TWO NOTATIONS:

i) $\vec{u} = \langle a_1, b_1, c_1 \rangle$, $\vec{v} = \langle a_2, b_2, c_2 \rangle$

ii) $\vec{u} = \langle u_1, u_2, u_3 \rangle$, $\vec{v} = \langle v_1, v_2, v_3 \rangle$

IN NOTATION i) $\vec{u}+\vec{v} = \langle a_1+a_2, b_1+b_2, c_1+c_2 \rangle$

IN NOTATION ii) $\vec{u}+\vec{v} = \langle u_1+v_1, u_2+v_2, u_3+v_3 \rangle$

NOTATION ii) IS GOOD WHEN WORKING IN MORE THAN 3 DIMENSIONS

BUT WHY DO IT?

<u>E.G.</u>: HOW EXCEL WORKS. COLUMNS IN EXCEL ARE <u>VECTORS</u>, CONTAINING AN ORDERED LIST OF NUMBERS. FOR EXAMPLE

STUDENTS IN ALPHAB. ORDER

$\vec{Q} = \langle \text{QUIZ AVG. IN \%} \rangle = \langle q_1, q_2, \ldots, q_{125} \rangle$

$\vec{W} = \langle \text{WEBWORK AVG IN \%} \rangle = \langle w_1, \ldots, w_{125} \rangle$

$\vec{F} = \langle \text{FINAL AVG IN \%} \rangle = \langle f_1, \ldots, f_{125} \rangle$

THEN $\vec{T} = \langle \text{TOTAL GRADES} \rangle$ IS

$$0.35\vec{Q} + 0.15\vec{W} + 0.5\vec{F} \quad !$$

PROPERTIES OF VECTOR OPERATIONS

- $\vec{u} + \vec{v} = \vec{v} + \vec{u}$ COMMUTATIVE
- $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$ ASSOCIATIVE
- $\vec{v} + \vec{0} = \vec{v}$ IDENTITY
- $c(d\vec{v}) = (cd)\vec{v}$
- $(c+d)\vec{v} = c\vec{v} + d\vec{v}$ DISTRIBUTIVE (i)
- $c(\vec{u} + \vec{v}) = c\vec{u} + c\vec{v}$ DISTRIBUTIVE (ii)
- $0\vec{v} = \vec{0}$
- $\|c\vec{v}\| = |c| \cdot \|\vec{v}\|$
- $\|\vec{u}\| = 0$ <u>IF AND ONLY IF</u> $\vec{u} = 0$

UNIT VECTORS

A VECTOR $\vec{v}$ IS A <u>UNIT VECTOR</u> IF $\|\vec{v}\| = 1$. UNIT VECTORS EXPRESS "ONLY DIRECTION".

IF $\vec{u}$ IS ANY <u>NON-ZERO</u> VECTOR, THEN $\dfrac{\vec{u}}{\|\vec{u}\|}$ IS THE UNIT VECTOR IN THE <u>DIRECTION OF $\vec{u}$</u>.

E.G.

$\vec{v} = \langle 3, -1 \rangle$, $\vec{w} = \langle 1, -2, -2 \rangle$

i) FIND THE UNIT VECTOR IN THE DIRECTION OF $\vec{v}$

ii) SAME FOR $\vec{w}$

iii) FIND A VECTOR IN THE DIRECTION OF $\vec{v}$ WITH MAGNITUDE 10

- $$\frac{\vec{v}}{\|\vec{v}\|} = \frac{1}{\sqrt{9+1}} \langle 3, -1 \rangle = \left\langle \frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}} \right\rangle$$

- $$\frac{\vec{w}}{\|\vec{w}\|} = \frac{1}{\sqrt{1+4+4}} \langle 1, -2, -2 \rangle = \left\langle \frac{1}{3}, -\frac{2}{3}, -\frac{2}{3} \right\rangle$$

- $$10 \frac{\vec{v}}{\|\vec{v}\|} = \frac{10}{\sqrt{10}} \langle 3, -1 \rangle = \sqrt{10} \langle 3, -1 \rangle$$

$$= \langle 3\sqrt{10}, -\sqrt{10} \rangle$$

TWO UNIT VECTORS $\vec{u}, \vec{v}$ ARE PARALLEL IF $\vec{u} = \pm \vec{v}$

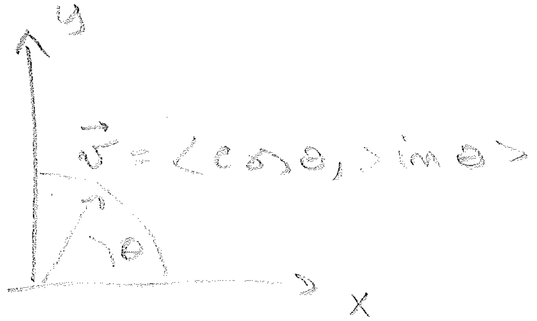
TWO VECTORS $\vec{u}, \vec{v} \neq \vec{0}$ ARE PARALLEL IF $\frac{\vec{u}}{\|\vec{u}\|} = \pm \frac{\vec{v}}{\|\vec{v}\|}$ (EQUIVALENTLY, IF $\vec{u} = c\vec{v}$ FOR SOME c)

