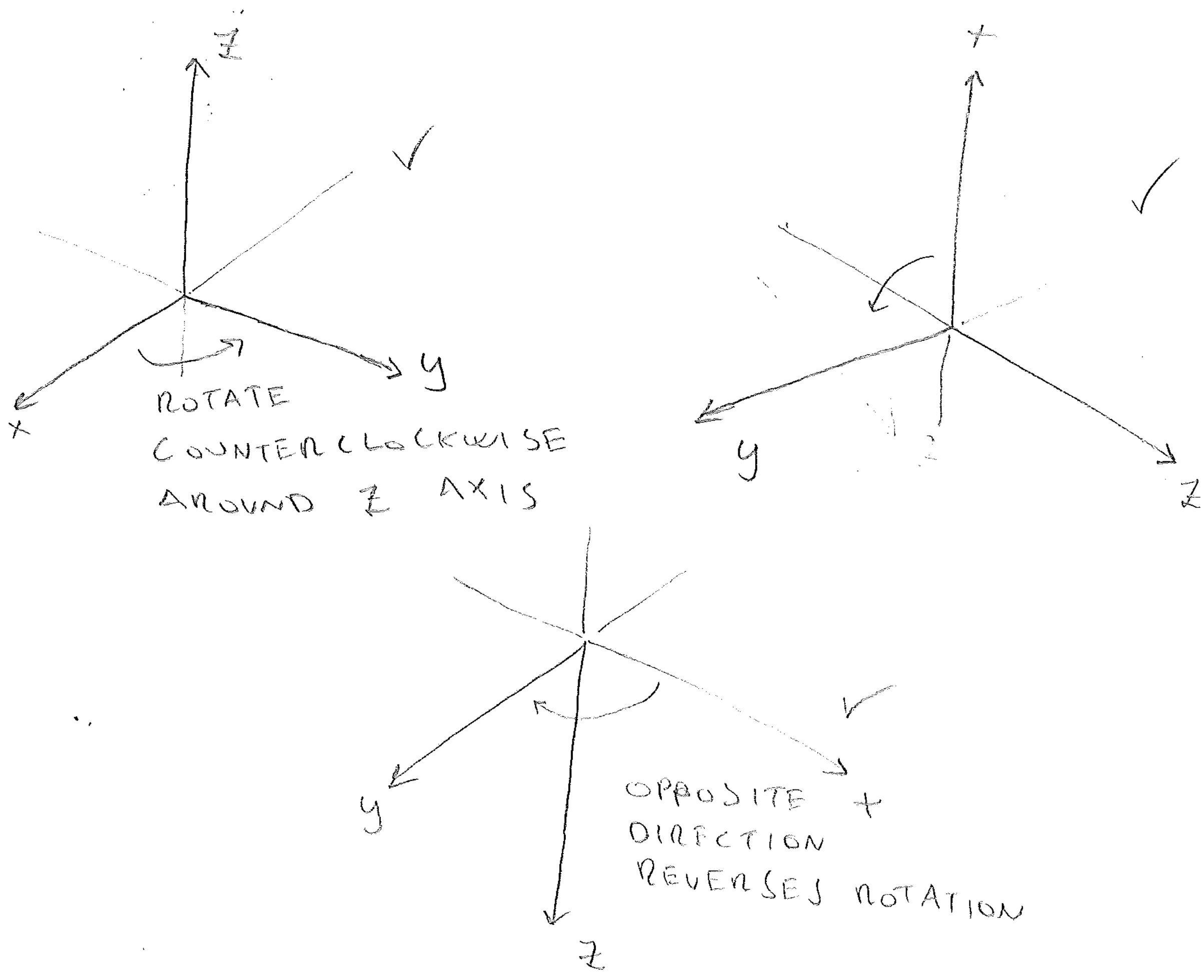
CARTESIAN COORDINATES IN 3D

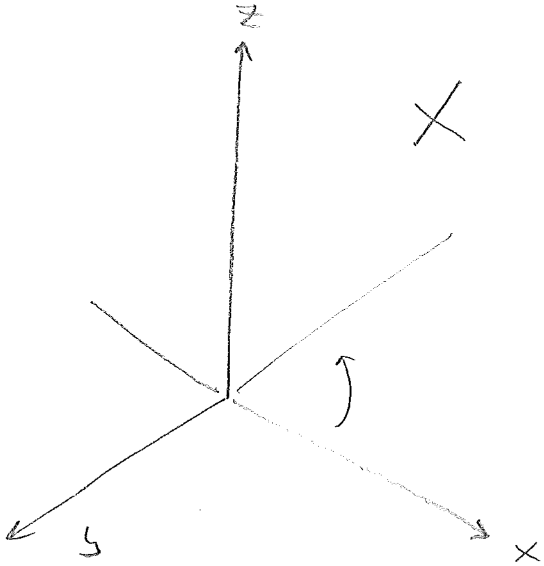
POINT $P = (a, b, c)$

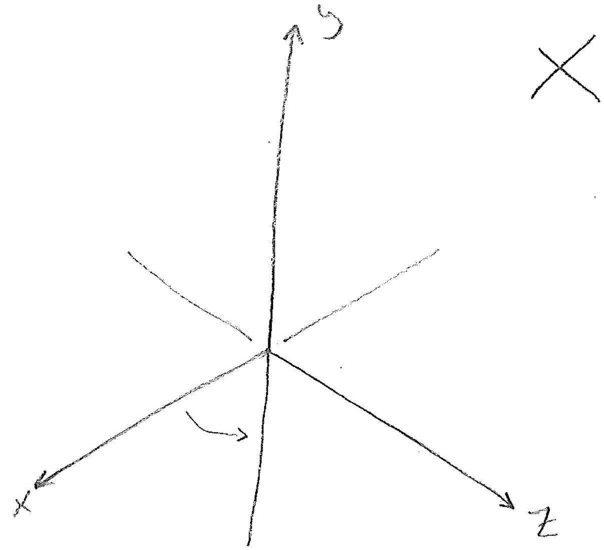
