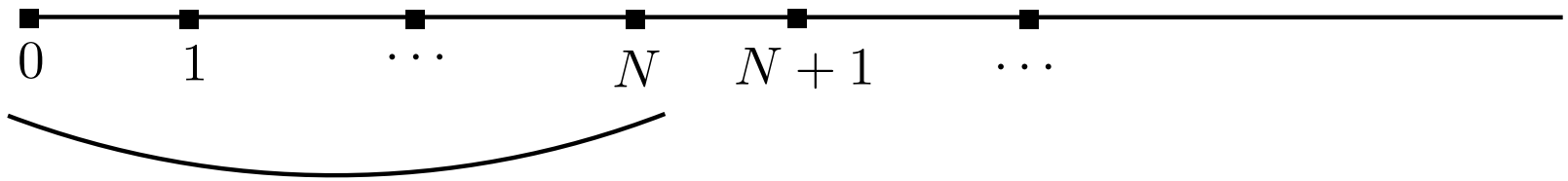
Performance guarantees

Theorem (Meyer, Girard, Witrant, HSCC 2015)

Let $(x^0, u^0, x^1, u^1, \dots)$ be a trajectory of S controlled with C_a^*.
Let $s^0, s^1, \dots$ such that $x^k \in s^k$, for all $k \in \mathbb{N}$. Then,

$$\sum_{k=0}^{+\infty} \lambda^k g(x^k, u^k) \leq J_a^0(s^0) +$$

Worst-case minimization on finite horizon:



$$\sum_{k=0}^{N} \lambda^k g_a(s^k, u^k) \leq J_a^0(s^0)$$

Performance guarantees

Theorem (Meyer, Girard, Witrant, HSCC 2015)

Let $(x^0, u^0, x^1, u^1, \dots)$ be a trajectory of S controlled with C_a^*.
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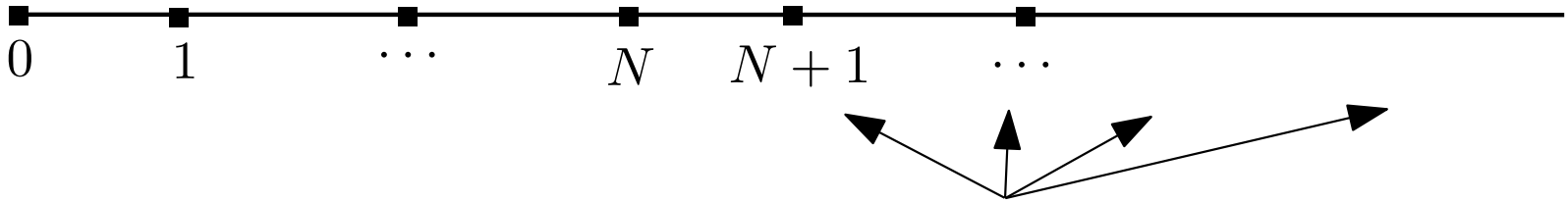
$$\sum_{k=0}^{+\infty} \lambda^k g(x^k, u^k) \leq J_a^0(s^0) + \frac{\lambda^{N+1}}{1-\lambda} M_a$$

Worst-case minimization of each remaining steps (receding horizon):



