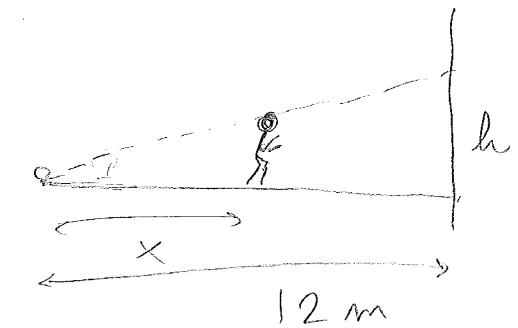
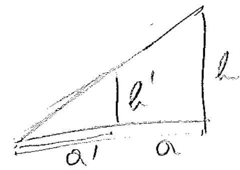
5) A LAMP SHINES LIGHT ON A WALL 12 m AWAY. A 2 m MAN WALKS TOWARDS THE WALL AT 1.6 m/sec. HOW FAST IS THE HEIGHT OF THE SHADOW DECREASING WHEN HE IS 4 m FROM WALL?



TALES:



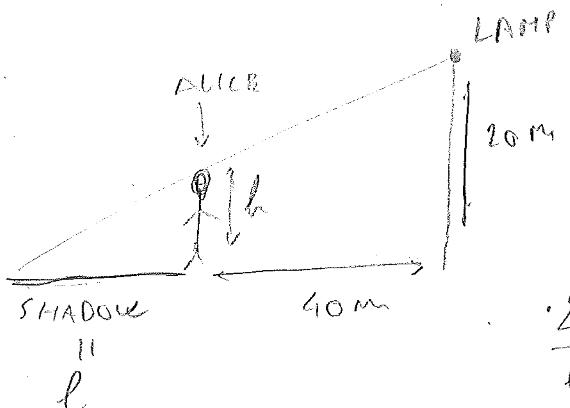
$$\frac{h}{h'} = \frac{a}{a'}$$

$$(h(t), x(t))$$

$$\frac{h}{2} = \frac{12}{x} \qquad h = \frac{24}{x} \qquad h' = -\frac{x' \cdot 24}{x^2}$$

$x' = 1.6$ m/s, $x = 8$ m $(12-4)$ $\qquad h' = \frac{(1.6)(24)}{64} = -\frac{24}{40} = -\frac{6}{10}$ m/s

6) ALICE IS IN WONDERAND. A COOKIE MAKES HER GROW AT 0.5 m/min. SHE IS 40 m AWAY FROM A LAMP THAT IS 20 m TALL. HOW FAST IS HER SHADOW INCREASING LENGTH WHEN SHE'S 15 m?



TALES AGAIN: $\quad (l(t), h(t))$

$$\frac{20}{h} = \frac{40+l}{l}$$

$$\frac{20}{h} = \frac{40}{l} + 1 \qquad 20l - lh = 40h$$

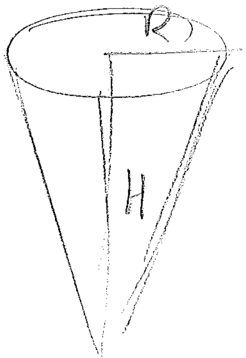
$$20l' - hl' - lh' = 40h' \sim 20l' - 15l' - 60 = 120.$$

$$\sim 5l' = 80 \sim l' = 16$$

WE NEED l WHEN $h = 15$

$$\frac{20}{15} = \frac{40}{l} + 1 \sim \frac{1}{3} = \frac{40}{l} \sim l = 120$$

7) A CONE-SHPED ICICLE IS MELTING. ITS VOLUME IS DECREASING BY 10 cm^3/hour, ITS RADIUS BY 0.4 cm/hour. CURRENTLY IT IS 5 cm IN RADIUS AND 14 cm TALL. HOW FAST IS THE HEIGHT CHANGING?



$$V = \frac{1}{3}\pi R^2 H \qquad (V(T), R(T), H(T))$$

$$\frac{dV}{dT} = \frac{1}{3}\pi\left(2RR'H + R^2H'\right)$$

$$-10 = \frac{1}{3}\pi\left(2\cdot 5\cdot \frac{-4}{10}\cdot 14 + 5^2H'\right) = \frac{1}{3}\pi\left(-56 + 25H'\right)$$

$$-10 + \frac{56\pi}{3} = \frac{25\pi}{3}H' \qquad H' = \frac{-30+56\pi}{25\pi}$$