x, y, z AXES: PERPENDICULAR AXES, POSITIVELY (I.E. ACCORDING TO RIGHT HAND RULE) ORIENTED.

a: POSITION OF P ALONG x AXIS
b: " " P " y "
c: " " P " z "

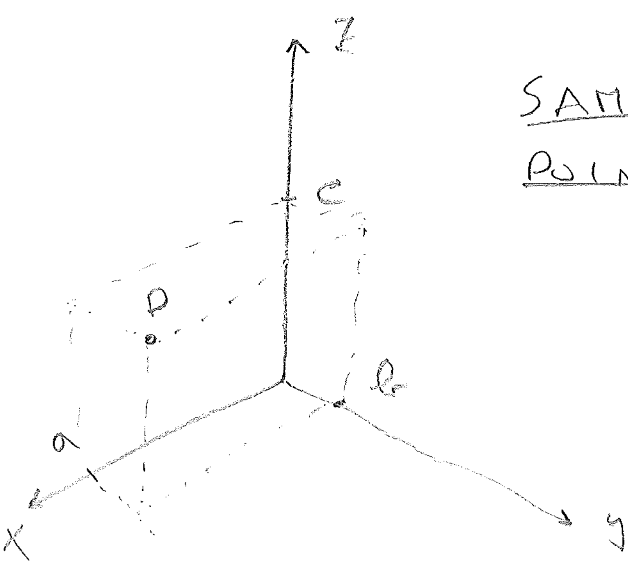
RIGHT HAND RULE:



