WARM-UP

① $f(x) = \dfrac{2x^2+8}{e^x}$

a) SHOW THAT f IS INVERTIBLE

b) FIND $(f^{-1})'(\frac{26}{e^3})$

SOL:

a) WE NEED TO SHOW THAT $f'(x)$ IS NEVER 0.

$$f'(x) = \left(e^{-x}(2x^2+8)\right)' = e^{-x}(4x) - e^{-x}(2x^2+8)$$

$$= -e^{-x}(2x^2-4x+8)$$

ALWAYS $\neq 0$

ROOTS

$\dfrac{2 \pm \sqrt{4-16}}{2}$ ← COMPLEX ROOTS → SO NO REAL ROOT → NEVER 0.

b) $y = f^{-1}$ $\quad f(y) = x \rightsquigarrow y' f'(y) = 1$

$$\rightsquigarrow y' = \frac{1}{f'(y)} = \frac{1}{-e^{-x}(2y^2-4y+8)}$$

$f(y) = \dfrac{26}{e^3} \rightsquigarrow \dfrac{2y^2+8}{e^y} = \dfrac{26}{e^3}$ TRY $y=3$ $\quad \dfrac{26}{e^3} = \dfrac{26}{e^3}$ OK

SO $y(\frac{26}{e^3}) = 3$

$$y'\left(\frac{26}{e^3}\right) = \frac{1}{-e^{-3}(18-12+8)} = \frac{-e^3}{14}$$

WARM-UP

(2) $y = \dfrac{x^{2y}}{(\sqrt{y})^4}$ FIND TANGENT AT $(1,1)$

Sol: $y = \dfrac{x^{2y}}{y^2}$ $\quad \ln y = 2y\ln x - 2\ln y$

$3\ln y = 2y\ln x \rightarrow 3\dfrac{d}{dx}\ln y = 2\dfrac{d}{dx}y\ln x$

$\rightarrow \dfrac{3y'}{y} = 2y'\ln x + 2\dfrac{y}{x}$

AT $(1,1)$ $\quad 3y' = 2 \quad y' = \dfrac{2}{3}$

TANGENT $\quad y = \dfrac{2}{3}(x-1)+1$

PRICE ELASTICITY OF DEMAND

THE PRICE ELASTICITY OF DEMAND, OR DEMAND ELASTICITY IS DEFINED AS

$$\epsilon = \frac{p}{q}\frac{dq}{dp} \leftarrow q(p)$$

IT CAN BE SEEN AS

$$\frac{\text{\% CHANGE IN QUANTITY DEMANDED}}{\text{\% CHANGE IN PRICE}}$$

NOTE: WE'RE THINKING OF PRICE AS A FREE VARIABLE SO

$$\frac{\text{\% CHANGE } q}{\text{\% CHANGE } p} = \frac{\frac{dq}{dp}\cdot\frac{1}{q}}{\frac{dp}{dp}\cdot\frac{1}{p}} = \frac{p}{q}\frac{dq}{dp}$$

IF THIS LOOKS GIMMICKY, YOU CAN IMAGINE P AS A FUNCTION OF SOME VARIABLE X (TIME, WEALTH...) THEN

$$\frac{\text{\% CHANGE } q}{\text{\% CHANGE } p} = \frac{\frac{dq}{dx}\cdot\frac{1}{q}}{\frac{dp}{dx}\cdot\frac{1}{p}} \overset{\text{CHAIN RULE}}{=} \frac{\frac{dq}{dp}\cdot\cancel{\frac{dp}{dx}}\cdot\frac{1}{q}}{\cancel{\frac{dp}{dx}}\cdot\frac{1}{p}} = \epsilon!$$

JUST LIKE MARGINAL COST, IT IS A CONCRETE MEASURE THAT ALLOWS US TO TAKE ACTUAL BUSINESS DECISIONS

RECALL THAT THE REVENUE $R(q)$ IS DEFINED BY $P \cdot q(P)$ (IF WE WRITE EVERYTHING IN TERMS OF PRICE).

