Julia Yulia Gordon MATH200-ALL_2017W2 Assignment Review_partials_and_gradients due 04/19/2018 at 11:59am PDT

1. (1 point) Library/Union/setMVderivatives/an14_3_3.pg

Let $z = \sqrt{4x + y}$. Then: The rate of change in *z* at (4,5) as we change *x* but hold *y* fixed is _____, and The rate of change in *z* at (4,5) as we change *y* but hold *x* fixed is _____.

2. (1 point) Library/Union/setMVderivatives/an14_7_15.pg

Consider the ellipsoid $2x^2 + 4y^2 + z^2 = 19$.

The implicit form of the tangent plane to this ellipsoid at (1,2,-1) is _____. The parametric form of the line through this point that is perpendicular to that tangent plane is L(t) =

3. (1 point) Library/Union/setMVderivatives/gradient-4/gradient-4.pg



The gradient vector field for a function $f: \mathbf{R}^2 \to \mathbf{R}$ is given at the left.

f has a relative minimum at ?. f has a relative maximum at ?. f is steepest at ?.

f has a saddle at ?.

4. (1 point) Library/Union/setMVtraces/trace-3b.pg Consider the function

$$f(x,y) = \frac{xy^2 - 3x^3}{6}.$$

Which graph below corresponds to the following traces:

____ 1. The trace for x = -0.3_____ 2. The trace for y = -1.9_____ 3. The trace for x = 1.25_____ 4. The trace for y = 0.75B B k h 1 0 1.0 1 0-1.0 1.0 -2 -2 2

B

5. (1 point) Library/Union/setMVtraces/trace-2a.pg Consider the function

$$f(x,y) = y - x.$$

С

D

Which graph below corresponds to the following traces:

-1. The trace for x = 1.1

Α

- 2. The trace for x = -2
- _____ 3. The trace for x = -0.5



10. (1 point) Library/Union/setMVlevelsets/levels-9a/levels-9a.pg Consider the level surface given by

$$x^2 - y^2 + z^2 = 2.$$

Match the slices with their correct plots below.

17. (1 point) Library/Union/setMVfunctions/sets-1.pg Consider the function $f(x,y) = (x,y,2x^2+y)$. Its graph is in \mathbb{R}^a where $a = \underline{\qquad}$. Its image is in \mathbb{R}^b where $b = \underline{\qquad}$.

Its **level sets** are in \mathbf{R}^c where c =____.

19. (1 point) Library/OSU/accelerated_calculus_and_analytic_geometry_iii/hmwk3/prob12.pg Find the maximum and minimum values of f(x, y) = 4x + y on the ellipse $x^2 + 9y^2 = 1$ maximum value: _______ minimum value: ______

20. (1 point) Library/UMN/calculusStewartET/s_14_7_29.pg Suppose $f(x,y) = x^2 + y^2 - 4x$ and *D* is the closed triangular region with vertices (4,0), (0,4), and (0,-4). Answer the following.

1. Find the absolute maximum of f(x, y) on the region *D*. Answer: _____

2. Find the absolute minimum of f(x, y) on the region *D*. Answer: _____

28. (1 point) Library/UMN/calculusStewartET/s_14_6_9.pg Suppose that $f(x,y,z) = x^2yz - xyz^3$ is a function of three variables. **1.** Find the gradient of f(x,y,z). Answer: $\nabla f(x,y,z) =$ ______ **2.** Evaluate the gradient at the point P(-2, -2, -2). Answer: $\nabla f(-2, -2, -2) =$ _____ **3.** Find the rate of change of f(x,y,z) at *P* in the direction of the vector $\mathbf{u} = \langle 0, \frac{4}{5}, -\frac{3}{5} \rangle$. Answer: $D_{\mathbf{u}} f(-2, -2, -2) =$ _____

32. (1 point) Library/UMN/calculusStewartET/s_14_4_prob01.pg Find the linearization L(x,y,z) of the $f(x,y,z) = 8\sqrt{x^3 + y^3 + z^3}$ at the point (1,2,3). Answer: L(x,y,z) =_____

34. (1 point) Library/UMN/calculusStewartET/s_14_7_43.pg Find three positive numbers whose sum is 200 and whose product is a maximum. Answer (separate by commas): ______

36. (1 point) Library/UMN/calculusStewartET/s_14_1_9.pg Let $g(x, y) = \cos(6x + 7y)$.

1. Evaluate g(1, -2). Answer: g(1, -2) =_____ 2. What is the range of g(x, y)? Answer (in interval notation): _____

39. (1 point) Library/UMN/calculusStewartET/s_14_8_3.pg Use Lagrange multipliers to find the minimum value of the function $f(x, y) = x^2 + y^2$ subject to the constraint xy = 2.

Minimum:

44. (1 point) Library/UMN/calculusStewartET/s_14_4_13.pg Find the linear approximation of the $f(x,y) = \frac{x}{x+y}$ at the point (1,2).

Answer: $f(x,y) \approx -$

45. (1 point) Library/UMN/calculusStewartET/s_14_6_22.pg Find the maximum rate of change of $f(s,t) = te^{st}$ at the point (0,1).

Answer: ____

47. (1 point) Library/UMN/calculusStewartET/s_14_7_7.pg Suppose f(x, y) = (x - y)(16 - xy). Answer the following.

1. Find the local maxima of f. List your answers as points in the form (a, b, c). Answer (separate by commas): _____

2. Find the local minima of f. List your answers as points in the form (a, b, c). Answer (separate by commas): _____

3. Find the saddle points of f. List your answers as points in the form (a, b, c). Answer (separate by commas): _____

Find the indicated partial derivative of the function $f(x,y) = \sin(4x+7y)$. Answer: $f_{yxy}(x,y) =$ _____

51. (1 point) Library/UMN/calculusStewartET/s_14_6_32.pg The temperature at a point (x, y, z) is given by

$$T(x, y, z) = 1300e^{-x^2 - 2y^2 - z^2}$$

where T is measured in $^{\circ}$ C and x, y, and z in meters.