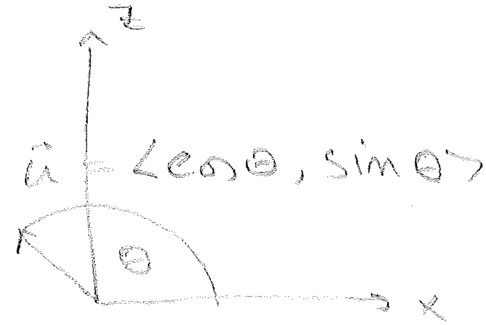
$\vec{u}$ AND $\vec{v}$ POINT IN THE <u>SAME DIRECTION</u>

IF $\dfrac{\vec{u}}{\|\vec{u}\|} = \dfrac{\vec{v}}{\|\vec{v}\|}$ (SAME AS $\vec{u} = c\vec{v}$, $c > 0$).

<u>NOTE:</u> IN 2D SPACE A UNIT VECTOR IS ALWAYS IN THE FORM

$\langle \cos\theta, \sin\theta \rangle$





WHERE θ IS THE ANGLE IT FORMS WITH THE POSITIVE 1ST COORDINATE.

IN $\mathbb{R}^3$ THE <u>STANDARD UNIT VECTORS</u>

ARE $\vec{i} = \langle 1, 0, 0 \rangle$, $\vec{j} = \langle 0, 1, 0 \rangle$,

$\vec{k} = \langle 0, 0, 1 \rangle$

<u>E.G.:</u> IF $\vec{v} = \langle -2, 3, 7 \rangle$ THEN

$\vec{v} = -2\vec{i} + 3\vec{j} + 7\vec{k}$.

THE DOT PRODUCT

$\vec{u} = \langle a_1, b_1, c_1 \rangle \quad \vec{v} = \langle a_2, b_2, c_2 \rangle$

THEN $\quad \vec{u} \cdot \vec{v} = \underbrace{a_1 a_2 + b_1 b_2 + c_1 c_2}_{\text{NUMBER}}$

WHY IS IT USEFUL?

$\vec{v} = \langle a, b, c \rangle \qquad \vec{v} \cdot \vec{v} = a^2 + b^2 + c^2 = \|\vec{v}\|^2$

ALGEBRAIC PROPERTIES OF THE DOT PROD:

- $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$
- $\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{w} \cdot \vec{w}$
- $c(\vec{u} \cdot \vec{w}) = (c\vec{u}) \cdot \vec{w} = \vec{u} \cdot (c\vec{w})$
- $\vec{0} \cdot \vec{v} = 0$
- $\vec{v} \cdot \vec{v} = \|\vec{v}\|^2$

NOTE: YOU CAN'T DO $\vec{u} \cdot \vec{v} \cdot \vec{w}$!

IT DEPENDS ON THE ORDER:

$$(\langle 1, 1, 0 \rangle \cdot \langle 0, 1, 1 \rangle) \langle 1, 0, 1 \rangle = \langle 1, 0, 1 \rangle$$

$$\neq$$

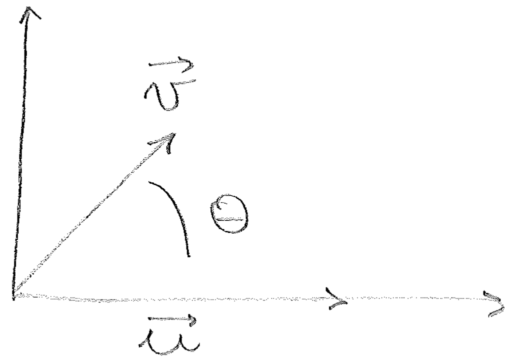
$$\langle 1, 1, 0 \rangle (\langle 0, 1, 1 \rangle \cdot \langle 1, 0, 1 \rangle) = \langle 1, 1, 0 \rangle$$

SO THE FORMULA $\vec{u} \cdot \vec{v} \cdot \vec{w}$ MAKES NO SENSE WITHOUT BRACKETS.

DOT PRODUCT AND ANGLES:

IN 2d: PICK $\vec{u} = a\vec{i} = \langle a, 0 \rangle$

$\vec{v} = b(\cos\theta\, \vec{i} + \sin\theta\, \vec{j}) = \langle b\cos\theta, b\sin\theta \rangle$



$\vec{u} \cdot \vec{v} = ab\cos\theta = \|\vec{u}\|\|\vec{v}\|\cos\theta$

IN GENERAL:

$\vec{u} = a\langle \cos\varphi, \sin\varphi \rangle$, $\vec{v} = b\langle \cos(\theta+\varphi), \sin(\theta+\varphi) \rangle$

$\vec{u} \cdot \vec{v} = a \cdot b\,(\underline{\cos^2\varphi}\cos\theta - \underbrace{\cos\theta \sin\varphi \sin\theta} +$

$\underline{\sin^2\varphi}\cos\theta + \underbrace{\sin\theta\sin\varphi\cos\theta}) =$

$a \cdot b\,(\cos\theta) = \|\vec{u}\|\|\vec{v}\|\cos\theta$!