$$g_a(s^k, u^k) \leq \max_{s \in Z_a} \min_{u \in C_a(s)} g_a(s, u) = M_a$$

Outline

1 Monotone control system

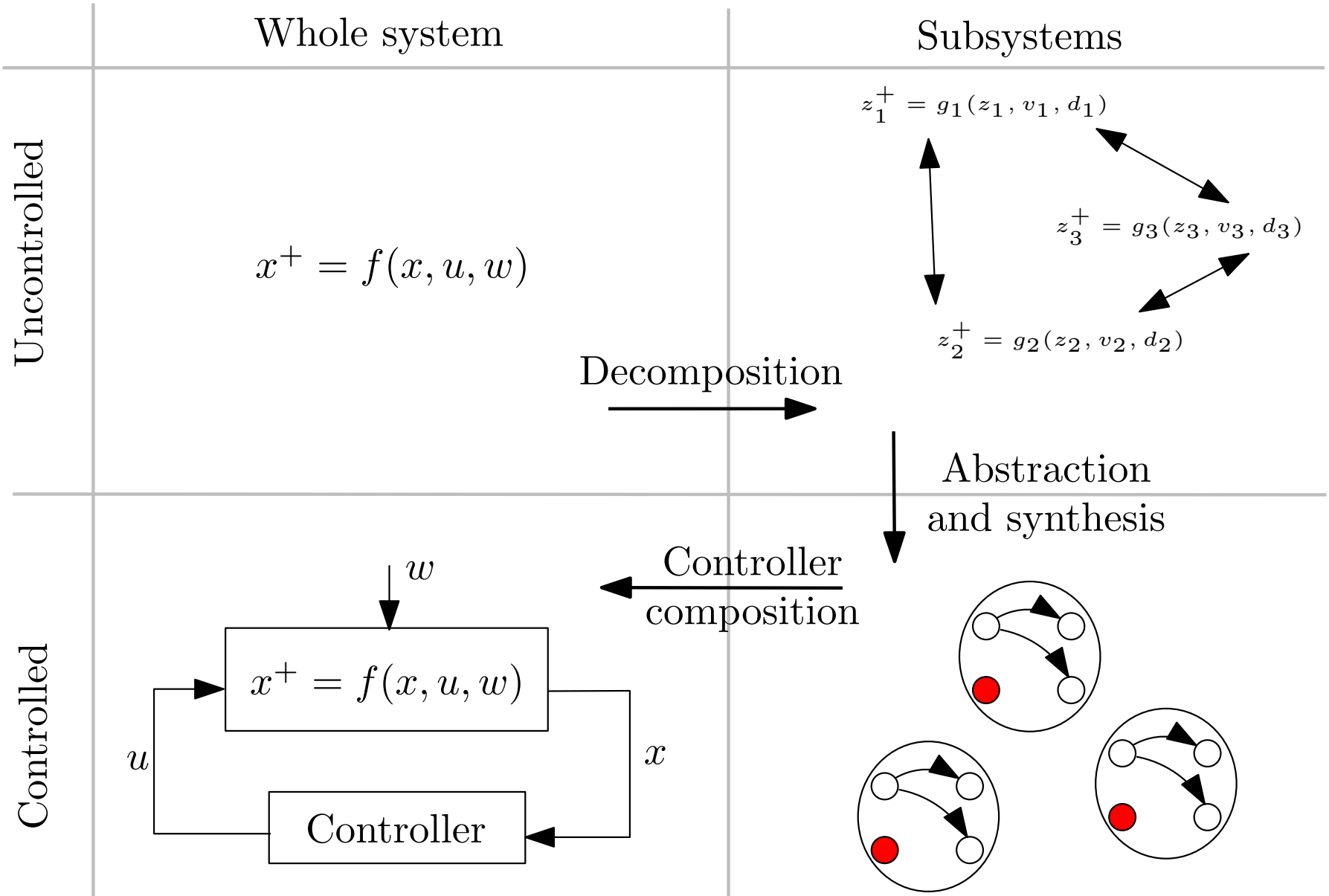
2 Robust controlled invariance

3 Symbolic control

4 Compositional approach

5 Control in intelligent buildings

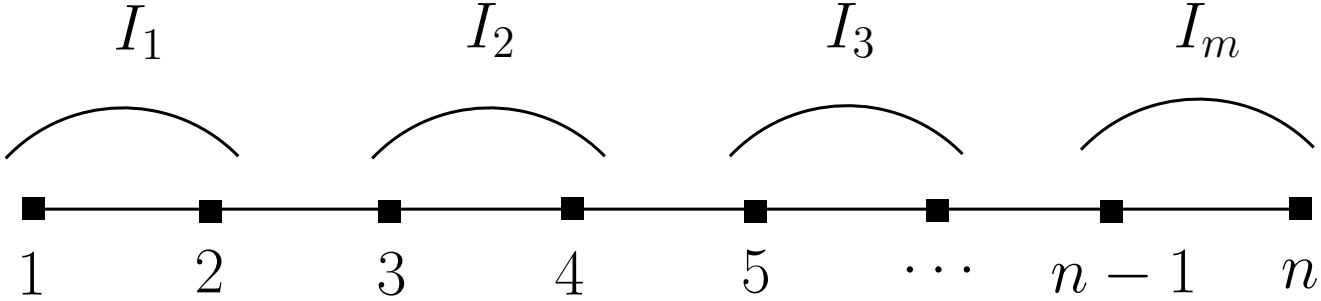
Compositional synthesis



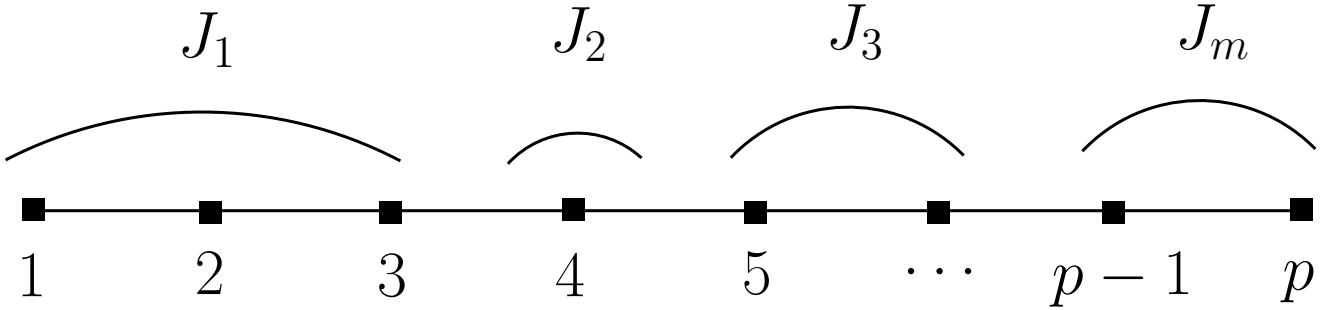
Decomposition

Decomposition into m subsystems:

Partition $(I_1, \ldots, I_m)$ of the **state** dimensions $\{1, \ldots, n\}$



Partition $(J_1, \ldots, J_m)$ of the **input** dimensions $\{1, \ldots, p\}$



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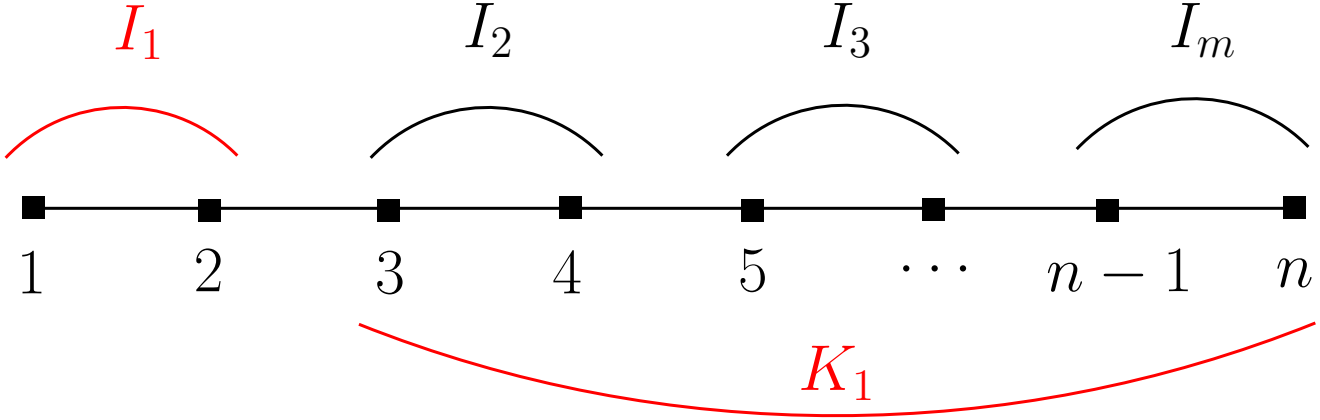
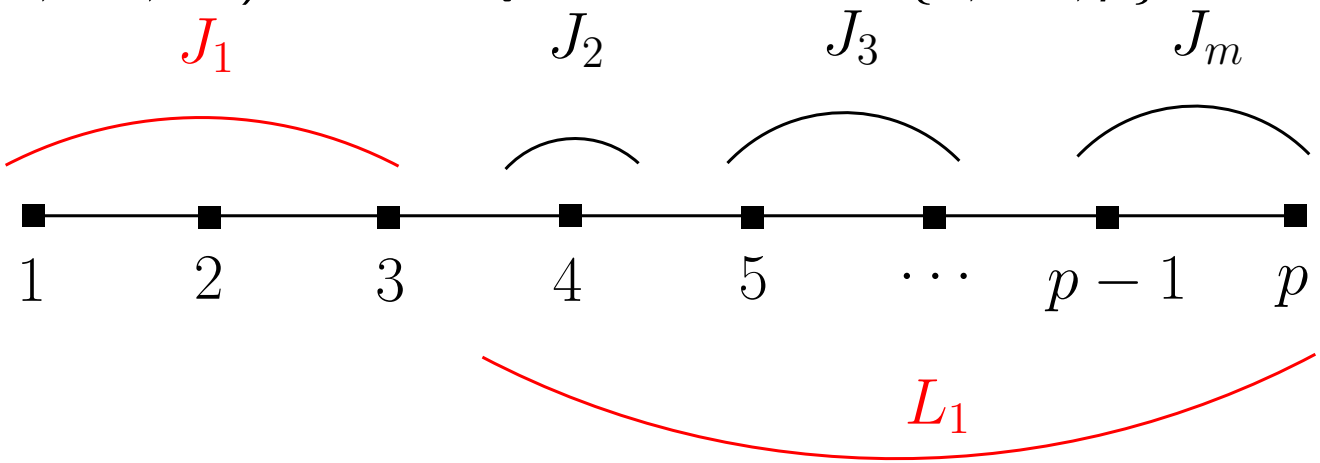


Partition $(J_1, \ldots, J_m)$ of the **input** dimensions $\{1, \ldots, p\}$



Control the states x_{I_1} using the inputs u_{J_1} with disturbances x_{K_1} and u_{L_1}

Abstraction

Symbolic abstraction $S_i = (X_i, U_i, \underset{i}{\longrightarrow})$ of subsystem $i \in \{1, \ldots, m\}$:

Classical method, but with an **assume-guarantee obligation**:

A/G Obligation (K_i)

Unobserved states: $x_{K_i} \in [\underline{x}_{K_i}, \overline{x}_{K_i}]$

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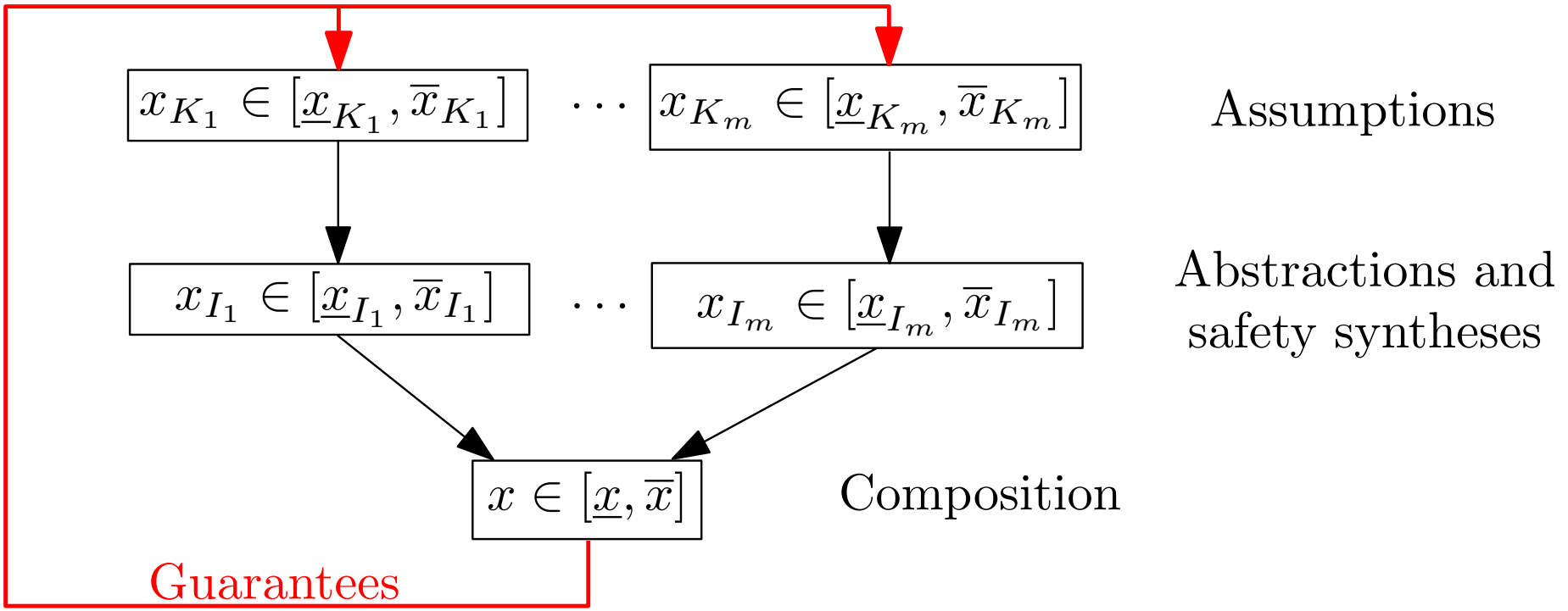

