MATH 200:921, Quiz 1

First Name: ______________________ Last Name: ______________________

Student-No: ____

Grade:

- Do not turn the page until instructed to do so.
- This test is closed book. No calculators or formula sheet allowed.
- You have 20 minutes to write this quiz.
- There are three questions in this quiz, worth a total of 10 points.

Long answer question—you must show your work

1. 3 marks Consider the points $P = (0, 2, 1), Q = (3, 3, -1)$. Find a point R on the z-axis such that the two vectors $\vec{PQ}$ and $\vec{PR}$ are perpendicular.

Solution: The vector $\vec{PQ}$ is equal to $\langle 3-0, 3-2, -1-1\rangle = \langle 3, 1, -2\rangle$.

A point R on the z-axis coordinates $(0, 0, c)$, so the vector $\vec{PR}$ has components $\langle 0, -2, c-1\rangle$.

For $\vec{PQ}$ and $\vec{PR}$ to be perpendicular we must have $\vec{PR} \cdot \vec{PQ} = 0$, that is, $-2 - 2c + 2 = 0$ which is satisfied when $c = 0$.

Long answer question—you must show your work

2. 3 marks In an appropriate coordinate system, a small zipline in an amusement park runs straight from point $P = (0, 20, 10)$ to the point $Q = (30, 30, -10)$ (in meters). A young boy whose mass is 30 kilograms is riding the attraction. Recall that the gravitational force is (approximately) equal to $(9.8 \cdot \text{mass})$ Newtons $(m \cdot kg/s^2)$, pointing down. There is also a strong wind, pushing the boy with a force of $\vec{F_w} = \langle 0, 20, 20\rangle$, in Newtons.

What is the work done by the total force acting on the boy as he rides the skyline from point P to point Q?

Solution:

The force of gravity is $\langle 0, 0, -9.8 \cdot 30\rangle$, so the total force $\vec{F_T}$ exerted by gravity and wind combined is, in Newtons, $\langle 0, 20, -9.8 \cdot 30 + 20\rangle = \langle 0, 20, -274\rangle$.

To find the work, we take the dot product of the displacement vector $\vec{PQ} = \langle 30, 10, -20\rangle$ and the force, obtaining $W = \vec{F_T} \cdot \vec{PQ} = 0 + 10 \cdot 20 + (-20)(-274) = 5680$ Joules.

Long answer question—you must show your work

3. [4 marks] Consider the vectors $\vec{v} = \vec{i} + \vec{j} - 2\vec{k}, \vec{w} = 4\vec{j} + 3\vec{k}$.

   1. Find the cross product $\vec{v} \times \vec{w}$.
   2. *Using the properties of cross products*, compute $(\vec{v} - \text{Proj}_{\vec{w}}\vec{v}) \times \vec{w} + \vec{w} \times \vec{v}$.

Solution:

- By the cross product formula we have

$$\vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -2 \\ 0 & 4 & 3 \end{vmatrix} = \left\langle \begin{vmatrix} 1 & -2 \\ 4 & 3 \end{vmatrix}, - \begin{vmatrix} 1 & -2 \\ 0 & 3 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 4 \end{vmatrix} \right\rangle = \langle 11, -3, 4 \rangle.$$

- By the distributive property $(\vec{v} - \text{Proj}_{\vec{w}}\vec{v}) \times \vec{w} = \vec{v} \times \vec{w} - \text{Proj}_{\vec{w}}\vec{v} \times \vec{w}$.

  Recall now that $\text{Proj}_{\vec{w}}\vec{v}$ is parallel to $\vec{w}$, and the cross product of two parallel vectors is zero, so $\vec{v} \times \vec{w} - \text{Proj}_{\vec{w}}\vec{v} \times \vec{w} = \vec{v} \times \vec{w}$.

  Finally, $\vec{v} \times \vec{w} = -\vec{w} \times \vec{v}$, so plugging in in the original formula we get

$$(\vec{v} - \text{Proj}_{\vec{w}}\vec{v}) \times \vec{w} + \vec{w} \times \vec{v} = -\vec{w} \times \vec{v} + \vec{w} \times \vec{v} = \vec{0}.$$