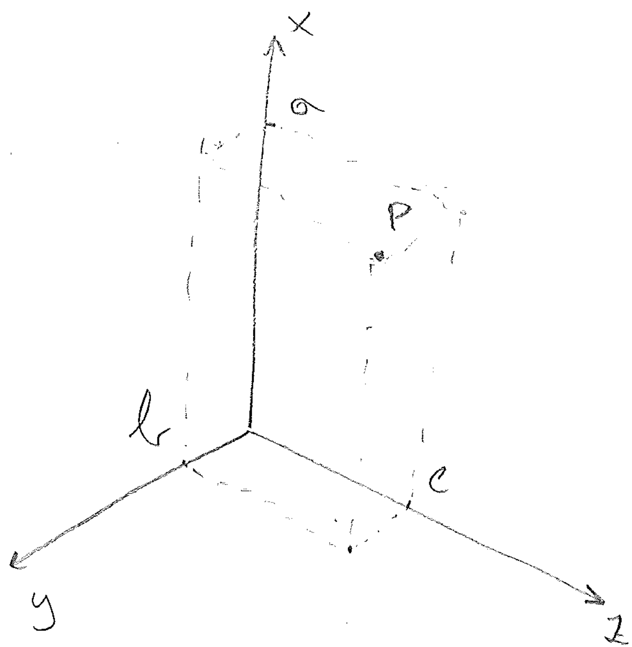




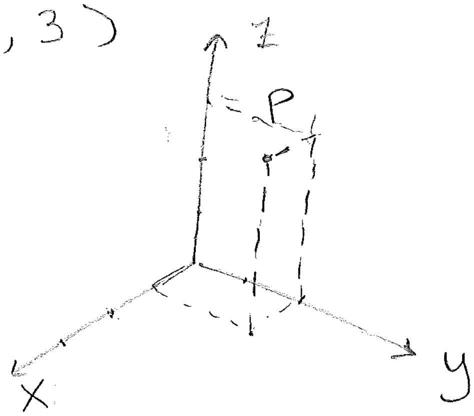
NOT USING THE RHR
<u>MESSES UP SIGNS</u>.



<u>SAME</u>
<u>POINT</u>



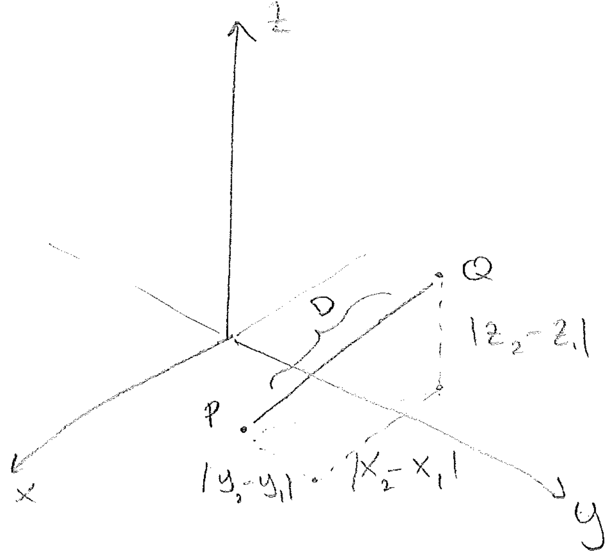
E.G.: P = (1, 2, 3)



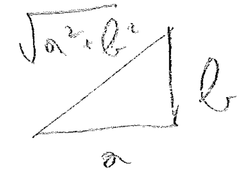
DISTANCE

WHAT IS THE DISTANCE BETWEEN TWO POINTS

$P = (x_1, y_1, z_1)$, $Q = (x_2, y_2, z_2)$?



PITHAGORAS' THM:

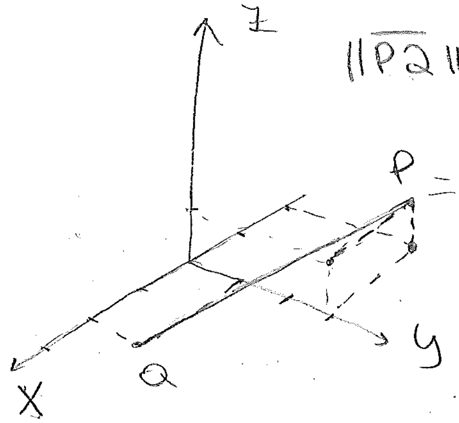


APPLY TWICE:

$$D(P,Q) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

E.G.:

$P = (-2, 3, 1)$, $Q = (2, 1, 0)$. DRAW THE SEGMENT PQ AND FIND ITS LENGTH.