Synthesis

Safety synthesis in the partition of $[\underline{x}_{I_i}, \overline{x}_{I_i}]$:

- maximal safe set: $Z_i \subseteq X_i$
- safety controller: $C_i : Z_i \to 2^{U_i}$

Performances optimization:

- cost function $g_i(s_{I_i}, u_{J_i})$, with $g_a(s, u) \leq \sum_{i=1}^{m} g_i(s_{I_i}, u_{J_i})$
- deterministic controller: $C_i^* : Z_i \to U_i$

Safety

Composition of safe sets and safety controllers:

- $Z_c = Z_1 \times \cdots \times Z_m$
- $\forall s \in Z_c,\ C_c(s) = C_1(s_{I_1}) \times \cdots \times C_m(s_{I_m})$

Theorem (Meyer, Girard, Witrant, ADHS 2015)

C_c is a safety controller for S in Z_c.

Safety

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Theorem (Meyer, Girard, Witrant, ADHS 2015)

C_c is a safety controller for S in Z_c.

Proposition (Safety comparison)

$Z_c \subseteq Z_a$.

Performance guarantees

- $\forall s \in Z_c,\ C_c^*(s) = (C_1^*(s_{I_1}), \ldots, C_m^*(s_{I_m}))$
- Let $M_i = \max_{s_i \in Z_i} \min_{u_i \in C_i(s_i)} g_i(s_i, u_i)$

Theorem (Meyer, Girard, Witrant, ADHS 2015)

Let $(x^0, u^0, x^1, u^1, \ldots)$ be a trajectory of S controlled with C_c^*. Let $s^0, s^1, \ldots$ such that $x^k \in s^k$, for all $k \in \mathbb{N}$. Then,

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Performance guarantees

- $\forall s \in Z_c,\ C_c^*(s) = (C_1^*(s_{I_1}), \dots, C_m^*(s_{I_m}))$
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Proposition (Guarantees comparison)

$$\forall s \in Z_c, \qquad J_a^0(s) + \frac{\lambda^{N+1}}{1-\lambda} M_a \leq \sum_{i=1}^{m} J_i^0(s_{I_i}) + \frac{\lambda^{N+1}}{1-\lambda} \sum_{i=1}^{m} M_i$$

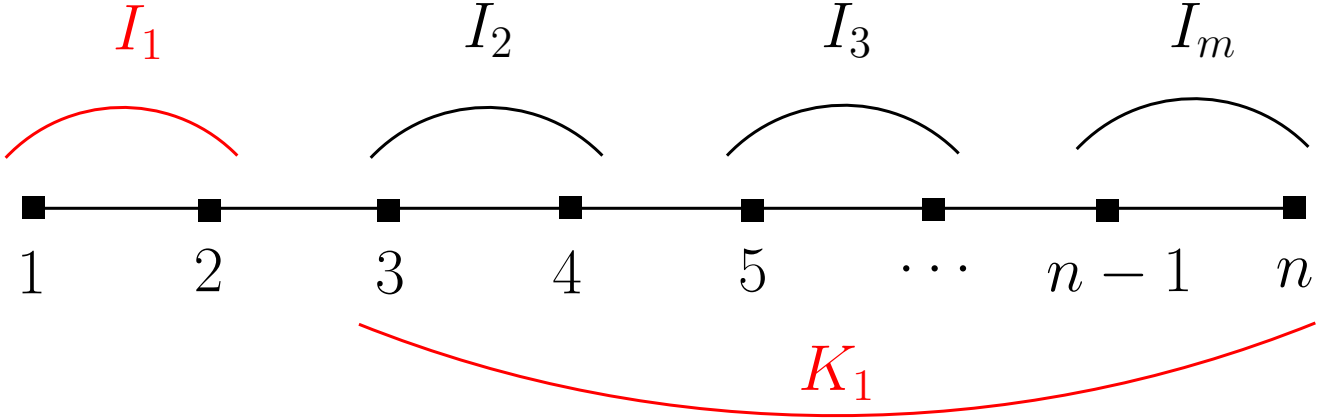
Complexity

- n: state space dimension
- p: control space dimension
- $\alpha_x \in \mathbb{N}$: number of symbols **per dimension** in the state partition
- $\alpha_u \in \mathbb{N}$: number of controls **per dimension** in the input discretization
- $|\cdot|$: cardinality of a set

	Method	
	Centralized	Compositional
Complexity	$\alpha_x^n \alpha_u^p$	$\sum_{i=1}^{m} \alpha_x^{\lvert I_i \rvert} \alpha_u^{\lvert J_i \rvert}$

Generalization

Decomposition into m subsystems:
Partition $(I_1, \ldots, I_m)$ of the **state** dimensions $\{1, \ldots, n\}$



Subsystem $i \in \{1, \ldots, m\}$:

- I_i: controlled states
- K_i: unobserved states (disturbances)

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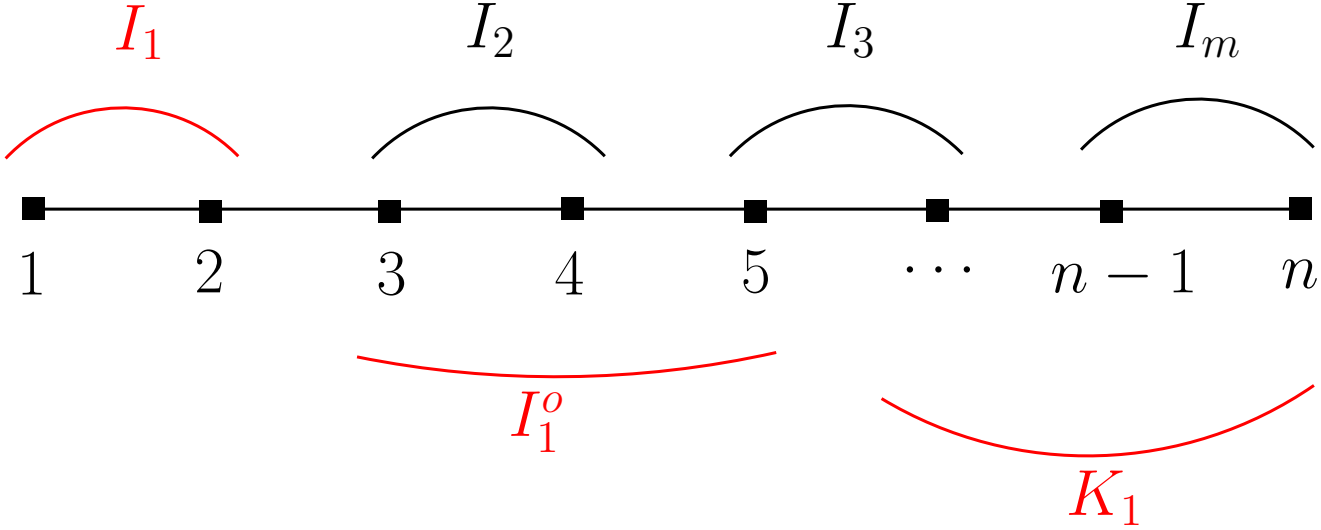