1. Find the rate of change of the temperature at the point P(2, -3, 2) in the direction toward the point Q(3, -5, 3).

Answer: $D_{\overrightarrow{PQ}} f(2, -3, 2) =$ _____ **2.** In what direction does the temperature increase fastest at *P*? Answer:

3. Find the maximum rate of increase at *P*. Answer: ____

57. (1 point) Library/UMN/calculusStewartET/s_12_1_5/s_12_1_5.pg Match the equations of the plane with one of the graphs below.

-1. x + y = -22. x + y = 23. x + z = 2-4. y-x=2

Note: You can click on the graphs to enlarge the images.

60. (1 point) Library/Rochester/setVmultivariable7MaxMin/ur_vc_7_14.pg

You are hiking the Inca Trail on the way to Machu Picchu. When you arrive at the highest point on the trail, which of the following are possibilities? In alphabetical order without punctuation or spacing, list the letters which indicate possibilities.

(A) The path passes through the center of a set of concentric contour lines.

- (B) The path is tangent to a contour line.
- (C) The path follows a contour line.
- (D) The path crosses a contour line.

Possibilities: ____

66. (1 point) Library/Rochester/setVmultivariable6Gradient/ur_vc_6_13.pg Suppose $f(x,y) = \frac{x}{y}$, P = (-3,4) and $\mathbf{v} = 4\mathbf{i} + 3\mathbf{j}$. A. Find the gradient of f. $\nabla f =$ _____ $\mathbf{i} +$ _____ \mathbf{j} Note: Your answers should be expressions of x and y; e.g. "3x - 4y" B. Find the gradient of f at the point P. $(\nabla f)(P) =$ _____ $\mathbf{i} +$ _____ \mathbf{j} Note: Your answers should be numbers C. Find the directional derivative of f at P in the direction of \mathbf{v} . $D_u f =$ ______ Note: Your answer should be a number D. Find the maximum rate of change of f at P.

Note: Your answer should be a number

E. Find the (unit) direction vector in which the maximum rate of change occurs at P.

u = _____ **i**+ _____ **j** Note: Your answers should be numbers

75. (1 point) Library/Dartmouth/setMTWCh2S6/problem_2.pg

Find the slope of the tangent line to the curve $\sqrt{2x+4y} + \sqrt{xy} = \sqrt{28} + \sqrt{24}$ at the point (8,3).

The slope is ______.

76. (1 point) Library/Dartmouth/setMTWCh2S6/problem_5.pg Suppose that z = f(x, y) is defined implicitly by an equation of the form F(x, y, z) = 0. Find formulas for the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ in terms of F_1, F_2, F_3 .

To enter your answer use F1, F2, F3 as the partial derviatives of F with respect to its first, second, and third variables.

 $\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}} = \underline{\qquad}$

79. (1 point) Library/Dartmouth/setStewartCh15S6/problem_14.pg Find the points on the surface $1x^2 + 4y^2 + 5z^2 = 1$ at which the tangent plane is parallel to the plane -4x + 3y - 1z = 0.

(_____, ____, ____) and

80. (1 point) Library/Dartmouth/setMTWCh2S4/problem_5.pg

Let $z = f(u, v) = \sin u \cos v$, $u = -2x^2 - 5y$, v = 4x + 2y, and put g(x, y) = (u(x, y), v(x, y)). The derivative matrix $\mathbf{D}(f \circ g)(x, y) =$

(______,____)

(Leaving your answer in terms of u, v, x, y)

81. (1 point) Library/Dartmouth/setStewartCh15S3/problem_1.pg

Suppose the f(x,y) is a smooth function and that its partial derivatives have the values, $f_x(-3,5) = 2$ and $f_y(-3,5) = 1$. Given that f(-3,5) = 0, use this information to estimate the value of f(-2,6). Note this is analogous to finding the tangent line approximation to a function of one variable. In fancy terms, it is the first Taylor approximation.

Estimate of (integer value) f(-2,6) _____

82. (1 point) Library/Dartmouth/setStewartCh15S4/problem_5.pg

Find the differential of the function $w = x^4 \sin(y^1 z^4)$ $dw = \underline{\qquad} dx + \underline{\qquad} dy + \underline{\qquad} dz$

83. (1 point) Library/Dartmouth/setMTWCh2S5/problem_3.pg

Consider the surface xyz = 60.

(______)

A. Find the unit normal vector to the surface at the point (5,3,4) with positive first coordinate.

B. Find the equation of the tangent plane to the surface at the given point. Express your answer in the form ax + by + cz + d = 0, normalized so that a = 12.

84. (1 point) Library/Dartmouth/setMTWCh2S5/problem_4.pg

Consider a function f(x, y) at the point (5,2). At that point the function has directional derivatives: $\frac{3}{\sqrt{40}}$ in the direction (parallel to) (6,2), and $\frac{5}{\sqrt{34}}$ in the direction (parallel to) (5,3). The gradient of *f* at the point (5,2) is (______, _____). 88. (1 point) Library/272/setStewart14_4/problem_2.pg

Find an equation of the tangent plane to the surface $z = -2x^2 - 2y^2 + 3x + 1y - 1$ at the point (3, 5, -55). z =______

90. (1 point) Library/272/setStewart14_4/problem_4.pg

Use the linear approximation to estimate the value of f(-3.1, -2.9) =_____

93. (1 point) Library/272/setStewart14_8/problem_3.pg

Find the maximum and minimum values of the function $f(x, y, z) = x^2y^2z^2$ subject to the constraint $x^2 + y^2 + z^2 = 9$.