$\|\overline{PQ}\|$ = LENGTH OF PQ $= D(P,Q)$

$$= \sqrt{(2+2)^2 + (1-3)^2 + (0-1)^2}$$

$$= \sqrt{21}$$

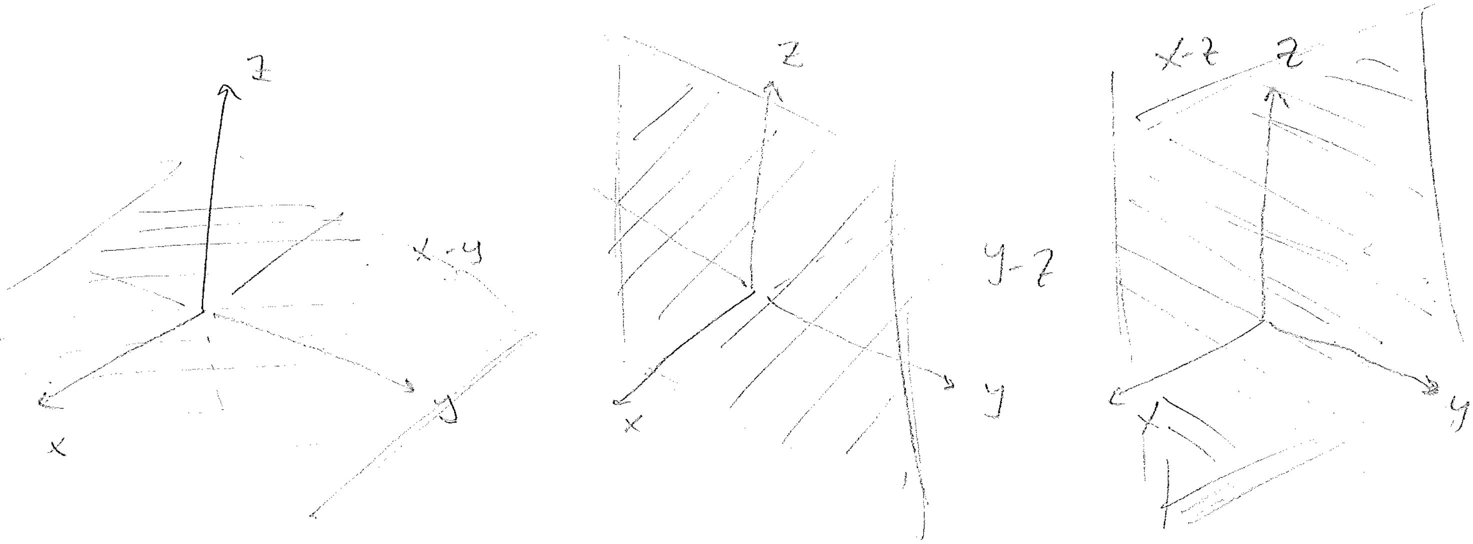
SPHERES, PLANES, CYLINDERS

PLANES:

X-Y PLANE: POINTS IN THE FORM $(a, b, 0)$. EQUATION $z = 0$

Y-Z PLANE: $(0, b, c)$. EQUATION $x = 0$.

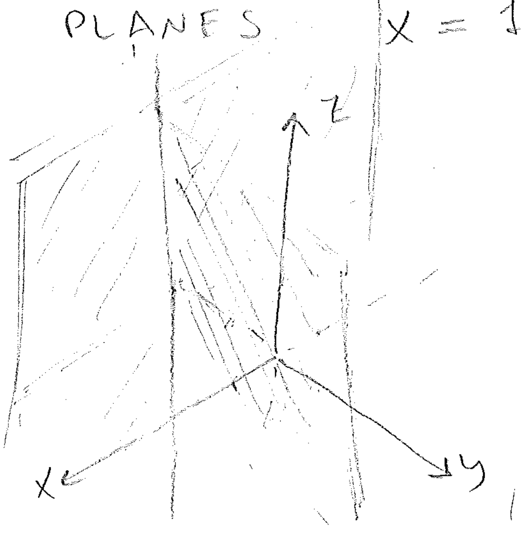
X-Z PLANE: $(a, 0, c)$. $y = 0$.