Subsystem $i \in \{1, \ldots, m\}$:

- I_i: controlled states
- I_i^o: observed but uncontrolled states
- K_i: unobserved states (disturbances)

Consequences

- Symbolic abstraction: needs two **assume-guarantee obligations**

A/G Obligation (K_i)

Unobserved states: $x_{K_i} \in [\underline{x}_{K_i}, \overline{x}_{K_i}]$

A/G Obligation (I_i^o)

Observed but uncontrolled states: $x_{I_i^o} \in [\underline{x}_{I_i^o}, \overline{x}_{I_i^o}]$

Consequences

- Symbolic abstraction: needs two **assume-guarantee obligations**

A/G Obligation (K_i)

Unobserved states: $x_{K_i} \in [\underline{x}_{K_i}, \overline{x}_{K_i}]$

A/G Obligation (I_i^o)

Observed but uncontrolled states: $x_{I_i^o} \in [\underline{x}_{I_i^o}, \overline{x}_{I_i^o}]$

- State composition with **overlaps**: $Z_c = Z_1 \Cap \cdots \Cap Z_m$
- Complexity depends on all **modeled states**: $\alpha_x^{|I_i \cup I_i^o|}$

Outline

1. Monotone control system
2. Robust controlled invariance
3. Symbolic control
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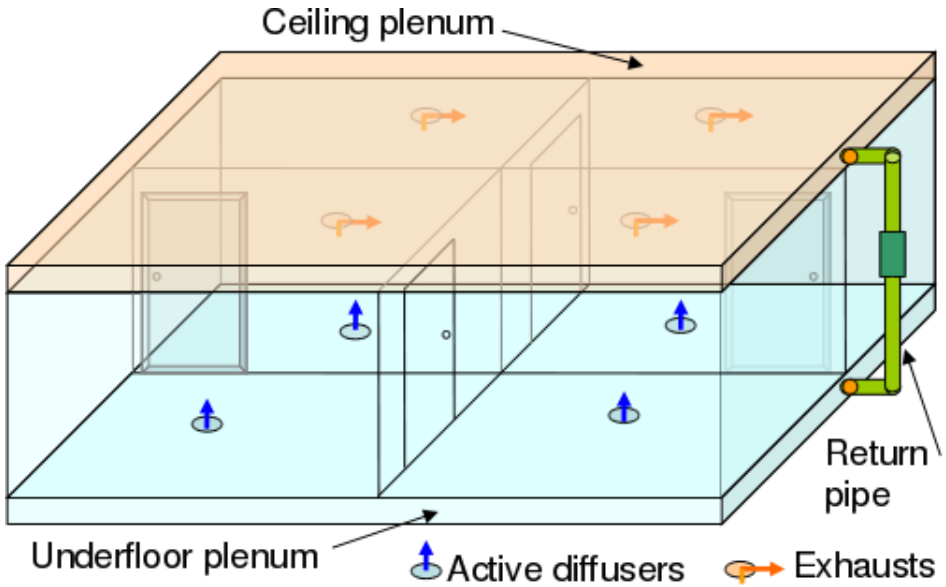
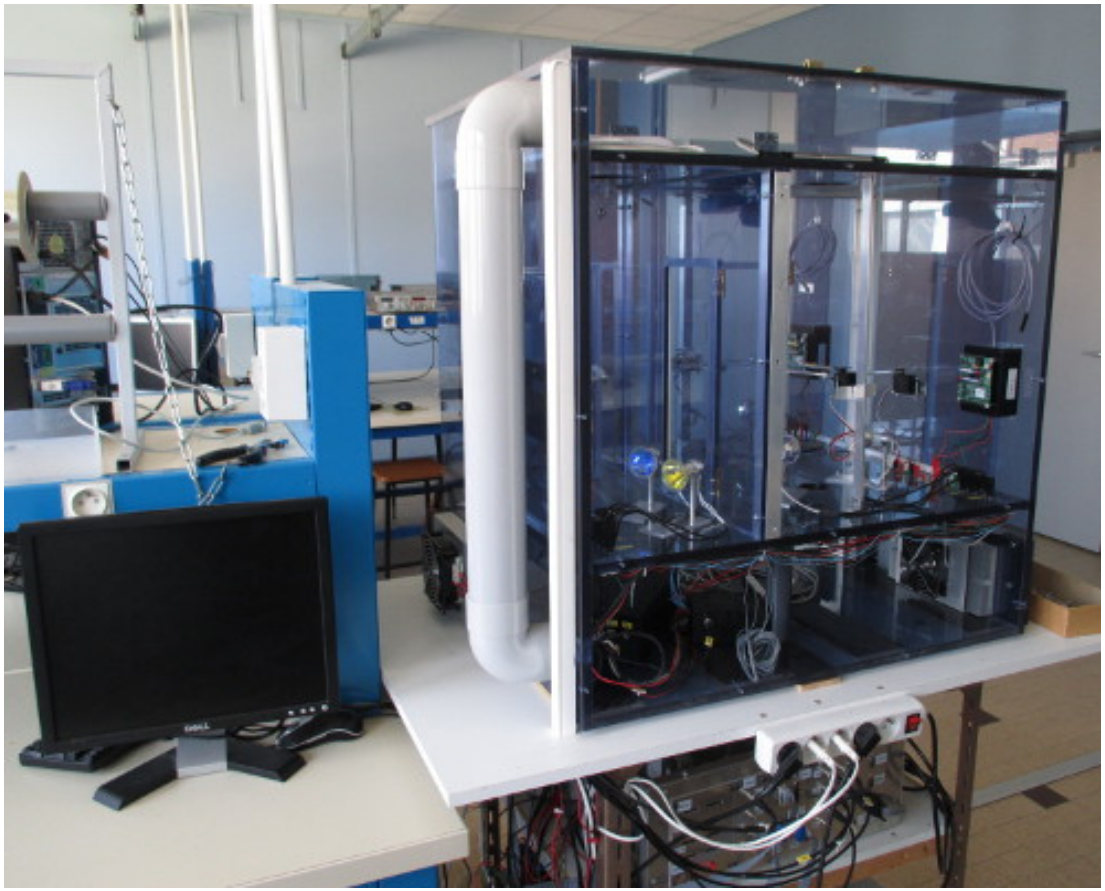
UnderFloor Air Distribution



- Underfloor air cooled down
- Sent into the rooms by fans
- Air excess pushed through the ceiling exhausts
- Returned to the underfloor

- Disturbances: heat sources; opening of doors

Experimental building



- $\approx 1m^3$
- 4 rooms with 4 doors
- 3 Peltier coolers
- Heat sources: lamps
- CompactRIO
- LabVIEW

Temperature model

Assume a **uniform temperature** in each room
Combine **energy and mass conservation** equations

Our model: $\dot{T} = f(T, u, w, \delta)$

- $T \in \mathbb{R}^4$: state (temperature)
- $u \in \mathbb{R}^4$: controlled input (fan air flow)
- w: exogenous input (other temperatures)
- δ: discrete disturbance

The model:

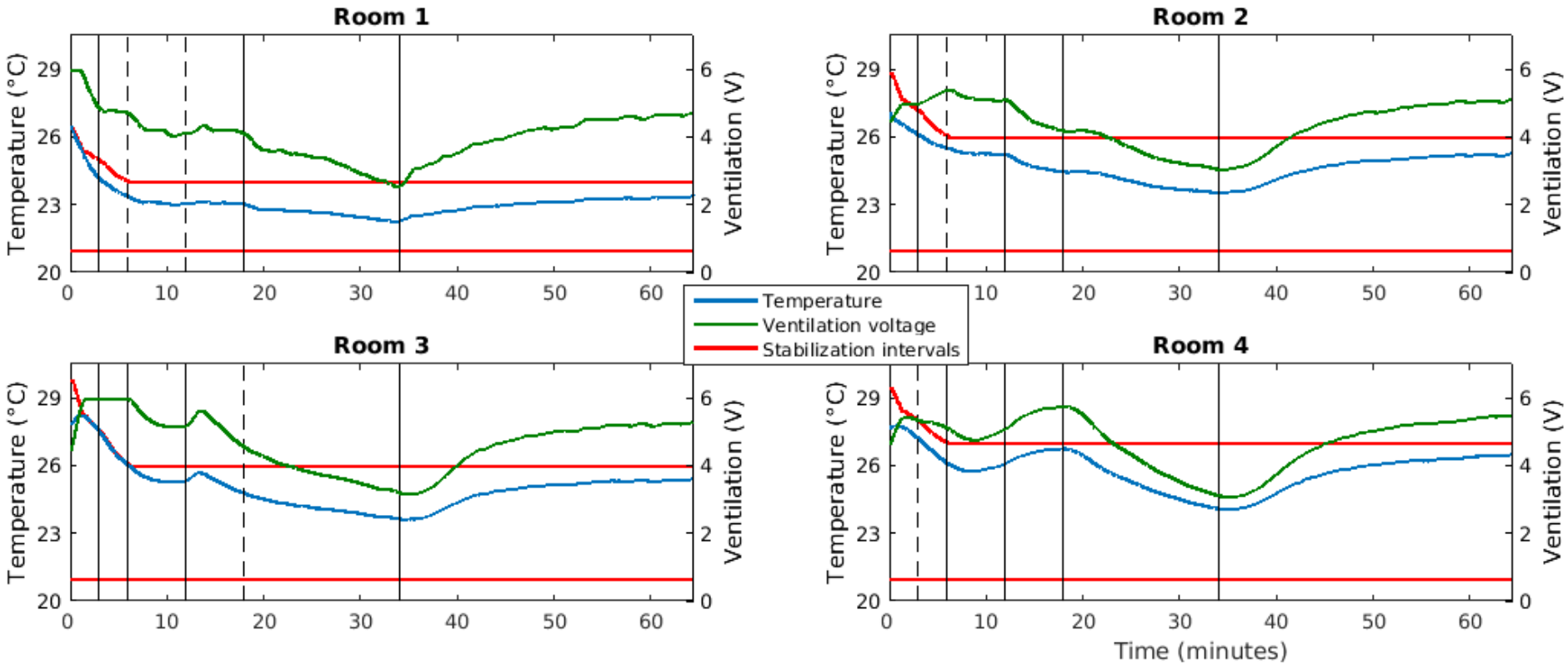
- is **monotone**
- has been **validated** by experimental data [1]
- its parameters are **identified** to match the experiment [1]

[1] Meyer, Nazarpour, Girard, Witrant, BuildSys 2013 & ECC 2014

Robust controlled invariance

Invariance controller: $\mathrm{u}_i(T) = \overline{u_i}\dfrac{T_i - \underline{T_i}}{\overline{T_i} - \underline{T_i}}$

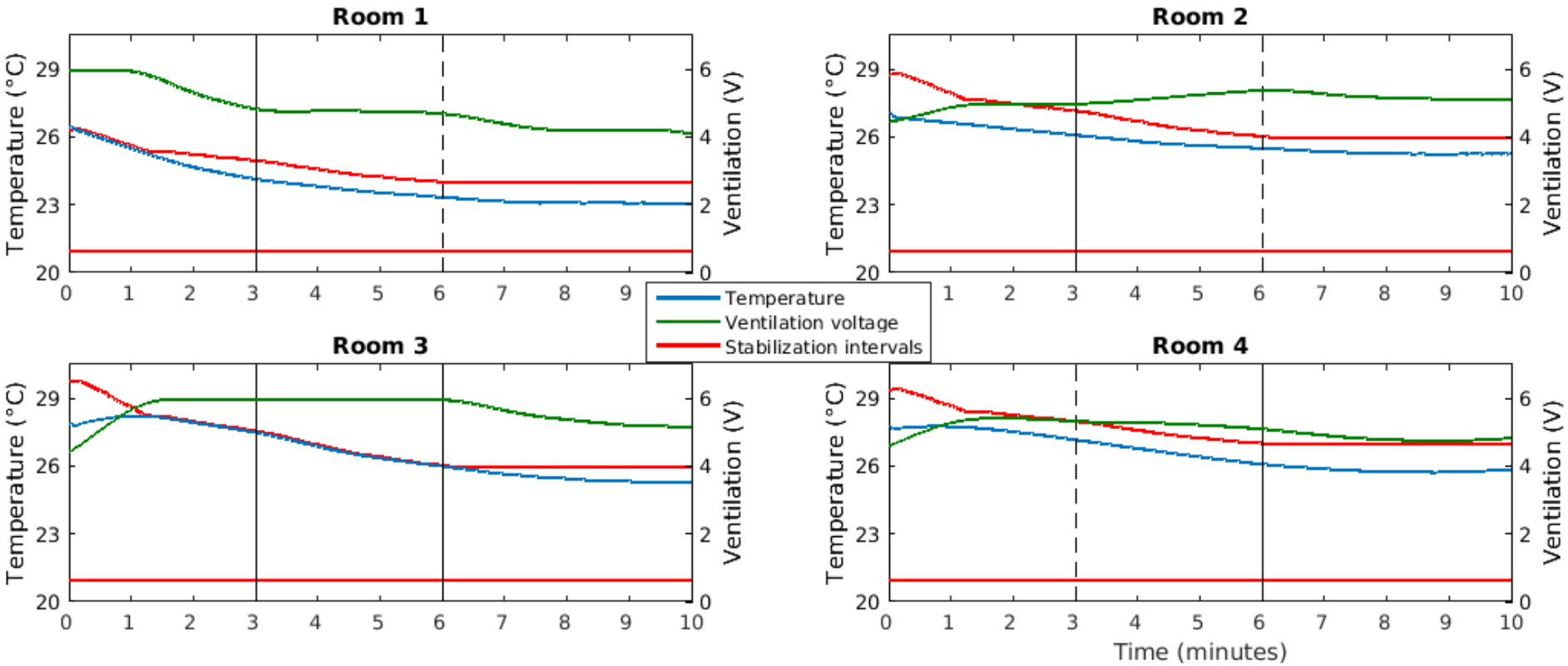
- $T_i = \overline{T_i}$: max ventilation
- $T_i = \underline{T_i}$: no ventilation



