1.1) MOTIVATING EXAMPLES STRAIGHT FROM BUSINESS!

it may look familiar, but we are armed with calculus differently from an intro economics course

INTRODUCING THE ACTORS:

- **P**ROFIT how much we take home
- **R**EVENUE how much goes through our hands
- **C**OST how much we invest in our business

Q: HOW ARE THEY TIED?

A: $P = R - C$

Q: WHO'S THE PROTAGONIST? THAT IS, WHICH ONE DO WE WANT TO MAXIMISE?

A: P, MOST OF THE TIME

OK, THESE ARE THE ACTORS. BUT WHO "WRITES THE SCRIPT"?

MEET THE PRODUCTION TEAM:

- PRICE how much we charge per unit
- DEMAND how many units we can actually sell

WE WRITE p FOR PRICE AND q FOR DEMAND (QUANTITY)

HOW IS EVERYTHING TIED?

NOTE: WE ARE NOT TAKING COMPETITORS INTO ACCOUNT. IT IS A VERY SIMPLIFIED MODEL.

Q: ARE p AND q TIED? HOW?

A: AS p INCREASES q SHOULD DECREASE. THE SIMPLEST (AND UNREALISTIC) RELATION IS LINEAR

$$ap + bq = c$$

NOTE: VERY SIMPLE. (See Apple)

HOW ABOUT THE ACTORS?

Q: R?

A: $R(q) = p \cdot q$ price per units sold

Q: C?

A: $C(q) = C_0 + C_1 \cdot q$ (LINEAR MODEL)

C_0: FIXED COST (RENT, R&D ...)

C_1: COST PER UNIT

NOTE: WE ARE NOT PRODUCING MORE THAN WE SELL

Q: P

A: $P(q) = R(q) - C(q)$

SO WE WANT TO MAKE $R(q) - C(q)$ POSITIVE - AND LARGE.

AN <u>EXPLICIT</u> EXAMPLE

SONNY CORP.

SONNY CORP PRODUCES THE POPULAR PAYSTATION 5, ABBREVIATED PAY5.

THE IN-HOUSE ECONOMIST SAYS:

- IF PRICE IS AT \$200/UNIT, DAILY DEMAND IS 5000 UNITS
- EVERY \$1 INCREASE DECREASES DEMAND BY 50
- FIXED COST IS \$100.000 / DAY
- PRODUCTION COST IS \$75 / UNIT

QUESTIONS:

1) FIND THE LINEAR DEMAND EQUATION FOR THE PAY5

2) FIND THE DAILY COST OF PRODUCING q PAY5s PER DAY

3) FIND THE DAILY REVENUE FUNCTION

4) FIND THE BREAK-EVEN POINTS (C = R)

5) FIND THE PROFIT

6) WHAT RANGE OF UNIT CAN WE MAKE PROFITABLY?

7) HOW TO MAXIMISE PROFIT?

(1) • WE WANT AN EQUATION IN THE FORM ...

$$q = ap + b$$

• WHEN $p = 200$ $q = 5000$ SO

$$5000 = 200a + b$$

• IF WE INCREASE p BY 1 THEN q DECREASES BY 50. THIS TELLS US THE SLOPE!

$$q = ap + b \quad \text{and} \quad q - 50 = a(p+1) + b$$

ARE BOTH TRUE. TAKE THE DIFFERENCE

$$q - q + 50 = a(p - p - 1) + b - b$$

$$50 = -a$$

$$a = -50$$

WITH NUMBERS

$$5000 = 200a + b$$

$$1450 = 201a + b$$

$$5000 - 1450 = (200 - 201)a + b - b$$

$$50 = -a$$

• SUBSTITUTING

$$5000 = 200(-50) + b$$

$$5000 = -10000 + b$$

$$b = 15000$$

• SO $\quad q = -50p + 15000 \approx p = \frac{q}{50} + 300$

(2) WE CAN ASSUME COST IS LINEAR GIVEN THE INFO

- $C(q) = C_0 + C_1 q = 100.000 + 75 \cdot q$

THIS IS NICE, BUT IN GENERAL THINGS ARE MORE COMPLICATED (e.g. COST DECREASES FOR LARGE NUMBER OF UNITS)