IN GENERAL: A PLANE IS DEFINED BY A LINEAR EQUATION

$$Ax + By + Cz + D = 0$$

E.G.: SKETCH THE INTERSECTION OF THE PLANES $x = 1$ AND $y = -2$



$$L = (1, -2, c)$$

SPHERES

A SPHERE WITH CENTER $C = (a, b, c)$ AND RADIUS r IS THE SET OF ALL POINTS WHOSE DISTANCE FROM P IS EXCTLY r.

IT HAS THE EQUATION

$$\|\overline{PC}\| = r \qquad P = (x, y, z)$$

$$\sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2} = r$$

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$$

E.G.:

IS $x^2 + 6x + y^2 + 2y + z^2 - 4z = 11$ THE EQUATION OF A SPHERE? IF SO, FIND C AND r.

WE COMPLETE SQUARES:

$$x^2 + 6x = (x+3)^2 - 9 \qquad z^2 - 4z = (z-2)^2 - 4$$

$$y^2 + 2y = (y+1)^2 - 1$$

$$x^2 + 6x + y^2 + 2y + z^2 - 4z = 11 \sim$$

$$(x+3)^2 + (y+1)^2 + (z-2)^2 - 14 = 11$$

$$(x+3)^2 + (y+1)^2 + (z-2)^2 = 25$$

$$C = (-3, -1, 2) \qquad r = \sqrt{25} = 5$$

E.G.:

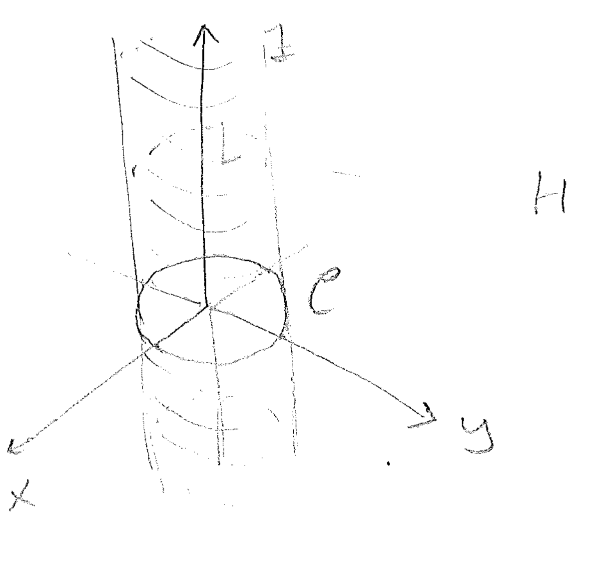
- $x^2 + xy + y^2 + z^2 = 9$ CANNOT BE THE EQUATION OF A SPHERE DUE TO THE MIXED TERM xy.
- $x^2 - 2y + y^2 - 2y + z = 12$ CANNOT BE THE EQUATION OF A SPHERE AS IT LACKS A z^2 TERM.

CYLINDERS:

LET H BE A PLANE, L A LINE NOT IN H AND $\mathcal{C}$ A CURVE ON H.
THE SET COMPOSED OF ALL LINES PARALLEL TO L AND PASSING THROUGH $\mathcal{C}$ IS CALLED A CYLINDER.

E.G.:

H = x-y PLANE, $\mathcal{C}$ = $x^2 + y^2 = 1$, L = z-AXIS.



