WORKSHEET

1) FIND THE DISTANCE FROM $P=(5,3,3)$ TO THE LINE THROUGH THE POINT $Q=(4,5,3)$ AND PARALLEL TO THE PLANES

$$H_1: x+z=0 \qquad H_2: y+2z=15$$

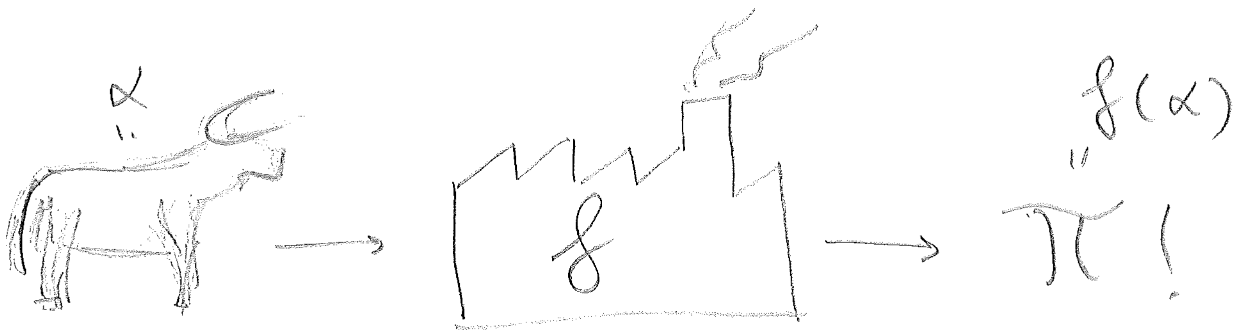
2) FUND THE DISTANCE BETWEEN THE LINES

$$\ell_1: x = y-1 = z-1$$

$$\ell_2: x = -y+1 = z+1$$

MULTIVARIABLE FUNCTIONS

WHAT'S A (REAL-VALUED) FUNCTION?



IT'S A "BLACK BOX" WHICH TAKES ELEMENTS IN SOME SET $\mathbb{D}$ AND PRODUCES REAL NUMBERS. IN OTHER WORDS, IT IS A WAY TO ASSIGN TO EACH α IN $\mathbb{D}$ A <u>UNIQUE</u> OUTPUT $f(\alpha)$ IN $\mathbb{R}$.

WE ARE USED TO

INPUT : IN $\mathbb{R}$ OUTPUT : IN $\mathbb{R}$

NEW SETTING

INPUT : IN $\mathbb{R}^2, \mathbb{R}^3$ OUTPUT : IN $\mathbb{R}$

WE CALL THE SET OF INPUTS OF f <u>THE DOMAIN</u> $\mathcal{D}(f)$, AND THE SET OF OUTPUTS OF f <u>THE RANGE</u> $\mathcal{R}(f)$.

THESE ARE "BUILT-IN" THE FUNCTION f; IT IS OFTEN COMMON THOUGH THAT WE ONLY HAVE A "FORMULA" FOR f AND WE NEED TO DESCRIBE A DOMAIN WHERE f IS DEFINED AS A FUNCTION.

<u>E.G.</u> $f(x,y,z)=\sqrt{4-x^2-y^2-3z^2}$

DOMAIN OF $f(x,y,z)$?

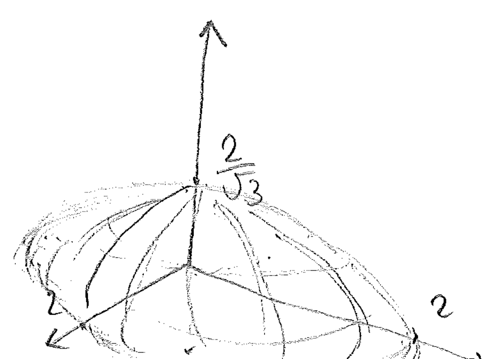
$D \subset \mathbb{R}^3$. WE NEED A POSITIVE (OR 0) NUMBER UNDER THE ROOT. THIS IS THE ONLY REQUIREMENT.

$4-x^2-y^2-3z^2 \geq 0$

$x^2+y^2+3z^2 \leq 4$

$\frac{x^2}{2^2}+\frac{y^2}{2^2}+\frac{z^2}{\left(\frac{2}{\sqrt{3}}\right)^2} \leq 1$

"FULL ELLIPSOID"



RANGE OF $f(x,y,z)$?

