Chocolate Problem 2

November 8, 2016

Try to solve any two of the two following questions:

1. Show that the function

$$\begin{cases} f(x) = x^2 \sin(\frac{1}{x}) & \text{if } x \neq 0 \\ 0 \quad \text{if } x = 0 \end{cases}$$

   Has a derivative $f'(x)$ everywhere, but the derivative is not a continuous function.

2. Show that an increasing or decreasing function must always admit a limit at $\pm\infty$.

3. Suppose $f(x)$ is differentiable everywhere, and the derivative $f'(x)$ is continuous. Using the I.V.T., show that if $\frac{f(b)-f(a)}{b-a} = c$ there is a point $a \leq x \leq b$ where $f'(x) = c$. (Suggestion: subtract a linear function from $f(x)$).

4. Show that a function whose derivative is always 0 must be a constant. (Suggestion: use the result in previous exercise).

5. Show that a differentiable function that is concave at all points and has a point x where $f'(x) \geq 0$ can never have a horizontal asymptote at $+\infty$, unless it's a constant.