WE WANT TO DISCUSS HOW THE REVENUE WILL CHANGE IF WE CHANGE PRICE.

- IF $R'(P) > 0$ REVENUE IS <u>INCREASING</u> AT P
- IF $R'(P) < 0$ REVENUE IS <u>DECREASING</u> AT P

CONSEQUENTLY WE CAN CHOOSE TO RESP. INCREASE OR DECREASE PRICE. LET'S COMPUTE $R'(P)$:

$$\frac{d R(P)}{dP} = q(P) \cdot \underbrace{\frac{dP}{dP}}_{=1} + P \frac{dq}{dP} = q(P)\left(1 + \frac{P}{q(P)} \frac{dq}{dP}\right)$$

$q(P) > 0$ SO THE SIGN DEPENDS ON

$$1 + \frac{P}{q} \frac{dq}{dP} = 1 + \epsilon$$

WE GENERALLY EXPECT ϵ TO BE NEGATIVE:

<u>LAW OF DEMAND</u>

DEMAND IS <u>INVERSELY PROPORTIONAL</u> TO PRICE, (THAT IS, $\frac{dq}{dP} \leq 0$)

SO FOR ORDINARY GOODS $\epsilon < 0$

SO IF $\epsilon < 0$ THEN THE SIGN OF $R'(P)$ WILL BE DETERMINED BY WETHER $|\epsilon| < 1$ OR $|\epsilon| > 1$ (OR $|\epsilon| = 1$).

TO CLARIFY THE CONCEPT, SUPPOSE A MANAGER WANTS TO FIND OUT HOW CONSUMERS REACT WHEN HE INCREASES THE PRICE OF A GOOD:

- WHEN $|\epsilon| > 1$, WE SAY THE <u>GOOD</u> IS <u>PRICE ELASTIC</u>: $\% \Delta Q > \% \Delta P$ SO FOR A 1% CHANGE IN PRICE THE CHANGE IN QUANTITY DEMANDED (A DECREASE IF GOOD FOLLOWS L.O.D.) IS GREATER THAN 1%
- WHEN $|\epsilon| < 1$, WE SAY THE <u>GOOD</u> IS <u>PRICE INELASTIC</u>: $\% \Delta Q < \% \Delta P$ SO FOR A 1% CHANGE IN PRICE THERE IS A LOWER THAN 1% CHANGE IN QUANTITY DEMANDED.
- WHEN $|\epsilon| = 1$, WE SAY THE GOOD IS <u>PRICE</u> <u>UNIT ELASTIC</u>. FOR A <u>1</u>% CHANGE IN PRICE WE GET A 1% CHANGE IN DEMAND.

THE LAST CASE TELLS US THAT WE HAVE THE <u>OPTIMAL PRICE</u>.

ACCORDING TO ECONOMIC THEORY, THE PRIMARY FACTOR OF PRICE ELASTICITY IS THE EXISTENCE OF ALTERNATIVES. IF MANY SUBSTITUTE GOODS ARE AVAILABLE, THEN THE GOOD WILL BE PRICE ELASTIC. FOR EXAMPLE, IF THE PRICE OF COKE INCREASED, A LOT OF PEOPLE WOULD JUST SWITCH TO PEPSI.

ON THE OTHER HAND, IF THE PRICE OF A GOOD WITH FEW SUBSTITUTES, SUCH AS RICE IN MUCH OF THE WORLD, OR SOME DRUGS, OR PIZZA* INCREASED, THE DEMAND WOULD NOT CHANGE MUCH AT ALL. THESE GOODS ARE HIGHLY PRICE INELASTIC.

EX 1:

THE DEMAND CURVE FOR oPADS IS

$$q = 500 - 10p$$

a) COMPUTE THE PRICE ELASTICITY

$$\epsilon = \frac{p}{q}\frac{dq}{dp} = \frac{-p}{500-10p} \cdot 10 = \frac{-p}{50-p}$$

b) WHAT'S THE ELASTICITY AT A PRICE OF \$30?