Maximum value is _____, occuring at _____ points (positive integer or "infinitely many"). Minimum value is _____, occuring at _____ points (positive integer or "infinitely many").

94. (1 point) Library/272/setStewart14_8/problem_2.pg

Find the absolute maximum and minimum of the function $f(x,y) = x^2 + y^2$ subject to the constraint $x^4 + y^4 = 1296$.

As usual, ignore unneeded answer blanks, and list points in lexicographic order.

Absolute minimum value: ______ attained at (____, ____), (____, ____), (____, ____), (____, ____). Absolute maximum value: _____ attained at (____, ____), (____, ____), (____, ____), (____, ____).

95. (1 point) Library/272/setStewart14_8/problem_4.pg Find the maximum and minimum values of the function $f(x, y) = 2x^2 + 3y^2 - 4x - 5$ on the domain $x^2 + y^2 \le 324$.

The maximum value of f(x, y) is: _____

List the point(s) where the function attains its maximum as an ordered pair, such as (-6,3), or a list of ordered pairs if there is more than one point, such as (1,3), (-4,7).

The minimum value of f(x, y) is: _____

List points where the function attains its minimum as an ordered pair, such as (-6,3), or a list of ordered pairs if there is more than one point, such as (1,3), (-4,7).

96. (1 point) Library/272/setStewart14_7/problem_16.pg

Find the volume of the largest rectangular box with edges parallel to the axes that can be inscribed in the ellipsoid

$$\frac{x^2}{49} + \frac{y^2}{36} + \frac{z^2}{16} = 1$$

Hint: By symmetry, you can restrict your attention to the first octant (where $x, y, z \ge 0$), and assume your volume has the form V = 8xyz. Then arguing by symmetry, you need only look for points which achieve the maximum which lie in the first octant.

Maximum volume: _____

97. (1 point) Library/272/setStewart14_6/problem_2.pg

Find the directional derivative of $f(x, y) = \sin(x + 2y)$ at the point (-4, 4) in the direction $\theta = \pi/4$.

The gradient of f is: $\nabla f = \langle __, __ \rangle$

 $\nabla f(-4,4) = \langle ___, ___ \rangle$

The directional derivative is:

98. (1 point) Library/272/setStewart14_6/problem_9.pg

Find the maximum rate of change of $f(x, y) = \ln(x^2 + y^2)$ at the point (2, -2) and the direction in which it occurs.

Maximum rate of change: ______ Direction (unit vector) in which it occurs: (_____, ____)

99. (1 point) Library/272/setStewart14_6/problem_12.pg

Find equations of the tangent plane and normal line to the surface $x = 3y^2 + 1z^2 - 17$ at the point (-5, 1, -3).

Tangent Plane: (make the coefficient of x equal to 1).

 $\underbrace{ \text{Normal line: } \langle -5, \underline{\qquad} \rangle = 0.}_{+t \langle 1, \underline{\qquad}, \underline{\qquad} \rangle.}$

100. (1 point) Library/272/setStewart14_6/problem_6.pg

Find the directional derivative of $f(x, y, z) = z^3 - x^2 y$ at the point (-5, -2, -4) in the direction of the vector $\mathbf{v} = \langle -4, 3, 1 \rangle$.

103. (1 point) Library/272/setStewart14_5/problem_7.pg Suppose $z = x^2 \sin y$, $x = -5s^2 + 3t^2$, y = -4st. A. Use the chain rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ as functions of x, y, s and t. $\frac{\partial z}{\partial s} =$ ______ $\frac{\partial z}{\partial t} =$ ______ B. Find the numerical values of $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ when (s,t) = (3,-1). $\frac{\partial z}{\partial s}(3,-1) =$ ______ $\frac{\partial z}{\partial t}(3,-1) =$ ______

104. (1 point) Library/272/setStewart14_5/problem_8.pg

The radius of a right circular cone is increasing at a rate of 5 inches per second and its height is decreasing at a rate of 4 inches per second. At what rate is the volume of the cone changing when the radius is 20 inches and the height is 30 inches?

_____ cubic inches per second

106. (1 point) Library/272/setStewart14_1/UR_VC_5_F.pg

On a map showing the Superstition mountains, the contour lines are:

- A. closely spaced
- B. far apart

On a map showing the Bonneville Flats of Utah, the contour lines are:

- A. closely spaced
- B. far apart

107. (1 point) Library/272/setStewart14_1/problem_4.pg

Match the functions with their contour plots labeled A - G. You may click on the thumbnail image to produce a larger image in a new window (sometimes exactly on top of the old one).

$$\begin{array}{c} _1. \ f(x,y) = 1/(1+x^2+y^2) \\ _2. \ f(x,y) = 3-x^2-y^2 \\ _3. \ f(x,y) = \sin(x) \\ _4. \ f(x,y) = \sin(x)\sin(y)e^{-x^2-y^2} \\ _5. \ f(x,y) = \cos(x^2+y^2)/(1+x^2+y^2) \\ _6. \ f(x,y) = (x-y)^2 \\ _7. \ f(x,y) = (x^2-y^2)^2 \end{array}$$

^{109. (1} point) Library/Hope/Multi2/09-01-Graphs-images-level-sets/Level-sets-04.pg

The figure shows level sets f(x,y) = 1 and g(x,y) = 0 for two different functions f(x,y) and g(x,y). Assuming that the graphs of f and g are both elliptic paraboloids with x = 0 and y = 0 as planes of reflection symmetry and that f(0,0) = 0 and g(0,0) = 1, find formulas for these functions.