Robust set stabilization

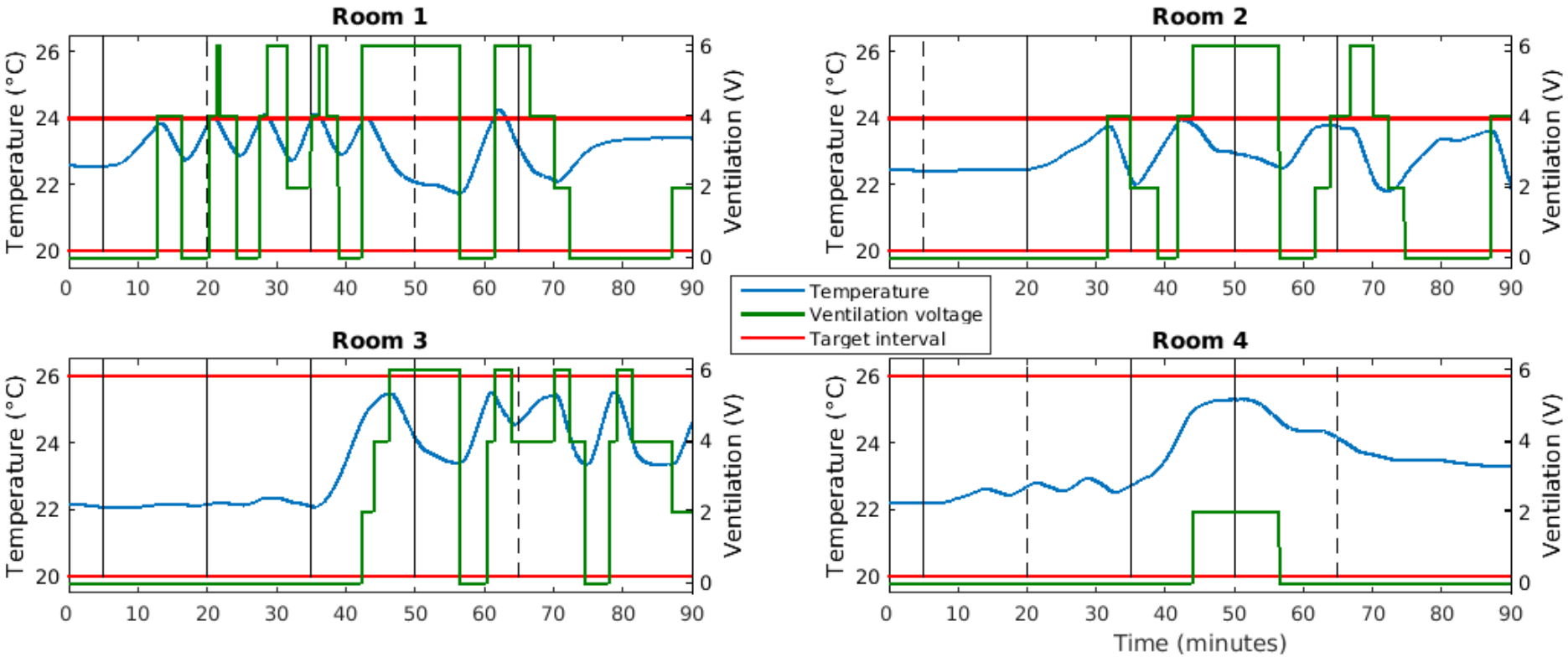
Stabilizing controller: $\mathrm{u}_i(T) = \overline{u}_i \dfrac{T_i - \underline{X}_i(\underline{\lambda}(T))}{\overline{X}_i(\overline{\lambda}(T)) - \underline{X}_i(\underline{\lambda}(T))}$

$[\underline{X}_i(\underline{\lambda}(T)), \overline{X}_i(\overline{\lambda}(T))]$: smallest RCI interval containing T



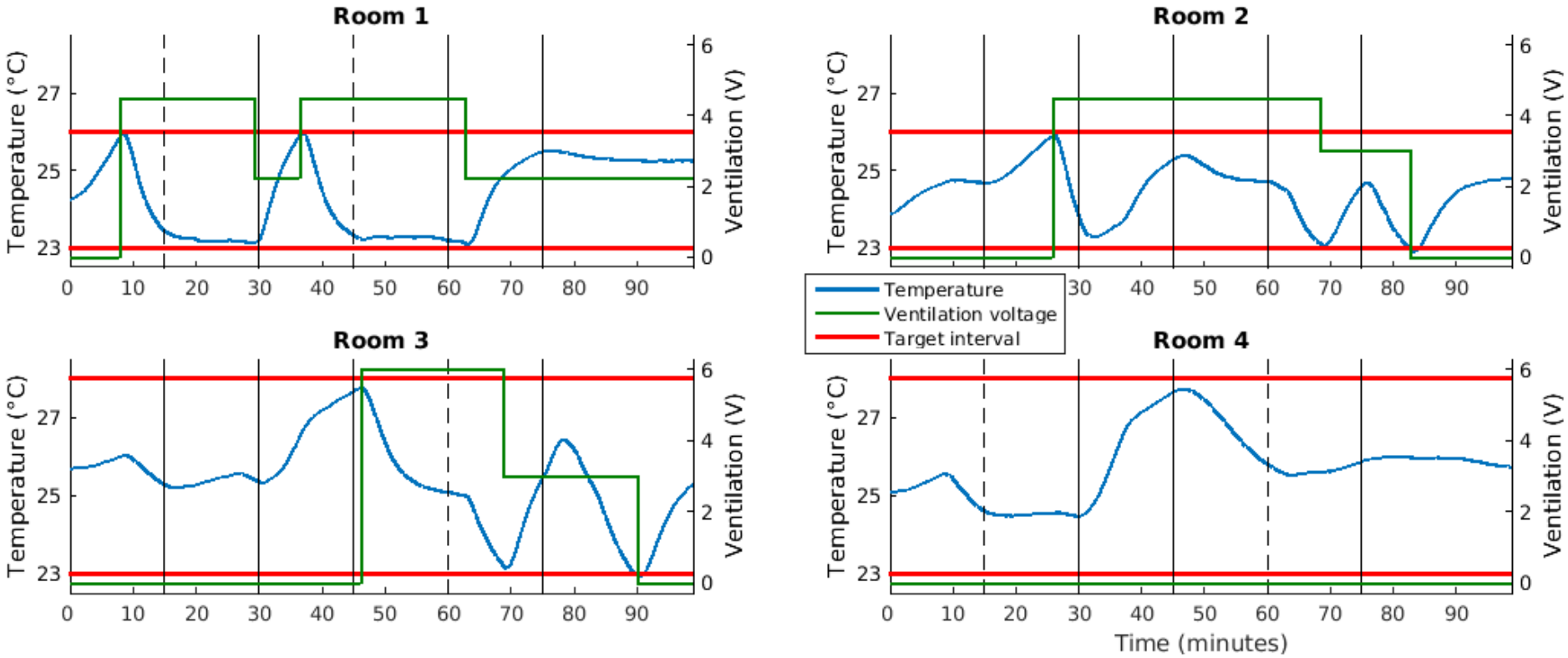
Centralized symbolic control

- $\alpha_x = 10$, $\alpha_u = 4$, $\tau = 34\ s$
- $g_a(s^k, u^k, u^{k-1}) = \|u^k\| + \|u^k - u^{k-1}\| + \|s_*^k - T_*\|$
- $N = 5$, $\lambda = 0.5$: $\lambda^{N+1} \approx 1.6\%$
- **Computation time: more than 2 days**



Compositional symbolic control

- $1D$ subsystems: $I_i = J_i = i$ and $I_i^o = \emptyset$
- $\alpha_x = 20$, $\alpha_u = 9$, $\tau = 10\ s$
- **Computation time:** $1.1\ s$



Conclusion

- Developed two approaches for the **robust control** of **monotone systems** with **safety specification**, based on:
  - **Invariance**, with an extension to **set stabilization**
  - **Symbolic abstraction**: both **centralized** and **compositional**
- Applied to the **temperature control** in a small-scale **experimental building**

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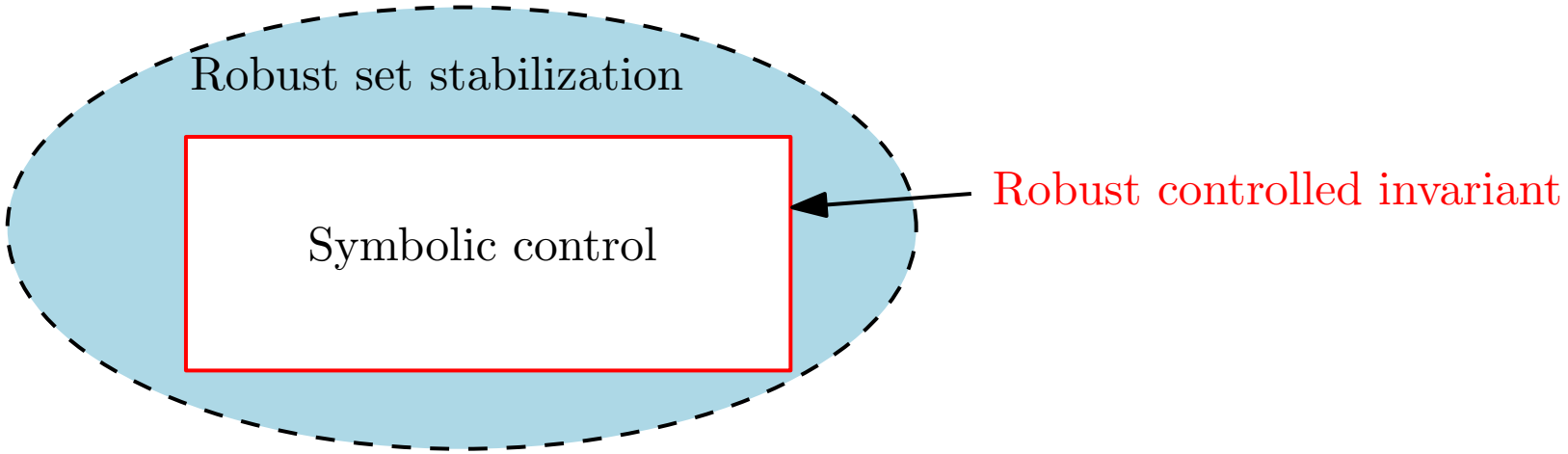