MAX VALUE UNDER $\sqrt{}$ IS 4

MIN VALUE UNDER $\sqrt{}$ IS 0

SO RANGE IS $[0,2]$

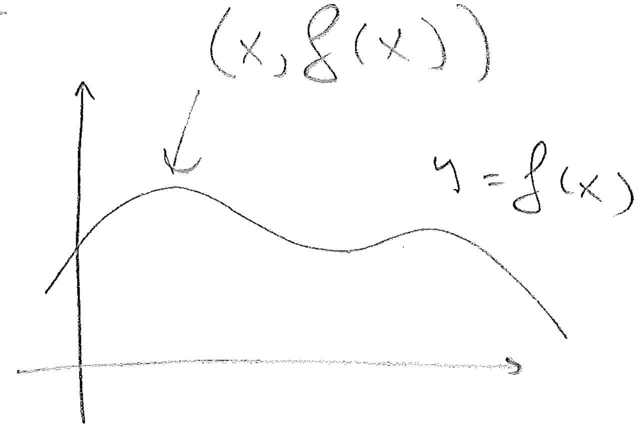
GRAPHS:

IN 1 VARIABLE

$y = f(x)$

↑

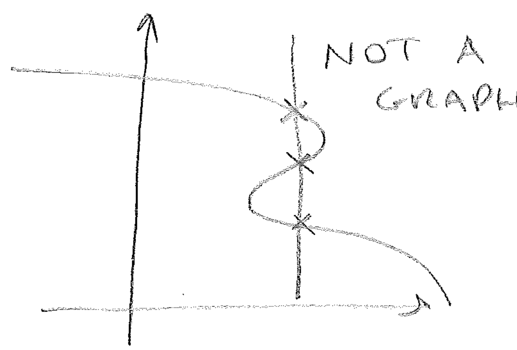
NEW COORDINATE



THE GRAPH LIVES IN

$D \times \mathbb{R} \subset \mathbb{R}^2$

↑ DOMAIN ↑ NEW VARIABLE



VERTICAL LINE TEST:

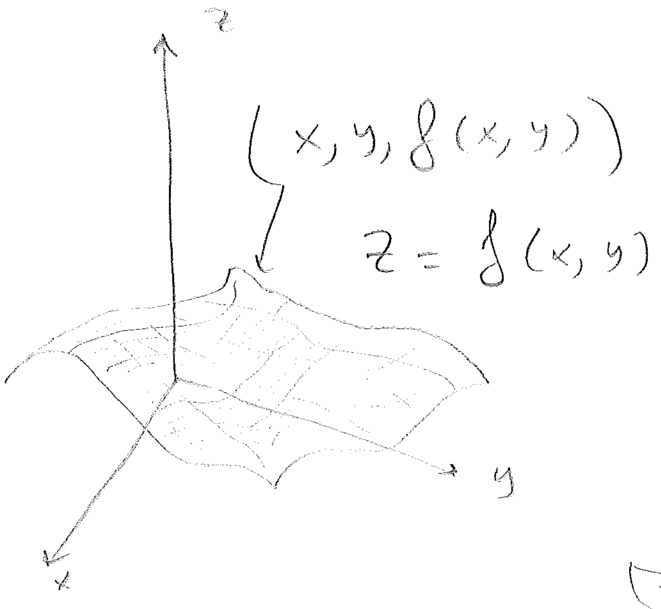
UNICITY OF OUTPUT MEANS A VERTICAL LINE MEETS GRAPH AT MOST IN 1 POINT.

IN 2 VARIABLES

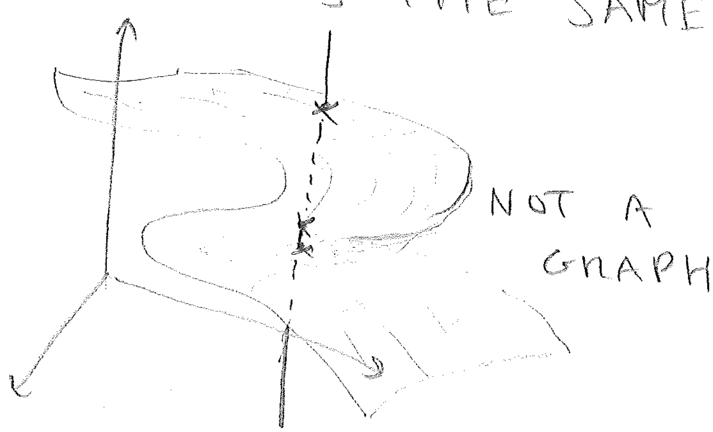
$D \times \mathbb{R} \subset \mathbb{R}^3$ (x,y) POINT IN DOMAIN

z EXTRA VARIABLE, SO THE GRAPH

IS A SURFACE IN $\mathbb{R}^3$.