111. (1 point) Library/WHFreeman/Rogawski_Calculus_Early_Transcendentals_Second_Edition/14_Differentia tion_in_Several_Variables/14.4_Differentiability_and_Tangent_Planes/14.4.35.pg

The volume of a cylinder of radius *r* and height *h* is $V = \pi r^2 h$. Calculate the percentage increase in *V* if *r* is increased by 3.9% and *h* is increased by 1.1%. Hint: Use the linear approximation to show that $\frac{\Delta V}{V} \approx \frac{2\Delta r}{r} + \frac{\Delta h}{h}$

 $\frac{\Delta V}{V} \times 100\% = \underline{\qquad} \%$

The volume of a certain cylinder V is determined by measuring r and h. Which will lead to a greater error in V:

- A. a 1% error in r
- B. a 1% error in h

• C. a 1% error in r is equivalent to a 1% error in h

112. (1 point) Library/WHFreeman/Rogawski_Calculus_Early_Transcendentals_Second_Edition/14_Differentia tion_in_Several_Variables/14.4_Differentiability_and_Tangent_Planes/14.4.39.pg

The volume V of a cylinder is computed using the values 8.4m for the diameter and 7.2m for the height. Use the linear approximation to estimate the maximum error in V if each of these values has a possible error of at most 9%.

Percentage error in V:_____%

113. (1 point) Library/WHFreeman/Rogawski_Calculus_Early_Transcendentals_Second_Edition/14_Differentia tion_in_Several_Variables/14.4_Differentiability_and_Tangent_Planes/14.4.20.pg

Find the linear approximation to $f(x, y, z) = \frac{xy}{z}$ at the point (1,1,2): $f(x, y, z) \approx$ _____

114. (1 point) Library/WHFreeman/Rogawski_Calculus_Early_Transcendentals_Second_Edition/14_Differentia tion_in_Several_Variables/14.4_Differentiability_and_Tangent_Planes/14.4.37.pg

The monthly payment for a home loan is given by a function f(P, r, N) where P is the principal (the initial size of the loan), r the interest rate, and N the length of the loan in months. Interest rates are expressed as a decimal: A

$$\frac{\partial f}{\partial P} = 0.005, \quad \frac{\partial f}{\partial r} = 6918, \quad \frac{\partial f}{\partial N} = -1.6748$$

Estimate:

(a) The change in monthly payment per 3500 increase in loan principal:
Δf ≈ ______ dollars
(b) The change in monthly payment if the interest rate changes from r = 0.09 to r = 0.075:
Δf ≈ ______ dollars
(c) The change in monthly payment if the length of the loan changes from 27 to 28 years:
Δf ≈ ______ dollars

115. (1 point) Library/WHFreeman/Rogawski_Calculus_Early_Transcendentals_Second_Edition/14_Differentia tion_in_Several_Variables/14.6_The_Chain_Rule/14.6.26.pg

Calculate the derivative using implicit differentiation:

$$\frac{\partial w}{\partial z}, \quad x^8 w + w^2 + wz^2 + 4yz = 0$$

 $\frac{\partial w}{\partial z} =$ _____

116. (1 point) Library/WHFreeman/Rogawski_Calculus_Early_Transcendentals_Second_Edition/14_Differentia tion_in_Several_Variables/14.3_Partial_Derivatives/14.3.7.pg

The plane y = 1 intersects the surface $z = x^3 + 5xy - y^6$ in a certain curve. Find the slope of the tangent line of this curve at the point P = (1, 1, 5).

m = _____

117. (1 point) Library/WHFreeman/Rogawski_Calculus_Early_Transcendentals_Second_Edition/14_Differentia tion_in_Several_Variables/14.3_Partial_Derivatives/14.3.68.pg

Given $u(x,t) = \frac{1}{t}e^{-\frac{x^2}{2t}}$, compute: $u_{xx} = \underline{\qquad}$

118. (1 point) Library/WHFreeman/Rogawski_Calculus_Early_Transcendentals_Second_Edition/14_Differentia tion_in_Several_Variables/14.5_The_Gradient_and_Directional_Derivatives/14.5.36.pg

Let $f(x, y) = xe^{x^2 - y}$ and P = (1, 1).

(a) Calculate $\|\nabla f_P\|$.

- (**b**) Find the rate of change of f in the direction ∇f_P .
- (c) Find the rate of change of f in the direction of a vector making an angle of 45° with ∇f_P . Answers :
- (a) _____
- (b) _____
- (c) _____

119. (1 point) Library/WHFreeman/Rogawski_Calculus_Early_Transcendentals_Second_Edition/14_Differentia tion_in_Several_Variables/14.5_The_Gradient_and_Directional_Derivatives/14.5.1.pg

Let $f(x,y) = x^3y^2$ and $c(t) = (2t^2, t^3)$ (a) Calculate: $\nabla f \cdot c'(t) =$ _____ (b) Use the Chain Rule for Paths to evaluate $\frac{d}{dt}f(c(t))$ at t = 1. $\frac{d}{dt}f(c(1)) =$ _____

120. (1 point) Library/WHFreeman/Rogawski_Calculus_Early_Transcendentals_Second_Edition/14_Differentia tion_in_Several_Variables/14.8_Lagrange_Multipliers-_Optimizing_with_a_Constraint/14.8.17.pg

The surface area of a right-circular cone of radius r and height h is $S = \pi r \sqrt{r^2 + h^2}$, and its volume is $V = \frac{1}{2}\pi r^2 h$.

(a) Determine h and r for the cone with given surface area S = 6 and maximal volume V.

 $h = \underline{\qquad}, r = \underline{\qquad}$

(b) What is the ratio h/r for a cone with given volume V = 6 and minimal surface area S? $\frac{h}{r} =$ _____

(c) Does a cone with given volume V and maximal surface area exist?

- A. yes
- B. no

121. (1 point) Library/WHFreeman/Rogawski_Calculus_Early_Transcendentals_Second_Edition/14_Differentia tion_in_Several_Variables/14.8_Lagrange_Multipliers-_Optimizing_with_a_Constraint/14.8.11.pg

Find the minimum and maximum values of the function f(x, y, z) = 3x + 2y + 4z subject to the constraint $x^2 + 2y^2 + 6z^2 = 16$.