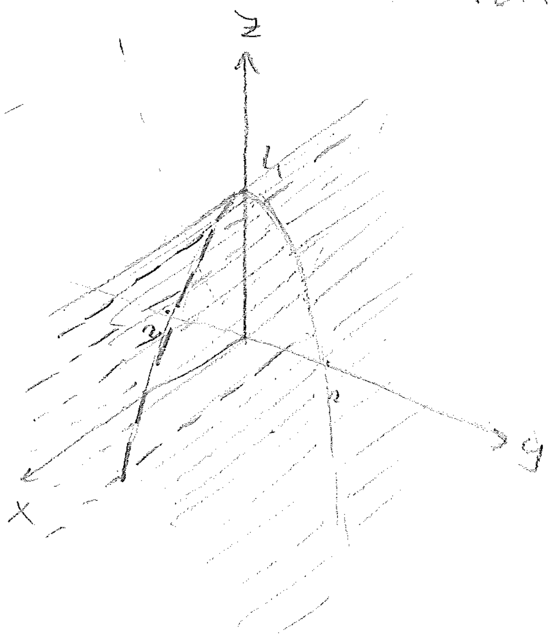
WE WILL ONLY CONSIDER CYLINDERS SUCH THAT:

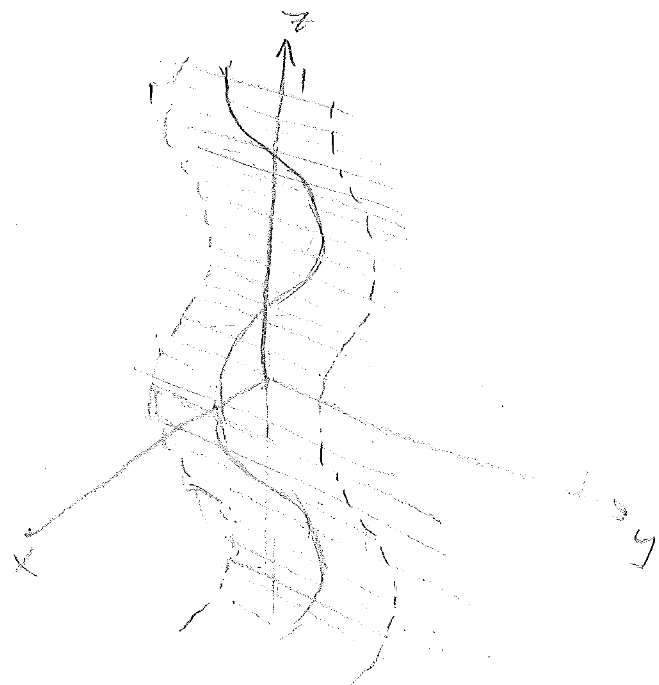
- H IS PARALLEL TO ONE OF THE COORD. PLANES
- L IS PERPENDICULAR TO H

SO WE BASICALLY ONLY NEED TO KNOW THE CURVE:

E.G.:

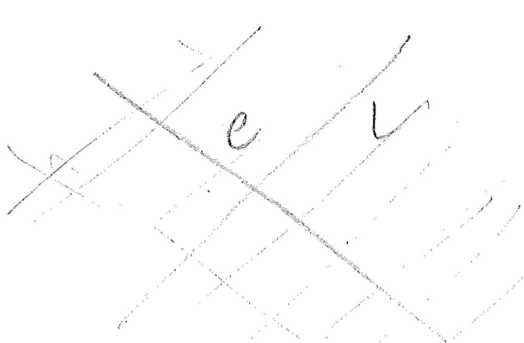
- $z = 4 - y^2$ y-z PLANE
- $x = \cos z$ x-z PLANE





NOTE:

PLANES ARE CYLINDERS AS WELL!



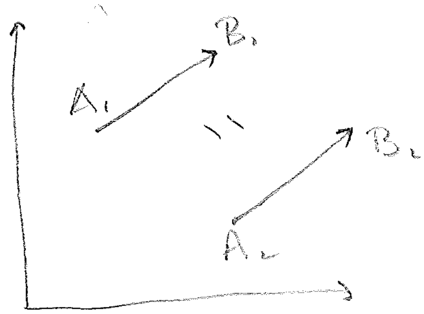
VECTORS

SOME QUANTITIES, SUCH AS FORCES, VELOCITIES, DISPLACEMENT, ETC... ARE COMPOSED OF BOTH A MAGNITUDE AND A DIRECTION. WE EXPRESS THESE QUANTITIES THROUGH VECTORS.