VERTICAL LINE TEST IS THE SAME



IN 3 DIMENSIONS

WE CANNOT REALLY DRAW STUFF IN 4 DIMENSIONS... $D \times \mathbb{R} \subset \mathbb{R}^4$

IN GENERAL, EVEN IN THE 2 DIMENSIONS CASE THE GRAPH CAN BE VERY HARD TO DRAW OR UNDERSTAND. WE NEED SOMETHING SIMPLER.

LEVEL CURVES (OR SURFACES)

FOR A FUNCTION $f(x,y)$ OF TWO VARIABLES THE SUBSET OF $^{*}\mathbb{R}^2$ DEFINED BY $f(x,y)=K$ FOR A FIXED K IS CALLED A LEVEL CURVE FOR f.

IN 3 DIMENSIONS THE SUBSET OF $^{*}\mathbb{R}^3$ GIVEN BY $f(x,y,z)=K$ IS CALLED A LEVEL SURFACE FOR f.

CONTOUR PLOT OF $f(x,y)$:

PLOT OF SEVERAL EQUALLY SPACED LEVEL CURVES

$f(x,y)=1$, $f(x,y)=2$ $f(x,y)=3$ ... $f(x,y)=10$

VERY USEFUL ON WOLFRAM ALPHA.

TRY : CONTOURPLOT(1/SQRT(x^2-y^2)

$*$ MORE PRECISELY OF $D \subseteq \mathbb{R}^2$

E.G. $f(x,y) = \dfrac{1}{\sqrt{x^2-y^2}}$

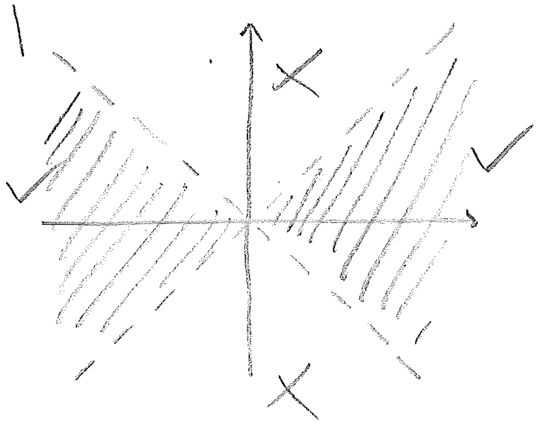
i) FIND THE DOMAIN OF $f(x,y)$

TWO REQUIREMENTS:

- NO NEGATIVE VALUE UNDER $\sqrt{}$
- NO 0 AT DENOMINATOR

SO $x^2-y^2>0$. $x^2>y^2 \sim \sqrt{x^2}>\sqrt{y^2}$

$\sim |x|>|y|$



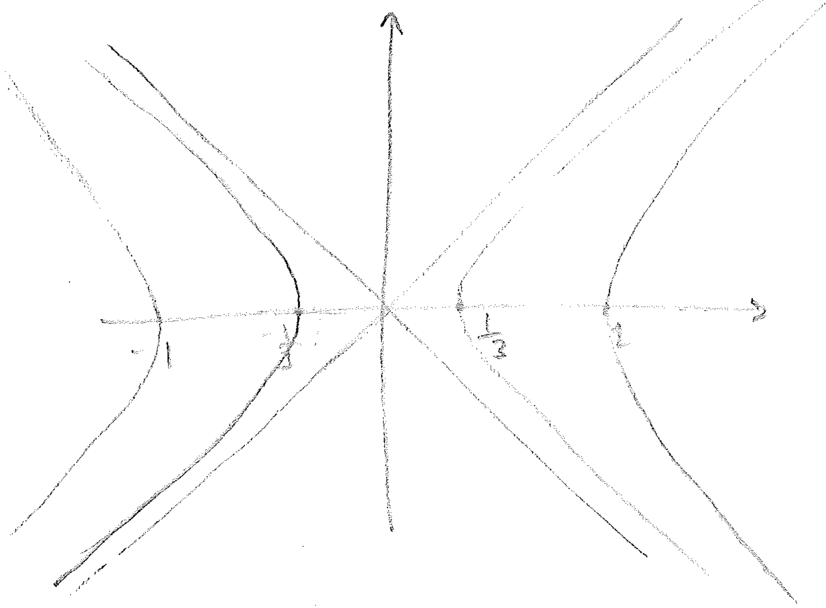
ii) SKETCH THE LEVEL CURVES

$f(x,y)=1$, $f(x,y)=3$

$$\frac{1}{\sqrt{x^2-y^2}}=3 \sim \sqrt{x^2-y^2}=\frac{1}{3} \sim x^2-y^2=\frac{1}{9}$$