 $f_{max} = _____$ $f_{min} = _____$

122. (1 point) Library/WHFreeman/Rogawski_Calculus_Early_Transcendentals_Second_Edition/14_Differentia tion_in_Several_Variables/14.8_Lagrange_Multipliers-_Optimizing_with_a_Constraint/14.8.15.pg

Use Lagrange multipliers to find the point (a,b) on the graph of $y = e^{2x}$, where the value *ab* is as small as possible.

 $P = _$

132. (1 point) Library/FortLewis/Calc3/12-3-Contour-diagrams/HGM4-12-3-21-Contour-diagrams/HGM4-12-3-2 1-Contour-diagrams.pg

The figure shows a hill with two paths, A and B.

(a) What is the elevation change along each path? ? feet

(b) Which path ascends more rapidly?

(c) On which path will you probably have a better view of the surrounding countryside (assuming that trees do not block your view)? ?

(d) Along which path is there more likely to be a stream?

(Click on graph to enlarge)

139. (1 point) Library/FortLewis/Calc3/12-5-Three-variable-functions/HGM4-12-5-23-Functions-of-three-variables.pg

Decide whether the level surfaces of each function are concentric circular cylinders, concentric spheres, cones, elliptical paraboloids, hyperbolic paraboloids, hyperboloids of one sheet, hyperboloids of two sheets, parabolic cylinders, or parallel planes.

 $\begin{array}{ll} ? & 1. \quad f(x,y,z) = \cos(3x+y+z) \\ ? & 2. \quad f(x,y,z) = 3y^2 - 4x^2 - z \\ ? & 3. \quad f(x,y,z) = \ln(\sqrt{y^2+z^2}) \\ ? & 4. \quad f(x,y,z) = 4x^2 - y \\ ? & 5. \quad f(x,y,z) = x+y-4z \\ ? & 6. \quad f(x,y,z) = \sin(\sqrt{8(x^2+y^2+z^2)}) \\ ? & 7. \quad f(x,y,z) = e^{-(x^2+y^2+z^2)} \\ ? & 8. \quad f(x,y,z) = 5x^2 + y^2 - z \end{array}$

144. (1 point) Library/FortLewis/Calc3/14-4-Gradients-in-plane/HGM4-14-4-34-Gradients-etc.pg Suppose $f(x,y) = \sqrt{\tan(x) + y}$ and u is the unit vector in the direction of $\langle 2, 1 \rangle$. Then,

- (a) $\nabla f(x,y) =$ _____
- (b) $\nabla f(0.8,7) =$ _____
- (c) $f_u(0.8,7) = D_u f(0.8,7) =$ _____

145. (1 point) Library/FortLewis/Calc3/14-4-Gradients-in-plane/HGM4-14-4-63-Gradients-etc.pg View the curve $(y-x)^2 + 2 = xy - 3$ as a contour of f(x, y).

(a) Use $\nabla f(2,3)$ to find a vector normal to the curve at (2,3).

(b) Use your answer to part (a) to find an implicit equation for the tangent line to the curve at (2,3).

146. (1 point) Library/FortLewis/Calc3/14-4-Gradients-in-plane/HGM4-14-4-01-Gradients-etc/HGM4-14-4-01-Gradients-etc.pg

Use the contour diagram of f to decide if the specified directional derivative is positive, negative, or approximately zero.

- ? 1. At the point (-2,2) in the direction of \vec{i} ,
- ? 2. At the point (-1,1) in the direction of $(-\vec{i}+\vec{j})/\sqrt{2}$,
- ? 3. At the point (0,2) in the direction of \vec{j} ,
- ?4. At the point (0, -2) in the direction of $(\vec{i} 2\vec{j})/\sqrt{5}$,
- ?5. At the point (-1,1) in the direction of $(-\vec{i}-\vec{j})/\sqrt{2}$,
- ? 6. At the point (1,0) in the direction of $-\vec{j}$,

147. (1 point) Library/FortLewis/Calc3/14-4-Gradients-in-plane/HGM4-14-4-40-Gradients-etc/HGM4-14-4-40 -Gradients-etc.pg

Use the contour diagram for f(x, y) shown below to estimate the directional derivative of f in the direction \vec{v} at the point P.

(a) At the point P = (2,2) in the direction $\vec{v} = \vec{j}$, the directional derivative is approximately _____

(b) At the point P = (2,3) in the direction $\vec{v} = -\vec{j}$, the directional derivative is approximately _____

(c) At the point P = (1,4) in the direction $\vec{v} = (\vec{i} + \vec{j})/\sqrt{2}$, the directional derivative is approximately _

(Click on graph to enlarge)

150. (1 point) Library/FortLewis/Calc3/12-1-Two-variable-functions/HGM4-12-1-13-Functions-of-two-variables/HGM4-12-1-13-Functions-of-two-variables.pg

(a) Describe a possible graph of the temperature, T, on an east-west line through Durango, Colorado, if the origin is at Durango, the positive x-axis corresponds to east of Durango, and the vertical z-axis is the temperature. For negative x within a few hundred miles of the origin, the sign of T'(x) is ?, while for positive x a few hundred miles from the origin, the sign of T'(x) is ?. We estimate that T(0) =? degrees Fahrenheit.

(b) Describe a possible graph of the temperature, T, on a north-south line through Des Moines, Iowa, if the origin is at Des Moines, the positive y-axis corresponds to north of Des Moines, and the vertical z-axis is the temperature. For negative y within a few hundred miles of the origin, the sign of T'(y) is ?, while for positive y a few hundred miles from the origin, the sign of T'(y) is ?. We estimate that T(0) = ? degrees Fahrenheit.

151. (1 point) Library/FortLewis/Calc3/14-7-Second-order-partials/HGM4-14-7-34-Second-order-partials.p g If z = f(x) + yg(x), what can we say about z_{yy} ?

