# MATH 200:921, Quiz 5

First Name: _____ Last Name: _____

Student-No: _____

| Grade: |
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- Do not turn the page until instructed to do so.
- This test is closed book. No calculators or formula sheet allowed.
- You have 20 minutes to write this quiz.
- There are three questions in this quiz, worth a total of 20 points.

## Short answer question

1. [4 marks] For each of the following statements write $T$ for true or $F$ for false next to it.

   1. We always have
   $$\int_0^1 \int_{a(x)}^{b(x)} h(x)g(y)\,\mathrm{d}y\,\mathrm{d}x = \left(\int_0^1 h(x)\,\mathrm{d}x\right)\left(\int_{a(0)}^{b(1)} g(y)\,\mathrm{d}y\right).$$

   2. If $f(x,y)$ is continuous then it is always true that
   $$\int_c^d \int_a^b f(x,y)\,\mathrm{d}x\,\mathrm{d}y = \int_a^b \int_c^d f(x,y)\,\mathrm{d}y\,\mathrm{d}x.$$

   3. We have
   $$\int_{-1}^1 \int_0^{\sqrt{1-x^2}} x\,\mathrm{d}y\,\mathrm{d}x = \int_0^\pi \int_0^1 r^2\cos(\theta)\,\mathrm{d}r\,\mathrm{d}\theta.$$

   4. If the density function is constant, the center of mass of a region $D$ must always lie inside of $D$.

## Long answer question—you must show your work

2. [8 marks] Consider the integral

$$\int_0^4 \int_{-\sqrt{4-x}}^{\sqrt{4-x}} f(x,y)\,\mathrm{d}y\,\mathrm{d}x$$

1. Sketch the domain of integration and rewrite the integral as a $\mathrm{d}x\,\mathrm{d}y$ integral.
2. Evaluate the integral when $f(x,y) = e^{8y-\frac{2}{3}y^3}$.

## Long answer question—you must show your work

3. [8 marks] Consider the triangle $T$ with vertices $A = (0, 0), B = (1, 1), C = (1, 0)$.

   1. Sketch $T$ and describe the side $\overline{CB}$ with polar coordinates equations $r = a(\theta), c \leq \theta \leq d$.
   2. *Using an integral in polar coordinates*, compute the area of the triangle.
   3. Assuming that the mass distribution on $T$ is a constant $\rho$, write integrals in polar coordinates that compute the coordinates of the center of mass of $T$. You do not need to evaluate them.

Name: _____ Student-No: _____