8) KOOLOG'S MAKS $q\cdot 1000$ PACKS OF FROUT LOOPS CEREAL PER WEEK, THE WHOLESALE PRICE P\$/BOX. THE SUPPLY EQUATION IS

$$6q^2 - 5qp + 2p^3 = 5$$

HOW FAST IS THE SUPPLY OF CEREALS CHANGING WHEN PRICE/BOX IS 6.50 \$, SUPPLY IS 10,000 BOXES AND PRICE/BOX INCREASES BY 0.10 \$/WEEK?

$$(P(T), q(T)) \qquad 6q^2 - 5qp + 2p^3 = 5$$

$$\frac{d(6q^2 - 5qp - 2p^3)}{dT} = 0 \sim 12qq' - 5(qp' + q'p) - 6p^2p' = 0$$

$$p' = 0.10,\ p = 6.5,\ q = 10 \qquad 120q' - 5(1 + 6.5q') - \frac{6(6.5)^2}{10} = 0$$

$$(120 - 32.5)q' = 5 + \frac{39(6.5)}{10} \qquad q' = \frac{50 + 39(6.5)}{87.5\cdot 10} = -0.23257$$

≈ -233 BOXES/WK

RATES OF CHANGE

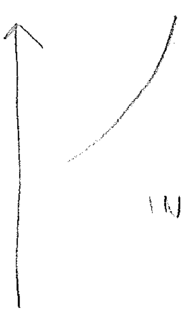
LET'S GET BACK FOR A MOMENT TO DERIVATIVES AS RATES OF CHANGE

$\dfrac{\Delta f}{\Delta x}$ AVERAGE RATE OF CHANGE

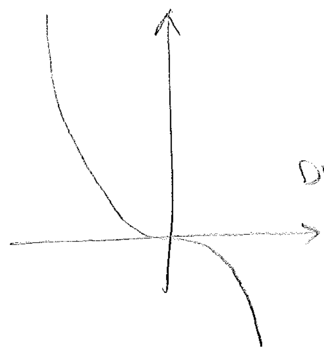
$\dfrac{df}{dx}$ INSTANTANEOUS RATE OF CHANGE = DERIVATIVE

WE RECENTLY INTRODUCED <u>INJECTIVE</u> FUNCTIONS. LET'S LOOK AT THEM FROM THE POINT OF VIEW

- A FUNCTION $f(x)$ IS <u>INCREASING</u> IF FOR ALL $b > a$ WE HAVE $f(b) > f(a)$
- A FUNCTION $f(x)$ IS <u>DECREASING</u> IF FOR ALL $b < a$ WE HAVE $f(b) < f(a)$

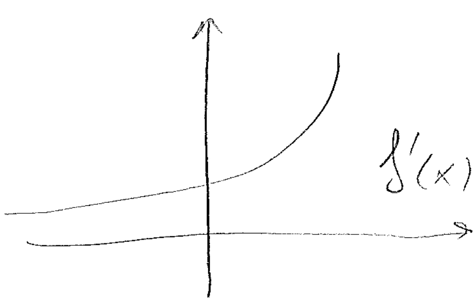


$f(x) = e^x$
INCREASING

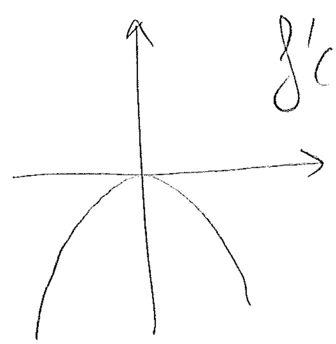


$f(x) = -x^3$
DECREASING

AN INCREASING (DECREASING) FUNCTION $f(x)$ THAT IS DIFFERENTIABLE SATISFIES $f'(x) \geq 0$ ($f'(x) \leq 0$) AT ALL POINTS