A. z_{yy} = 0
B. z_{yy} = y
C. z_{yy} = g(x)
D. z_{yy} = z_{xx}
E. We cannot say anything

153. (1 point) Library/CSUN/Calculus/Tangent_plane_1.pg

Find the equation of the plane that passes through the point (2,1,0) and is tangent to the surface

$$x^2 + 4y^3 + 3z^2 = 8$$

at (2,1,0). Write it in the form indicated below.

155. (1 point) Library/ASU-topics/setCalculus/stef16_6p3.pg

Consider x = h(y,z) as a parametrized surface in the natural way. Write the **equation** of the tangent plane to the surface at the point (-4, -5, -3) given that $\frac{\partial h}{\partial y}(-5, -3) = -3$ and $\frac{\partial h}{\partial z}(-5, -3) = -3$.

162. (1 point) Library/Michigan/Chap14Sec4/Q51.pg

For each of the following pairs of functions f and g, determine if the level curves of the functions cross at right angles, and find their gradients at the indicated point.

(a) f(x,y) = 4x + 4y, g(x,y) = 4x - 4y. Do the level curves of f and g cross at right angles? [?/yes/no] $\nabla f(6,2) =$ ______ $\nabla g(6,2) =$ ______ (b) $f(x,y) = x^4 - y$, $g(x,y) = 4y + \ln(|x|)$. Do the level curves of f and g cross at right angles? [?/yes/no] $\nabla f(6,2) =$ ______ $\nabla g(6,2) =$ ______

163. (1 point) Library/Michigan/Chap14Sec4/Q29.pg Find the gradient ∇f of the function f given the differential.

$$df = \left(5x^4 + 1\right)ye^x dx + x^5 e^x dy$$

 $\nabla f =$ _____

165. (1 point) Library/Michigan/Chap14Sec7/Q44.pg A contour diagram for the smooth function z = f(x, y) is shown below.

(a) Is *z* an increasing or decreasing function of *x*? [?/increasing/decreasing] Of *y*? [?/increasing/decreasing]

(b) Is f_x positive or negative? [?/positive/negative]

How about f_y ? [?/positive/negative]

(c) Is f_{xx} positive or negative? [?/positive/negative]

How about f_{yy} ? [?/positive/negative]

(d) In what direction does ∇f point at point *P*?

in the direction ____

(Give a vector that points generally in the right direction.)

In what direction does ∇f point at point Q?

166. (1 point) Library/Michigan/Chap14Sec7/Q42.pg

Assume z is a smooth function of x and y. If $z_{xy} = -2y$, what can you say about each of the following?

(a) $z_{yx} =$ _____

(b) $z_{xyx} =$ _____

(c) $z_{xyy} = -$

167. (1 point) Library/Michigan/Chap14Sec7/Q27.pg

Use the level curves of the function z = f(x, y) to decide the sign (positive, negative, or zero) of each of the partial derivatives at the point *P*. Assume the *x*- and *y*-axes are in the usual positions.

168. (1 point) Library/Michigan/Chap15Sec3/Q43.pg

For each value of λ the function $h(x, y) = x^2 + y^2 - \lambda(2x + 6y - 19)$ has a minimum value $m(\lambda)$.

(a) Find $m(\lambda)$

 $m(\lambda) = -$

(Use the letter **L** for λ in your expression.)

(b) For which value of λ is $m(\lambda)$ the largest, and what is that maximum value?

 $\lambda =$ _____

maximum $m(\lambda) =$ _

(c) Find the minimum value of $f(x, y) = x^2 + y^2$ subject to the constraint 2x + 6y = 19 using the method of Lagrange multipliers and evaluate λ .

minimum $f = _$

 $\lambda =$ _____

(How are these results related to your result in part (b)?)

169. (1 point) Library/Michigan/Chap14Sec5/Q23.pg

Find the directional derivative of $f(x, y, z) = yz + x^3$ at the point (2,3,1) in the direction of a vector making an angle of $3\pi/4$ with $\nabla f(2,3,1)$.

 $f_{\vec{u}} = -$

170. (1 point) Library/Michigan/Chap14Sec5/Q25.pg

Check that the point (1, -1, 2) lies on the given surface. Then, viewing the surface as a level surface for a function f(x, y, z), find a vector normal to the surface and an equation for the tangent plane to the surface at (1, -1, 2).

$$2x^2 - y^2 + 4z^2 = 17$$

vector normal = _____ tangent plane:

z = -

171. (1 point) Library/Michigan/Chap14Sec5/Q35.pg If the gradient of f is $\nabla f = 2x\tilde{i} + y\tilde{j} - 3zy\tilde{k}$ and the point P = (-8, 10, 2) lies on the level surface f(x, y, z) = 0, find an equation for the tangent plane to the surface at the point P.

 $z = _$

172. (1 point) Library/Michigan/Chap14Sec5/Q55.pg

A differentiable function f(x, y) has the property that f(4, 3) = 4 and $f_x(4, 3) = 3$ and $f_y(4, 3) = 7$. Find the equation of the tangent plane at the point on the surface z = f(x, y) where x = 4, y = 3.

z = -

173. (1 point) Library/Michigan/Chap14Sec5/Q17.pg Find the gradient of the function $f(x, y, z) = x^4 \ln(zy)$, at the point (2, *e*, 1) $\nabla f(2, e, 1) =$ ______

174. (1 point) Library/Michigan/Chap14Sec3/Q03.pg Find the equation of the tangent plane to

$$z = e^y + x + x^2 + 9$$

at the point (4,0,30).

z = -

175. (1 point) Library/Michigan/Chap14Sec3/Q13.pg Find the differential of the function $f(x,y) = ye^{-x}$ at (0,-1). df = _____