DEF: A VECTOR IS A DIRECTED LINE SEGMENT.

GIVEN POINTS A, B WE CALL $\overrightarrow{AB}$ THE VECTOR GOING FROM A TO B.

TWO VECTORS ARE EQUIVALENT IF THEY HAVE THE SAME DIRECTION AND MAGNITUDE, I.E. IF THEY ARE TRANSLATES OF EACH OTHER.



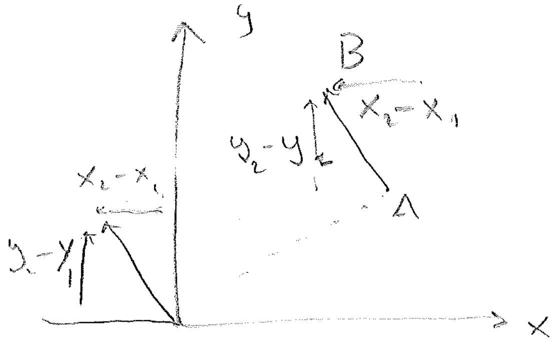
THE MAGNITUDE, NORM OR LENGTH OF $\overrightarrow{AB}$ IS THE DISTANCE FROM A TO B.

THE STANDARD WAY TO REPRESENT A VECTOR $\vec{v} = \overrightarrow{AB}$ IS TO SEE IT AS THE "DISPLACEMENT" FROM A TO B, OR EQUIVALENTLY TO MOVE A TO (0,0,0) (OR (0,0) IN $\mathbb{R}^2$) AND ONLY REMEBER B.

$A = (x_1, y_1, z_1)$ $B = (x_2, y_2, z_2)$

$\overrightarrow{AB} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$

COMPONENTS



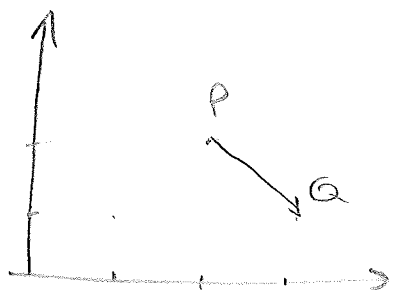
E.G.: $A = (3,5,2)$, $B = (2,6,2)$

$\overrightarrow{AB} = \langle -1, 1, 0 \rangle$

E.G.: $A = (1,1,0)$, $B = (2,2,-1)$

$\overrightarrow{AB} = \langle 1, 1, -1 \rangle$

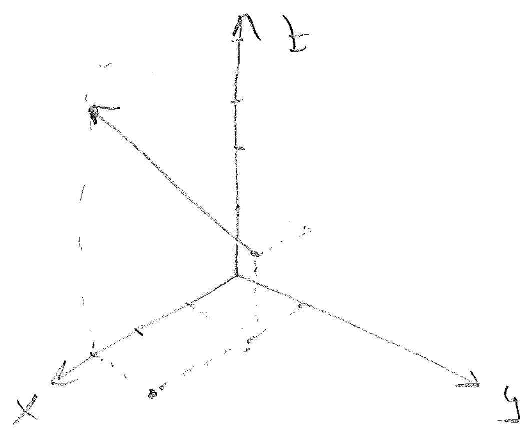
E.G.: SKETCH THE VECTOR $\vec{v} = \langle 1, -1 \rangle$ STARTING AT $P = (2,2)$



$P = (2, 2)$

$Q = (3, 1)$

E.G.: SKETCH THE VECTOR $\vec{v} = \langle 2, -1, 3 \rangle$ STARTING AT $A = (1,1,1)$ AND FIND ITS MAGNITUDE.



