MATH 200:921, Quiz 4

First Name: ———————————————— Last Name: ————————————————

Student-No: ————————————————————————————————

Grade:

- Do not turn the page until instructed to do so.
- This test is closed book. No calculators or formula sheet allowed.
- You have 20 minutes to write this quiz.
- There are three questions in this quiz, worth a total of 20 points.

Long answer question—you must show your work

1. $\boxed{6 \text{ marks}}$ Consider the plane
$$H : x - y + z = 1.$$

   1. Let $O = (0,0,0)$ be the origin of the axes in $\mathbb{R}^3$. Given a point P in the plane H, write the square norm $\|\vec{OP}\|^2$ as a function $f(x,y)$ of two variables.

   2. Classify each critical point of $f(x,y)$ as either local maximum, local minimum, saddle point or undetermined.

Long answer question—you must show your work

2. [8 marks] On the plane H from the previous question consider the triangle T whose vertices are

$$A = (1, 2, 2), B = (1, -1, -1), C = (-2, -1, 2).$$

Let T' be the projection of T on the xy plane.

1. Sketch T' and write down its sides and vertices. A side should be described by an equation and a range, such as $y = 2x, 0 \leq x \leq 2$ or $x = 3, 1 \leq y \leq 5$.

2. Make a list of the points in the segment $\overline{AB}$ which could realize the minimum or maximum distance from the origin. You do not need to evaluate the function at these points.

3. If you wanted to find the minimum and maximum distance from a point of T to the origin, at how many points in T' would you need to evaluate the distance function at most? Explain your reasoning.

Long answer question—you must show your work

3. ☐ 6 marks ☐ Consider the cylinder given in three dimensional space by

$$S : x^2 + y^2 = 5.$$

Let R be the curve obtained by intersecting S with the plane H from the first two questions.

1. Set up the Lagrange multiplier system of the equation needed to find the points in R which minimize or maximize the square distance from the origin. Do not solve it.

2. For each of the following points, determine if it is possible or not that they minimize or maximize the square distance from R to the origin:

$$P_1 = (0, \sqrt{5}, 1 + \sqrt{5}) \quad P_2 = (2, 1, 2) \quad P_3 = (0, -1, 0)$$

Name: _____ Student-No: _____