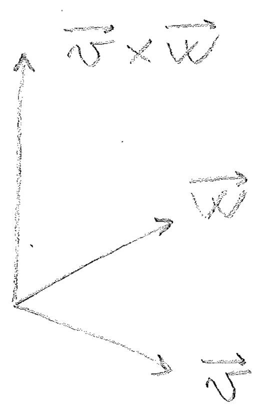
b) $\vec{d} = -5 \langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle$ $\vec{F} = -19.6 \langle 0, 1 \rangle$

$$\vec{F} \cdot \vec{d} = (-5 \cdot -19.6) \langle \tfrac{\sqrt{3}}{2}, \tfrac{1}{2} \rangle \cdot \langle 0, 1 \rangle$$

$$= \frac{5 \cdot 19.6}{2} = 49 \text{ (JOULES)}$$

CROSS PRODUCT

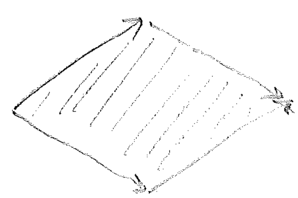


GEOMETRICALLY, $\vec{v} \times \vec{w}$ IS A VECTOR PERPENDICULAR TO THE PLANE CONTAINING $\vec{v}, \vec{w}$

(OR ZERO IF $\vec{v} \parallel \vec{w}$)

WITH DIRECTION DETERMINED BY RHR. LENGTH IS $\|\vec{v}\| \cdot \|\vec{w}\| \cdot \sin \alpha$

= AREA OF



NOTE: i) IF $\vec{v} \parallel \vec{w}$ THE $\sin \alpha = 0$ AND $\vec{v} \times \vec{w} = 0$

ii) $\vec{w} \times \vec{v} = -\vec{v} \times \vec{w}$ (CROSS PRODUCT ANTI-COMMUTES)

iii) THE CROSS PRODUCT "ONLY WORKS" IN 3 DIMENSIONS.

ALGEBRAICALLY:

$$\vec{v} = \langle a_1, b_1, c_1 \rangle, \quad \vec{w} = \langle a_2, b_2, c_2 \rangle$$

THEN

A BIT OF AN ABUSE OF NOTATION...

$$\vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \vec{i} \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} - \vec{j} \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$

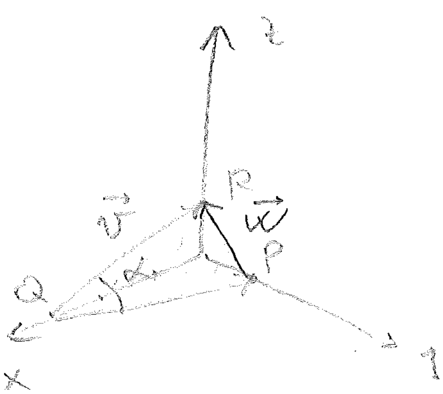
$$+ \vec{k} \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

$$= (b_1 c_2 - c_1 b_1)\vec{i} - (a_1 c_2 - c_1 a_2)\vec{j} +$$

$$(a_1 b_2 - b_1 a_2)\vec{k}$$

$$= \langle b_1 c_2 - c_1 b_1, -a_1 c_2 + c_1 a_2, a_1 b_2 - b_1 a_2 \rangle$$

<u>E.G.:</u> FIND A VECTOR $\perp$ TO THE PLANE CONTAINING $P = (0,1,0)$, $Q = (3,0,0)$, $R = (0,0,1)$ AND THE AREA OF $\widehat{PQR}$.



$$\vec{v} = \vec{QR} \qquad \vec{w} = \vec{PR}$$

$$= \langle -3, 0, 1 \rangle \qquad = \langle 0, -1, 1 \rangle$$

$\vec{v} \times \vec{w}$ IS $\perp$ TO PLANE

$$\vec{i} \begin{vmatrix} 0 & 1 \\ -1 & 1 \end{vmatrix} - \vec{j} \begin{vmatrix} -3 & 1 \\ 0 & 1 \end{vmatrix} + \vec{k} \begin{vmatrix} -3 & 0 \\ 0 & -1 \end{vmatrix} = \langle 1, 3, 3 \rangle$$

AND $\|\vec{v} \times \vec{w}\| = \|\vec{v}\|\|\vec{w}\| \sin \alpha$

$= 2 \text{ AREA}(\widehat{PQR})$

VISUALLY:

SO AREA $= \frac{1}{2}\sqrt{1^2+3^2+3^2} = \frac{\sqrt{19}}{2}$

ANOTHER WAY TO COMPUTE $\vec{v} \times \vec{w}$

$\vec{v} = \langle a_1, b_1, c_1 \rangle \quad \vec{w} = \langle a_2, b_2, c_2 \rangle$

$$\vec{v} \times \vec{w} = \begin{array}{cc} a_1 & a_2 \\ b_1 & b_2 \\ c_1 & c_2 \\ a_1 & a_2 \\ b_1 & b_2 \\ c_1 & c_2 \end{array} \begin{array}{c} \\ \longrightarrow \\ \longrightarrow \\ \longrightarrow \\ \\ \end{array} \begin{bmatrix} b_1 c_2 - c_1 b_2 \\ c_1 a_2 - a_1 c_2 \\ a_1 b_2 - b_1 a_2 \end{bmatrix}$$

$= \langle b_1 c_2 - c_1 b_2, \; c_1 a_2 - a_1 c_2,$

$a_1 b_2 - b_1 a_2 \rangle$

PROPERTIES OF CROSS PRODUCT

i) $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

ii) $(c\vec{a}) \times \vec{b} = c(\vec{a} \times \vec{b}) = \vec{a} \times (c\vec{b})$

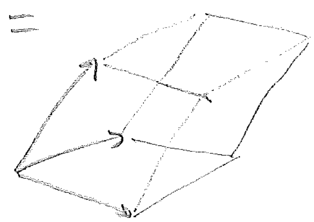
iii) $\vec{a} \times (\vec{b} + \vec{d}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{d}$

iv) $\vec{a} \times (\vec{b} \times \vec{d}) = (\vec{a} \cdot \vec{d})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

$$\neq$$

$$(\vec{a} \times \vec{b}) \times \vec{d} \, !$$

v) $\vec{a} \cdot (\vec{b} \times \vec{d}) = (\vec{a} \times \vec{b}) \cdot \vec{d} =$



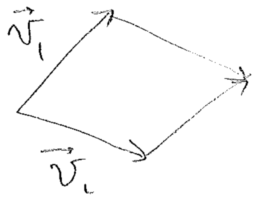
$\pm$ VOLUME OF THE BOX

"SCALAR TRIPLE PRODUCT"

WHY DO THE AREA / VOLUME FORMULAS WORK?

FACT: $\vec{v}_1 = \langle a_1, b_1 \rangle \qquad \vec{v}_2 = \langle a_2, b_2 \rangle$

THEN $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = \pm$ AREA OF

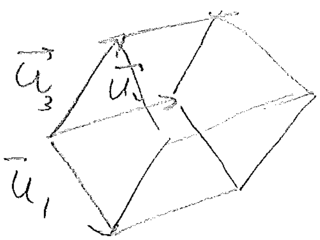


$\vec{u}_1 = \langle a_1, b_1, c_1 \rangle, \; \vec{u}_2 = \langle a_2, b_2, c_2 \rangle$

$, \; \vec{u}_3 = \langle a_3, b_3, c_3 \rangle$

THEN

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \pm \text{ VOLUME OF THE BOX}$$



PARALLELOGRAM FORMULA:

MOVE $\vec{v}, \vec{u}$ SO THAT THEY LIE ON THE X-Y PLANE. THEN

$$\vec{v} = \langle a_1, b_1, 0 \rangle, \quad \vec{u} = \langle a_2, b_2, 0 \rangle$$

$$\vec{v} \times \vec{u} = \vec{i} \underbrace{\begin{vmatrix} b_1 & 0 \\ b_2 & 0 \end{vmatrix}}_{=0} - \vec{j} \underbrace{\begin{vmatrix} a_1 & 0 \\ a_2 & 0 \end{vmatrix}}_{=0} + \vec{k} \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

$$= \pm \vec{k} \text{ (AREA OF PARALLELOGRAM)}$$

BOX FORMULA: $\vec{v} = \langle a_1, b_1, c_1 \rangle$

$\vec{u} = \langle a_2, b_2, c_2 \rangle, \quad \vec{w} = \langle a_3, b_3, c_3 \rangle$

$$(\vec{u} \times \vec{v}) \cdot \vec{w} = \left(\vec{i} \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} - \vec{j} \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} + \vec{k} \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \right) \cdot \vec{w}$$