$$\frac{1}{\sqrt{x^2-x^2}}=1 \sim \sqrt{x^2-y^2}=1 \sim x^2-y^2=1$$

HYPERBOLAS



iii) FIND THE RANGE OF $f(x,y)$

$$f(x,y) = K \sim \frac{1}{\sqrt{x^2 - y^2}} = \underset{\text{MUST BE} > 0}{K} \sim \sqrt{x^2 - y^2} = \frac{1}{K}$$

$$x^2 - y^2 = \frac{1}{K^2} \qquad \left(\frac{1}{K}, 0\right) \text{ IS A SOLUTION.}$$

SO THE RANGE IS $(0, +\infty)$.

EQUIVALENTLY: $f(t, 0) = \dfrac{1}{\sqrt{t^2}} = \dfrac{1}{|t|}$

IS A CONTINUOUS FUNCTION WHOSE RANGE IS $(0, +\infty)$, SO

$R(f)$ MUST BE $(0, +\infty)$ AS WELL.

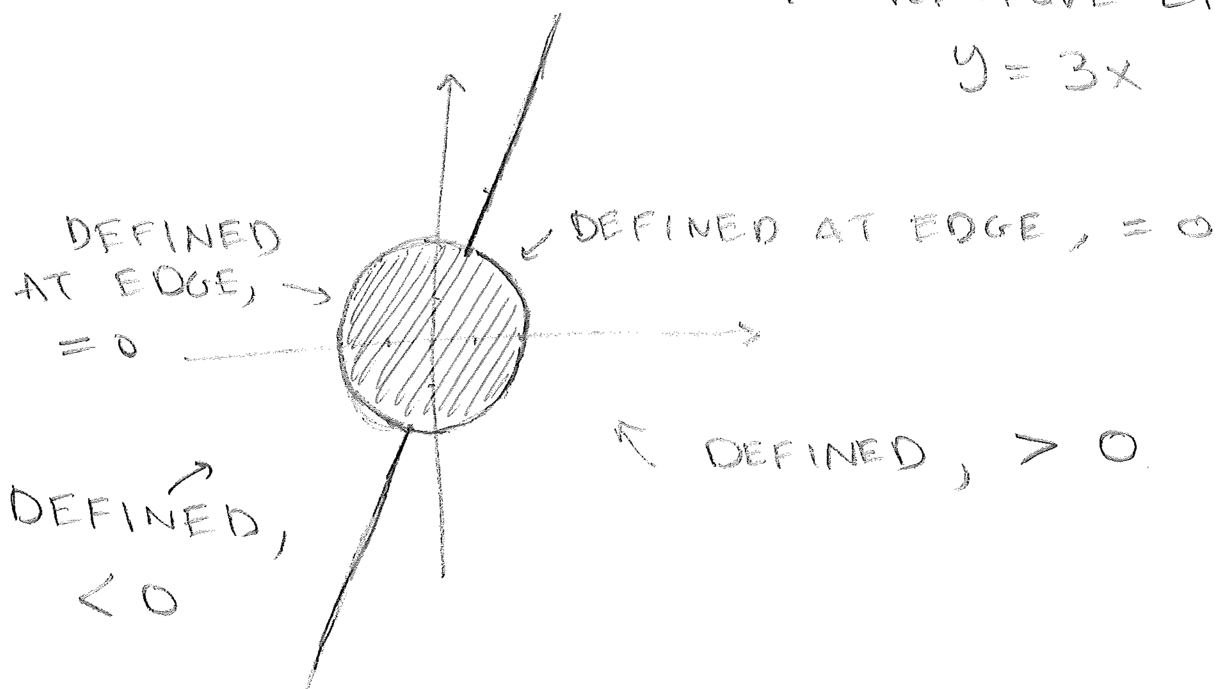
E.G. $f(x,y) = \dfrac{\sqrt{x^2+y^2-4}}{3x-y}$

i) DOMAIN OF $f(x,y)$?

WE NEED THE STUFF UNDER ROOT TO NOT BE NEGATIVE, AND THE DENOMINATOR TO NOT BE 0

SO $x^2+y^2-4 \geq 0$ ← REMOVE INSIDE OF CIRCLE OF RADIUS 2

$3x-y \neq 0$ ← REMOVE LINE $y = 3x$



ii) RANGE OF $f(x,y)$?

$f(x,y)$ CAN BE, $0, >0, <0$ AND GOES TO $\pm\infty$, SO $R(f) = (-\infty, \infty) = \mathbb{R}$