Perspectives

- Monotonicity appears in various other fields:
  - biology, chemistry, economy, population dynamics, ...
- Extension of the symbolic compositional approach
  - to non-monotone systems
  - to other specifications than safety
- Adaptive symbolic control framework:
  - measure the disturbance; tight estimation of its future bounds
  - synthesize compositional controller on the more accurate abstraction
  - apply controller until the next measure

$\Longrightarrow$ increased precision and robustness, local monotonicity

Publications

Authors: P.-J. Meyer, H. Nazarpour, A. Girard and E. Witrant

Journal paper:

- Robust controlled invariance for monotone systems: application to ventilation regulation in buildings. **Automatica**, provisionally accepted.

International conference:

- Safety control with performance guarantees of cooperative systems using compositional abstractions. **ADHS**, 2015.
- Experimental Implementation of UFAD Regulation based on Robust Controlled Invariance. **ECC**, 2014.
- Controllability and invariance of monotone systems for robust ventilation automation in buildings. **CDC**, 2013.

Conference poster:

- Symbolic Control of Monotone Systems, Application to Ventilation Regulation in Buildings. **HSCC**, 2015.
- Robust Controlled Invariance for UFAD Regulation. **BuildSys**, 2013.

Stabilization functions

Robust set stabilization from $[\underline{x_0}, \overline{x_0}]$ to $[\underline{x_f}, \overline{x_f}]$

Linear function	$\overline{X}(\lambda) = \lambda\overline{x_0} + (1-\lambda)\overline{x_f}$
Static input-state characteristic $k_x : \mathcal{U} \times \mathcal{W} \to \mathcal{X}$ $\exists \overline{u_f},\ \overline{u_0} \mid \underline{u} < \overline{u_f} < \overline{u_0}, \quad \overline{x_0} = k_x(\overline{u_0}, \overline{w}), \quad \overline{x_f} = k_x(\overline{u_f}, \overline{w})$	
Family of equilibria	$\overline{U}(\lambda) = \lambda\overline{u_0} + (1-\lambda)\overline{u_f}$ $\overline{X}(\lambda) = k_x(\overline{U}(\lambda), \overline{w})$
Trajectory $\overline{x_f} \to \overline{x_0}$	$\overline{X}(\lambda) = \Phi(\frac{\lambda}{1-\lambda}, \overline{x_f}, \overline{u_0}, \overline{w})$
Trajectory $\overline{x_0} \to \overline{x_f}$	$\overline{X}(\lambda) = \Phi(\frac{1-\lambda}{\lambda}, \overline{x_0}, \overline{u_f}, \overline{w})$

Symbolic abstraction

State partition $\mathcal{P}$ of $[\underline{x}, \overline{x}] \subseteq \mathbb{R}^n$ into α_x identical intervals per dimension

$$\mathcal{P} = \left\{ \left[\underline{s}, \underline{s} + \frac{\overline{x} - \underline{x}}{\alpha_x} \right] \mid \underline{s} \in \left(\underline{x} + \frac{\overline{x} - \underline{x}}{\alpha_x} * \mathbb{Z}^n \right) \cap [\underline{x}, \overline{x}] \right\}$$

Input discretization U_a of $[\underline{u}, \overline{u}] \subseteq \mathbb{R}^p$ into $\alpha_u \geq 2$ values per dimension

$$U_a = \left(\underline{u} + \frac{\overline{u} - \underline{u}}{\alpha_u - 1} * \mathbb{Z}^p \right) \cap [\underline{u}, \overline{u}]$$

Sampling period

Guidelines for the **viability kernel**[2] (maximal invariant set):

$$2L\tau^2 \sup_{x \in [\underline{x}, \overline{x}]} \|f(x, \overline{u}, \overline{w})\| \geq \frac{\|\overline{x} - \underline{x}\|}{\alpha_x}$$

- τ: sampling period
- $\frac{\|\overline{x} - \underline{x}\|}{\alpha_x}$: step of the state partition
- L: Lipschitz constant
- $\sup_{x \in [\underline{x}, \overline{x}]} \|f(x, \overline{u}, \overline{w})\|$: supremum of the vector field

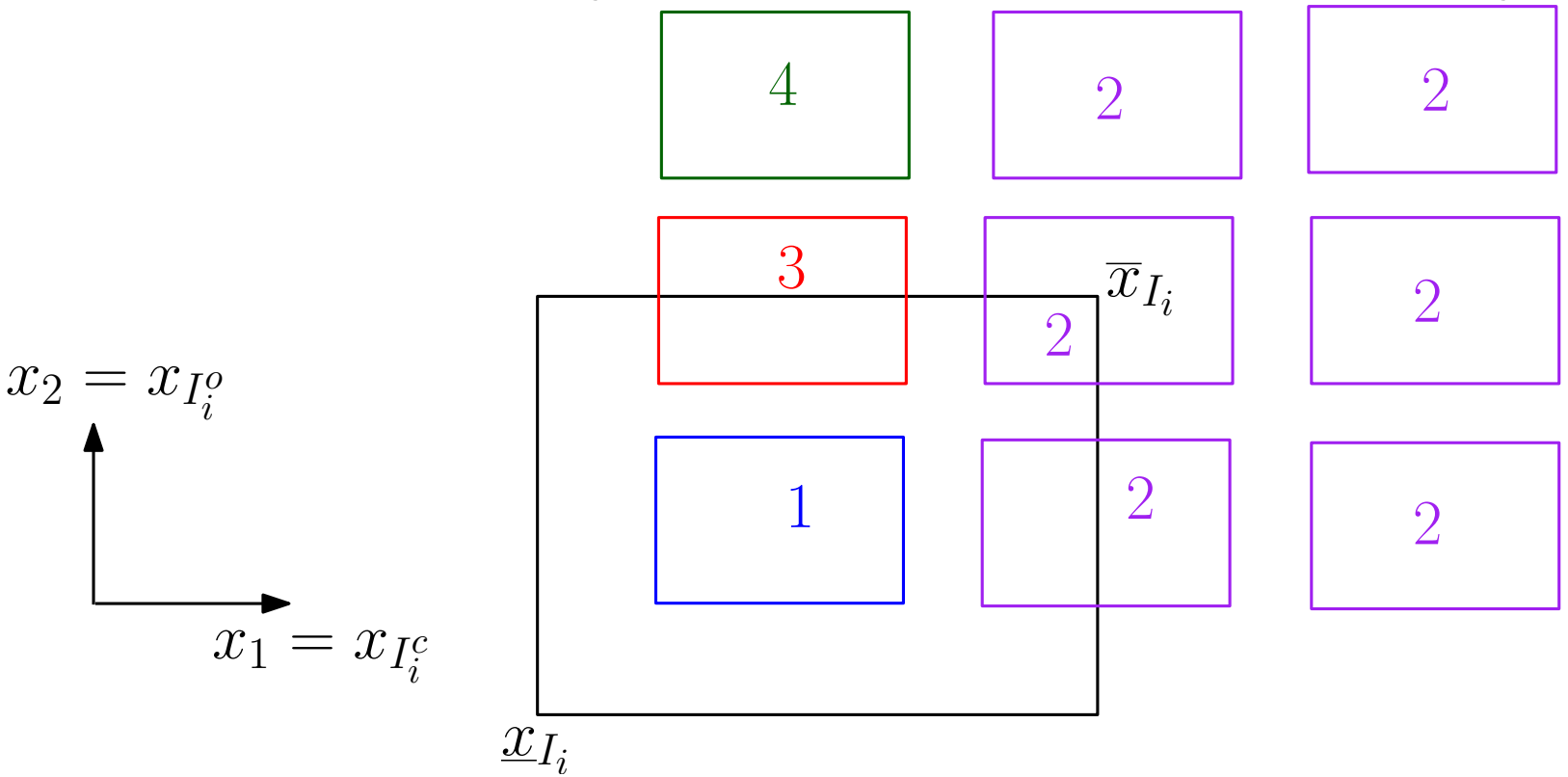
[2] P. Saint-Pierre. Approximation of the viability kernel. *Applied Mathematics and Optimization*, 29(2):187–209, 1994.