$f'(x) = e^x$



$f'(x) = -3x^2$

NOW RECALL THAT IF

$f'(x)$ = RATE OF CHANGE OF $f(x)$

THEN

$f''(x)$ = ACCELERATION OF $f(x)$

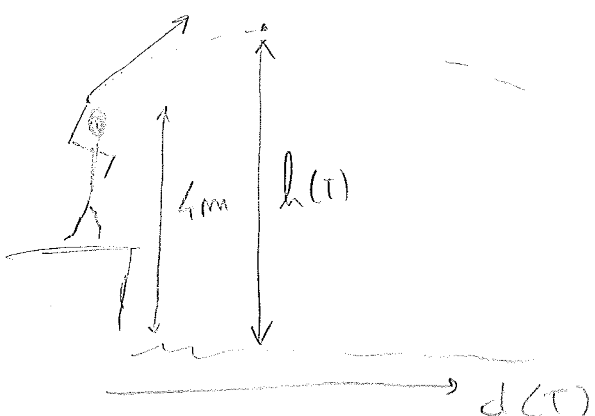
THAT CAN DESCRIBE THE MOTION OF A PARTICLE/PLANET /CAR/ $\$$ /POPULATION

EXAMPLE

$S(T)$ = POSITION AT TIME T

VELOCITY
- AVERAGE $\frac{\Delta S}{\Delta T}$ ↓ limit
- INSTANTANEOUS $\frac{dS}{dT} = S'(T)$ $\frac{m}{sec}$

ACCELERATION
- AVERAGE $\frac{\Delta v}{\Delta T}$ ↓ limit
- ISTANTANEOUS $\frac{dS'(T)}{dT}$ $\frac{m}{sec^2}$



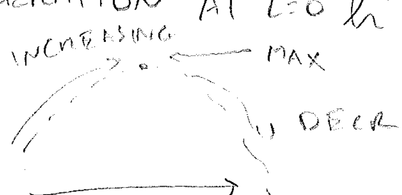
WE THROW A STONE TOWARDS THE SEA (ON MARS, IN THE FUTURE)

$h(T) = -1{,}85T^2 + 10T + 4$ $\qquad d(T) = 10T$

VERTICAL SPEED AT $T=0$ $\quad h'(0) = 10 \frac{m}{s}$

VERTICAL ACCELERATION AT $T=0$ $\quad h''(T) = -3{,}70 \frac{m}{s^2}$

MAX HEIGHT?



SO WE FIND WHEN $h'(T) = -3{,}70T + 10 = 0$

$T = \frac{10}{3{,}7}$, MAX HEIGHT $= -1.85\left(\frac{10}{3.7}\right)^2 + 10 \cdot \frac{10}{3.7} + 4$

WHY?

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

INCREASING : SIGN OF $f(x+h) - f(x)$ IS SAME AS SIGN OF h

DECREASING: SIGN OF $f(x+h) - f(x)$ IS OPPOSITE TO SIGN OF h

<u>CONVERSELY</u>

- IF $\underline{f'(x) > 0}$ $f(x)$ IS <u>INCREASING</u> ON SOME INTERVAL $(x-a, x+a)$
- IF $\underline{f'(x) < 0}$ $f(x)$ IS <u>DECREASING</u> ON SOME INTERVAL $(x-a, x+a)$

WHY?

$f'(x) > 0$ MEANS FOR h CLOSE TO 0 $f(x+h) - f(x)$ HAS SAME SIGN OF h $\sim$ f <u>INCREASING</u>

$f'(x) < 0$ " " " " " " $f(x+h) - f(x)$ " OPPOSING SIGN TO h $\sim$ f <u>DECREASING</u>.

SPEED WHEN STONE HITS WATER?

- WE SOLVE

$$h(T_0)=0$$

- THEN WE EVALUATE $h'(T_0)$

WE WANT THE POSITIVE SOL

$$-1{,}85\,T_0^2+10\,T_0+4=0 \qquad T_0=\frac{-10\pm\sqrt{100+1{,}85\cdot 16}}{-3{,}7}\approx \frac{-10-11.38}{-3.7},\ \frac{-10+11.38}{-3.7}$$

$$\text{SO}\quad T_0=\frac{-10-\sqrt{100+1{,}85\cdot 16}}{-3.7}\approx\frac{21.38}{3.7}$$

$$h'(T_0)=-3{,}70\left(\frac{-10-\sqrt{100+1{,}85\cdot 16}}{-3.7}\right)+10\approx -22.38+10=-12.38\ m/s$$

DISTANCE WHE STONE HITS WATER?

$$d(T_0)=10\cdot\left(\frac{10+\sqrt{100+1.85\cdot 16}}{3.7}\right)\approx 223.8\ m$$

SOME OTHERS RATES

$P(T)$ = POPULATION AT TIME T

$P'(T)$ = POP. GROWTH AT TIME T

$\frac{\Delta P}{\Delta T}$ = AVERAGE POP. GROWTH

$C(q)$ = COST AT QUANTITY Q

$C'(q)$ = COST VARIATION AT QUANTITY q