USING (a) WE GET $\frac{-30}{20} = -1.5$

SO THE PRICE IS ELASTIC, AND WE SHOULD LOWER IT.

* 1 IN 3 IS A JOKE.

c) WHAT'S THE (EXPECTED) PERCENT CHANGE IN DEMAND IF THE PRICE IS \$30 AND WE RAISE IT BY 0.45 %

$$\epsilon = \frac{\Delta\% \text{ DEMAND}}{\Delta\% \text{ PRICE}} \quad \text{SO} \quad \Delta\% \text{ DEMAND} = \epsilon \, \Delta\% \text{ PRICE}$$

$$= -1.5 \cdot 0.045 = -0.0675$$

DEMAND DECREASES BY 0.675 %

NOTE: WE ARE USING A GENERAL PRINCIPLE HERE; FOR SMALL PERTURBATIONS THE VARIATION OF A FUNCTION

Δ PRICE

Δ DEMAND

IS WELL APPROXIMATED BY ITS DERIVATIVE.
(FOR LINEAR FUNCTIONS THIS IS LITERALLY TRUE)

d) SUPPOSE WE INCREASED PRICE BY 0.45% AND SAY A DECREASE IN DEMAND BY 0.675 %. WHAT DOES THIS SUGGEST?

$\Delta\%$ DEMAND $>$ $\Delta\%$ PRICE SO PRICE IS ELASTIC. WE SHOULD DECREASE PRICE.

EX 2:

WE SELL CALCULATORS. THE DEMAND FUNCTION IS GIVEN BY $q = 400 - 2p^2$. FIND THE OPTIMAL PRICE TO MAXIMISE REVENUE.

SOL:

WE HAVE

$$\epsilon = \frac{p}{400-2p^2}\frac{dq}{dp} = \frac{p}{400-2p^2}(-4p)$$

WE WANT $\epsilon = -1$ (UNIT ELASTIC)

$$\frac{-4p^2}{400-2p^2} = -1 \sim -4p^2 = 2p^2 - 400$$

$$-6p^2 = -400 \sim p^2 = \frac{200}{3} \sim p = \sqrt{\frac{200}{3}}$$

EX 3: THE DEMAND FOR UPPER BOWL TICKETS TO WATCH THE VANCOUVER CANUCKS IS GIVEN BY $p = \left(100 - \frac{q}{10}\right)^2$.

FIND THE ELASTICITY AND DETERMINE WETHER THE PRICE SHOULD GO UP OR DOWN FROM \$ 100.

SOL: OPTION (1) $\sqrt{p} = 100 - \frac{q}{10} \sim q = 1000 - 10\sqrt{p}$

$$\frac{dq}{dp} = -\frac{5}{\sqrt{p}} \quad \text{SO} \quad \epsilon = \frac{-500}{\sqrt{p}(1000-10\sqrt{p})}$$

$p = 100$

$$\epsilon = \frac{-500}{10{,}900} = -\frac{1}{18}$$

$\Rightarrow$ INCREASE!

OPTION (2) IMPLICIT DIFF $1 = 2q'\left(100 - \frac{q}{10}\right)$

$p = 100 \Rightarrow q = 900$; SUBSTITUTE TO SOLVE.

WRAP_UP

- OPAD'S DEMAND CURVE IS

$$q = 1000\, e^{-p/200}$$

a) FIND THE ELASTICITY OF DEMAND ϵ AT THE CURRENT PRICE OF \$100

b) USE ELASTICITY TO ESTIMATE THE PERCENTAGE DECLINE IN SALES IF PRICE IS RAISED 1%.

- CURRENTLY 1800 PPL/DAY RIDE A PASSENGER FERRY AT \$4/ TICKET. THE DEMAND IS

$$q = \left(\frac{q - 3000}{600}\right)^2$$

SHOULD THE COMPANY INCREASE OR DECREASE PRICE?