2nd A/G obligation

A/G Obligation (I_i^o)

Observed but uncontrolled states: $x_{I_i^o} \in [\underline{x}_{I_i^o}, \overline{x}_{I_i^o}]$

Remove transitions where only uncontrolled states violate the safety



Complexity

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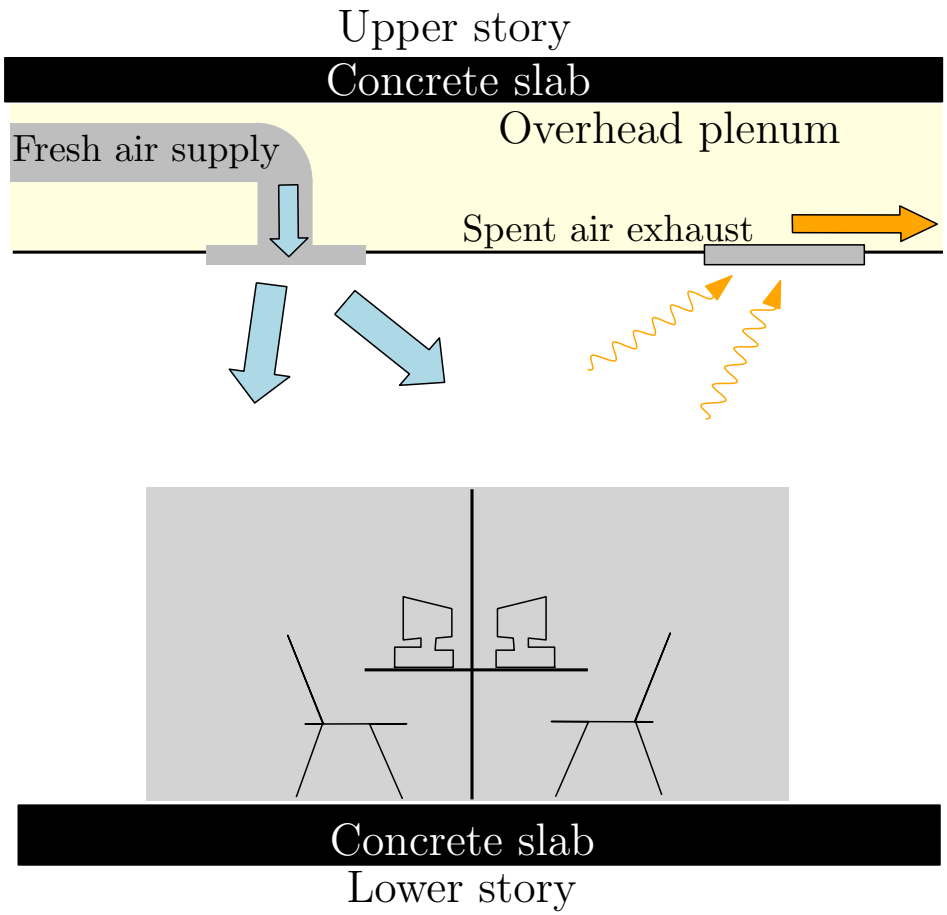
- $|\cdot|$: cardinality of a set

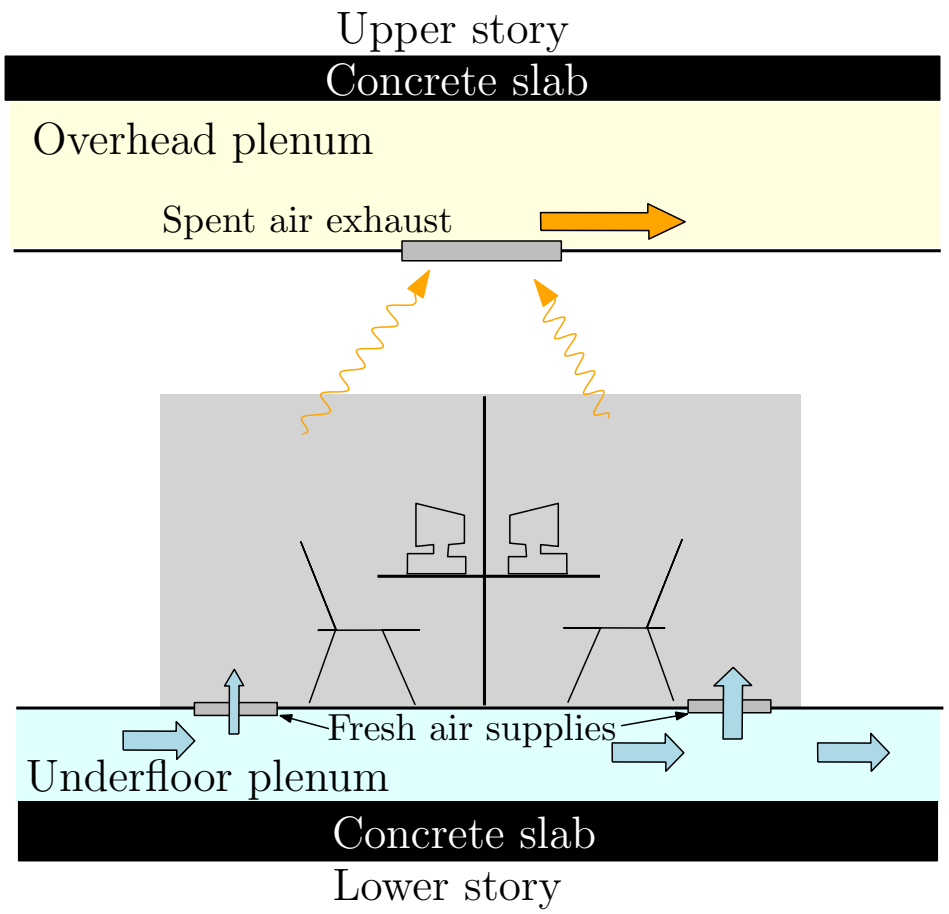
	Method	
	Centralized	Compositional
Abstraction (successors computed)	$2\alpha_x^n \alpha_u^p$	$\sum_{i=1}^{m} 2\alpha_x^{\lvert I_i \rvert} \alpha_u^{\lvert J_i \rvert}$
Dynamic programming (max iterations)	$N\alpha_x^{2n} \alpha_u^p$	$\sum_{i=1}^{m} N\alpha_x^{2\lvert I_i \rvert} \alpha_u^{\lvert J_i \rvert}$

UnderFloor Air Distribution

Overhead ventilation



UnderFloor Air Distribution



UFAD model

- $\mathcal{N}_i$: indices of neighbor rooms of room i
- $\mathcal{N}_i^*$: $\mathcal{N}_i$, underfloor, ceiling, outside
- δ_{s_i}, $\delta_{d_{ij}}$: discrete state of heat source and doors
- $\dot{m}_{u\to i}$: mass flow rate forced by the underfloor fan (control input)

$$\rho V_i C_v \frac{dT_i}{dt} = \sum_{j\in\mathcal{N}_i^*} \frac{k_{ij}A_{ij}}{\Delta_{ij}}(T_j - T_i) \qquad \text{(Conduction)}$$

$$+ \delta_{s_i}\varepsilon_{s_i}\sigma A_{s_i}(T_{s_i}^4 - T_i^4) \qquad \text{(Radiation)}$$

$$+ C_p\dot{m}_{u\to i}(T_u - T_i) \qquad \text{(Ventilation)}$$

$$+ \sum_{j\in\mathcal{N}_i} \delta_{d_{ij}} C_p \rho A_d \sqrt{2R}\max(0, T_j - T_i)^{3/2} \qquad \text{(Open doors)}$$