176. (1 point) Library/Michigan/Chap14Sec3/Q27.pg

The gas equation for one mole of oxygen relates its pressure, P (in atmospheres), its temperature, T (in K), and its volume, V (in cubic decimeters, dm³):

$$T = 16.574 \cdot \frac{1}{V} - 0.52754 \cdot \frac{1}{V^2} - 0.3879P + 12.187VP.$$

(a) Find the temperature T and differential dT if the volume is 20 dm³ and the pressure is 0.5 atmosphere. $T = ____$ $dT = ___$

(b) Use your answer to part (a) to estimate how much the volume would have to change if the pressure increased by 0.25 atmosphere and the temperature remained constant. change in volume = _____

177. (1 point) Library/Michigan/Chap14Sec3/Q29.pg

A fluid moves through a tube of length 1 meter and radius $r = 0.006 \pm 0.0002$ meters under a pressure $p = 3 \cdot 10^5 \pm 1000$ pascals, at a rate $v = 0.5 \cdot 10^{-9}$ m³ per unit time. Use differentials to estimate the maximum error in the viscosity η given by

$$\eta = \frac{\pi}{8} \frac{pr^4}{v}.$$

maximum error \approx _____

178. (1 point) Library/Michigan/Chap15Sec2/Q23.pg

What is the shortest distance from the surface $xy + 6x + z^2 = 41$ to the origin?

distance = _____

179. (1 point) Library/Michigan/Chap14Sec2/Q45.pg Consider the partial derivatives

$$f_x(x,y) = 5x^4y^6 - 10x^4y,$$

$$f_y(x,y) = 6x^5y^5 - 2x^5.$$

Is there a function *f* which has these partial derivatives? [?/Yes/No] If so, what is it?

f = _____ (Enter **none** if there is no such function.) Are there any others?

[?/Yes/No]

180. (1 point) Library/Michigan/Chap12Sec3/Q17.pg

Match each of the tables shown below with the contour diagrams below them.

Table (a) : graph

- •?
- 1

• 2

• 3

• 4

- 5
- 6

• none of the graphs

Table (**b**) : graph

• ?

- 1
- 2
- 3
- 4
- 5
- 6
- none of the graphs

Table (c) : graph

- ?
- 1
- 2
- 3
- 4
- 5
- 6
- none of the graphs

Table (d) : graph

- ?
- 1
- 2
- 3
- 4
- 5
- 6
- none of the graphs

181. (1 point) Library/Michigan/Chap14Sec6/Q24.pg

Given z = f(x, y), x = x(u, v), y = y(u, v), with x(2, 6) = 1 and y(2, 6) = 4, calculate $z_u(2, 6)$ in terms of some of the values given in the table below.

$$\begin{array}{|c|c|c|c|c|c|c|}\hline f_x(2,6) = s & f_y(2,6) = p & x_u(2,6) = r & y_u(2,6) = q \\ \hline f_x(1,4) = c & f_y(1,4) = -5 & x_v(2,6) = 3 & y_v(2,6) = a \\ \hline \end{array}$$

 $z_u(2,6) =$ ______

182. (1 point) Library/Michigan/Chap12Sec5/Q05.pg Write the level surface $5 = \frac{3x^2+3y}{z}$ as the graph of a function f(x,y). f(x,y) =______ 183. (1 point) Library/Michigan/Chap12Sec5/Q23.pg

The surface *S* is the graph of $f(x,y) = \sqrt{4-y^2}$.

Be sure you can explain why S is the upper half of a circular cylinder of radius 2, centered along the x-axis. Write three or four sentences on a sheet of paper that clearly demonstrate this.

Find a level surface g(x, y, z) = c representing *S*.

 $g(x, y, z) = _$

with $c = _$

(Note that because your answers for g and c are interdependent, you cannot get partial credit for this problem.)

184. (1 point) Library/Michigan/Chap12Sec5/Q21.pg

Find a formula for a function g(x, y, z) whose level surfaces are planes parallel to the plane z = 3x + 6y - 8. g(x, y, z) =_____

185. (1 point) Library/Michigan/Chap12Sec5/Q31.pg

Describe in words the level surfaces of $g(x, y, z) = e^{-(x^2 + (y+2)^2 + z^2)}$. Think what the intersections of these surfaces with the three coordinate planes look like.

(a) In symbolic form, these level surfaces are given by $e^{-(x^2+(y+2)^2+z^2)} = c$ where c is a constant. For what values of c are level surfaces defined?

 $c \in _$

(*Give your answer as an interval or list of intervals; for example, if c is less than 1 or greater than or equal to 2, enter* (-inf,1),[2,inf).)

(b) Pick a level surface that intersects the *xz*-plane. What is your value of c? $c = ___$

Write the equation for the intersection, as an expression set equal to a constant:

(c) Pick a level surface that intersects the yz-plane. What is your value of c (which might be different from that you used in (a)?

 $c = _$

Write the equation for the intersection, as an expression set equal to a constant:

(d) Pick a level surface that intersects the *xy*-plane. What is your value of c? $c = ____$

Write the equation for the intersection, as an expression set equal to a constant:

187. (1 point) Library/Michigan/Chap12Sec2/Q05.pg

On a piece of paper, sketch each of the following surfaces:

(i) 4 = x + y

(ii) $z = x^2 + y^2 + 4$

=

Use your graphs to fill in the following descriptions of cross-sections of the surfaces.