$B = (1+2, 1-1, 1+3) =$

$(3, 0, 4)$

$$\|\overline{AB}\| = \sqrt{(3-1)^2 + (0-1)^2 + (4-1)^2} =$$

$$\sqrt{2^2 + (-1)^2 + 3^2} = \sqrt{14}$$

↑

THESE ARE JUST THE <u>COMPONENTS</u> OF $\vec{v}$! WE DO NOT NEED A, B.

VECTOR ALGEBRA

JUST LIKE THE QUANTITIES THEY REPRESENT, VECTORS CAN BE ADDED UP AND MULTIPLIED BY REAL NUMBERS:

DEF:

LET $\vec{u}, \vec{v}$ BE (2 OR 3-DIMENSIONAL) VECTORS. THEN $\vec{u}+\vec{v}$ IS THEIR COMPONENT-BY COMPONENT SUM:

$$\langle a_1, b_1, c_1 \rangle + \langle a_2, b_2, c_2 \rangle =$$

$$\langle a_1 + a_2, b_1 + b_2, c_1 + c_2 \rangle$$

$$\langle a_1, b_1 \rangle + \langle a_2, b_2 \rangle = \langle a_1 + a_2, b_1 + b_2 \rangle$$

$\vec{u} - \vec{v}$ IS THEIR COMPONENT BY COMPONENT DIFFERENCE:

$$\langle a_1, b_1, c_1 \rangle - \langle a_2, b_2, c_2 \rangle =$$

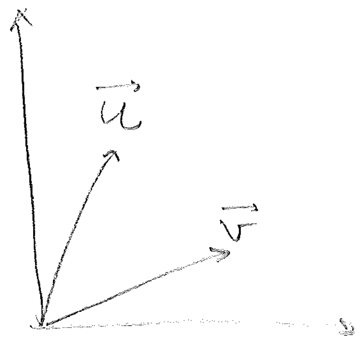
$$\langle a_1 - a_2, b_1 - b_2, c_1 - c_2 \rangle$$

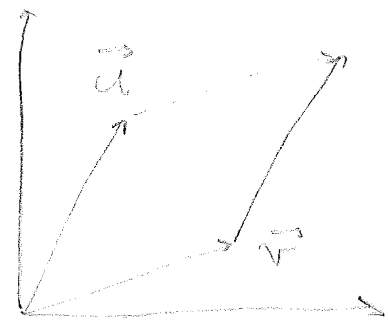
IF c IS A REAL NUMBER THEN $c \cdot \vec{v}$ IS THE COMPONENT WISE PRODUCT:

$$c \cdot \langle x, y, z \rangle = \langle cx, cy, cz \rangle$$

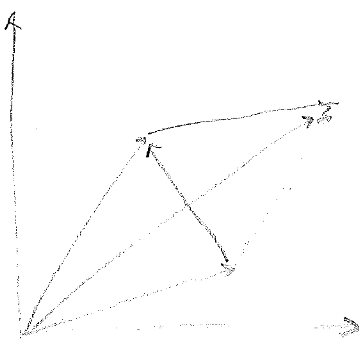
E.G. "PARALLELOGRAM RULE"

TO SKETCH THE SUM OF TWO VECTORS IN 2D SPACE ONE CAN DO THIS:





PLACE TAILS AT (0,0), COMPLETE PARALLELOGRAM



DIAG STARTING AT 0: $\vec{u}+\vec{v}$

DIAG STARTING AT END OF $\vec{v}$: $\vec{u}-\vec{v}$

E.G. THE ZERO VECTOR $\vec{0}$ IS THE VECTOR WHERE ALL COMPONENTS ARE 0. IT IS THE ONLY VECTOR SUCH THAT $\vec{0}+\vec{v}=\vec{v}$.

E.G. $\vec{u}=\langle 3,1,-1\rangle$, $\vec{v}=\langle 2,0,3\rangle$, $w=\langle 0,1,2\rangle$. FIND $\vec{u}-\vec{v}+2\vec{w}$

SOL: $\langle 3-2+0,\ 1-0+2,\ -1-3+4\rangle = \langle 1,3,0\rangle$.