(a) For (i) (4 = x + y):

Cross sections with *x* fixed give

- ?
- a horizontal line in a plane parallel to the yz-plane
- an empty set, or one or two vertical lines in a plane parallel to the yz-plane
- a vertical line in a plane parallel to the yz-plane
- two vertical lines in a plane parallel to the yz-plane
- a line in a plane parallel to the yz-plane
- an upward opening parabola in a plane parallel to the yz-plane
- a downward opening parabola in a plane parallel to the yz-plane
- a circle centered on the origin in a plane parallel to the yz-plane
- a point in a plane parallel to the yz-plane
- an empty set, or a circle centered on the origin in a plane parallel to the yz-plane

Cross sections with y fixed give

- ?
- a horizontal line in a plane parallel to the xz-plane
- an empty set, or one or two vertical lines in a plane parallel to the xz-plane
- a vertical line in a plane parallel to the xz-plane
- two vertical lines in a plane parallel to the xz-plane
- a line in a plane parallel to the xz-plane
- an upward opening parabola in a plane parallel to the xz-plane
- a downward opening parabola in a plane parallel to the xz-plane
- a circle centered on the origin in a plane parallel the xz-plane
- a point in a plane parallel to the xz-plane
- an empty set, or a circle centered on the origin in a plane parallel to the xz-plane

Cross sections with z fixed give

- •?
- a horizontal line in a plane parallel to the xy-plane
- two horizontal lines in a plane parallel to the xy-plane
- an empty set, or one or two horizontal lines in a plane parallel to the xy-plane
- a vertical line in a plane parallel to the xy-plane
- two vertical lines in a plane parallel to the xy-plane
- a line in a plane parallel to the xy-plane
- an upward opening parabola in a plane parallel to the xy-plane
- a downward opening parabola in a plane parallel to the xy-plane
- a circle centered on the origin in a plane parallel to the xy-plane
- a point in a plane parallel to the xy-plane
- an empty set, or a circle centered on the origin in a plane parallel to the xy-plane
- an empty set, or a plane parallel to the xy-plane

(b) For **(ii)** $(z = x^2 + y^2 + 4)$:

Cross sections with x fixed give

- ?
- a horizontal line in a plane parallel to the yz-plane
- an empty set, or one or two vertical lines in a plane parallel to the yz-plane
- a vertical line in a plane parallel to the yz-plane
- two vertical lines in a plane parallel to the yz-plane
- a line in a plane parallel to the yz-plane
- an upward opening parabola in a plane parallel to the yz-plane
- a downward opening parabola in a plane parallel to the yz-plane
- a circle centered on the origin in a plane parallel to the yz-plane
- a point in a plane parallel to the yz-plane
- an empty set, or a circle centered on the origin in a plane parallel to the yz-plane

Cross sections with y fixed give

- ?
- a horizontal line in a plane parallel to the xz-plane
- an empty set, or one or two vertical lines in a plane parallel to the xz-plane
- a vertical line in a plane parallel to the xz-plane
- two vertical lines in a plane parallel to the xz-plane
- a line in a plane parallel to the xz-plane
- an upward opening parabola in a plane parallel to the xz-plane
- a downward opening parabola in a plane parallel to the xz-plane
- a circle centered on the origin in a plane parallel the xz-plane
- a point in a plane parallel to the xz-plane
- an empty set, or a circle centered on the origin in a plane parallel to the xz-plane

Cross sections with z fixed give

- •?
- a horizontal line in a plane parallel to the xy-plane
- two horizontal lines in a plane parallel to the xy-plane
- an empty set, or one or two horizontal lines in a plane parallel to the xy-plane
- a vertical line in a plane parallel to the xy-plane
- two vertical lines in a plane parallel to the xy-plane
- a line in a plane parallel to the xy-plane
- an upward opening parabola in a plane parallel to the xy-plane
- a downward opening parabola in a plane parallel to the xy-plane
- a circle centered on the origin in a plane parallel to the xy-plane
- a point in a plane parallel to the xy-plane
- an empty set, or a circle centered on the origin in a plane parallel to the xy-plane
- an empty set, or a plane parallel to the xy-plane

188. (1 point) Library/Michigan/Chap12Sec2/Q23.pg

By setting one variable constant, find a plane that intersects the graph of $z = 6x^2 - 3y^2 + 5$ in a:

(a) Parabola opening upward: the plane ____ = ___

(*Give your answer by specifying the variable in the first answer blank and a value for it in the second.*) (b) Parabola opening downward: the plane ____ = ___

(Give your answer by specifying the variable in the first answer blank and a value for it in the second.)

(c) Pair of intersecting straight lines: the plane ____ = _____ (*Give your answer by specifying the variable in the first answer blank and a value for it in the second.*)

189. (1 point) Library/Michigan/Chap12Sec2/Q17.pg

For each of the following functions, decide whether its graph could be a bowl, a plate, or neither. Consider a plate to be any fairly flat surface and a bowl to be anything that could hold water, assuming the positive *z*-axis is up.

(a) x + y + z = 1 : [?/bowl/plate/neither] (b) $z = 1 - x^2 - y^2$: [?/bowl/plate/neither] (c) z = 3 : [?/bowl/plate/neither] (d) $z = \frac{xy}{|xy|}$: [?/bowl/plate/neither]

191. (1 point) Library/Michigan/Chap16Sec2/Q41.pg

The region W lies below the surface $f(x,y) = 4e^{-(x-1)^2-y^2}$ and above the disk $x^2 + y^2 \le 4$ in the xy-plane.

(a) Think about what the contours of f look like. You may want to using f(x,y) = 1 as an example. Sketch a rough contour diagram on a separate sheet of paper.

(b) Write an integral giving the area of the cross-section of W in the plane x = 1.

Area = $\int_a^b \underline{\qquad} d \underline{\qquad}$, where $a = \underline{\qquad}$ and $b = \underline{\qquad}$

(c) Use your work from (b) to write an iterated double integral giving the volume of W, using the work from (b) to inform the construction of the inside integral.

Volume = $\int_a^b \int_c^d \underline{\qquad} d \underline{\qquad} d \underline{\qquad} d$, where $a = \underline{\qquad}, b = \underline{\qquad} c = \underline{\qquad}$ and $d = \underline{\qquad}$

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