(3) DAILY REVENUE IS

$R = p \cdot q$, SO

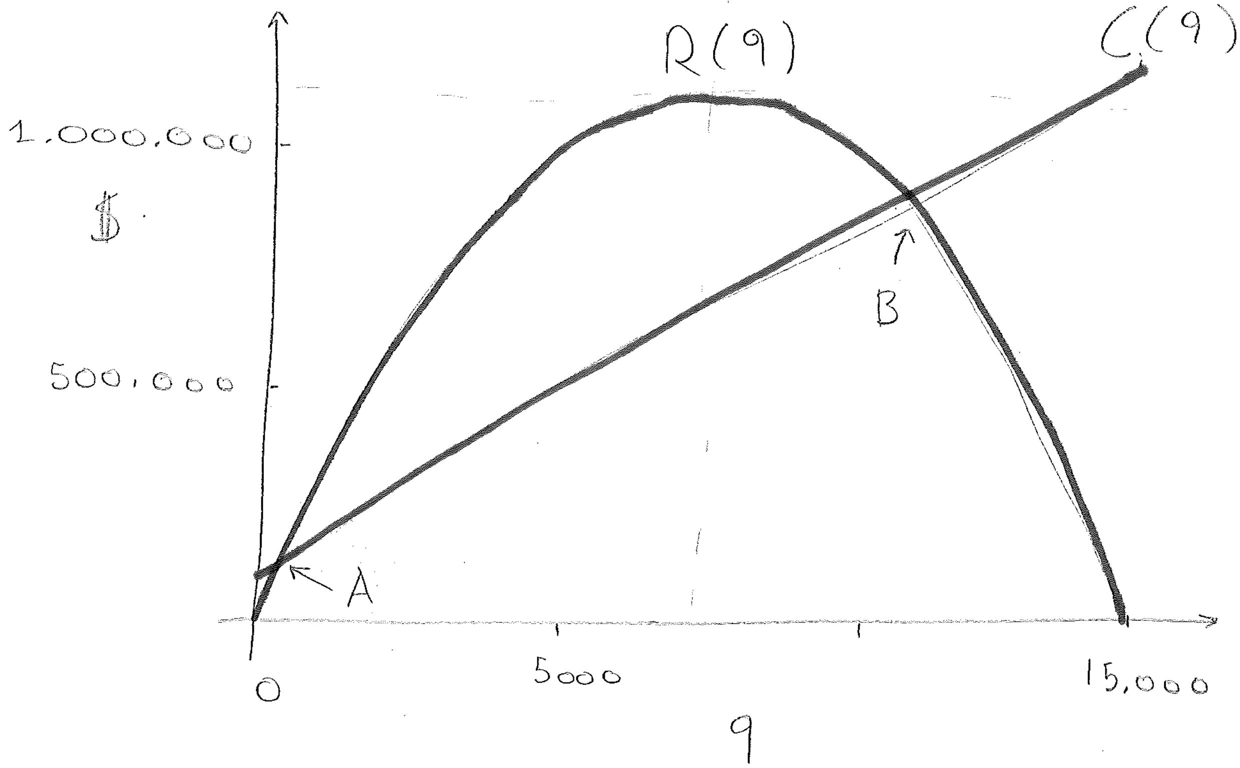
- $R(q) = p \cdot q = q\left(300 - \frac{q}{50}\right)$

AS A FUNCTION OF p

- $R(p) = p \cdot q = p \cdot (15.000 - 300p)$

FIRST EQUATION IS OFTEN MORE USEFUL

(4) WE DRAW A SKETCH (next page)



$R(q) = q \cdot (300 - q/50)$ $\quad C(q) = 10.000 + 75q$

BREAK EVEN POINTS:

A - LOW DEMAND, HIGH PRICE

B - HIGH DEMAND, LOW PRICE

- EXACT SOLUTION

$C(q) = R(q) \qquad 10.000 + 75q = 300q - q^2/50$

$10.000 + q(75 - 300) + q^2/50 = 0$

$10.000 - 225q + q^2/50 = 0$

$q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{225 \pm \sqrt{225^2 - 800}}{2/50} = 5625 \pm 125\sqrt{1705}$

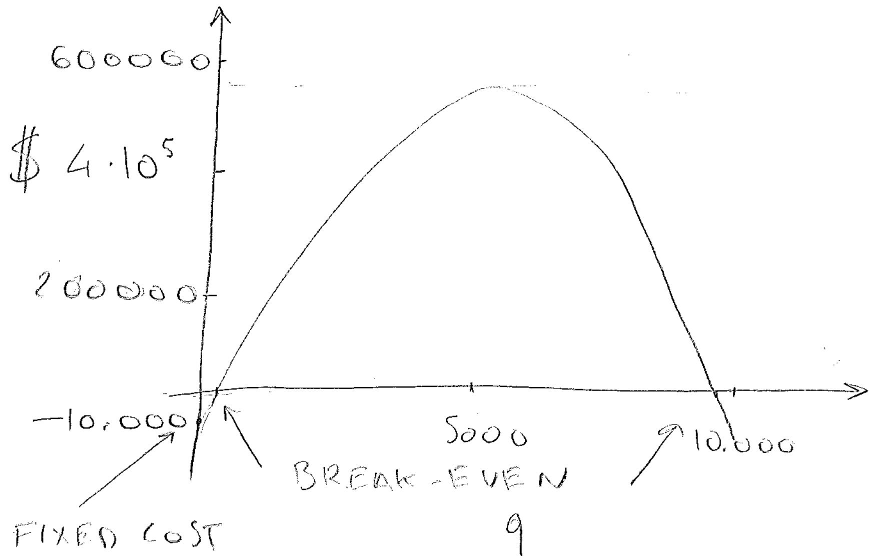
$\approx 463{,}544 \quad , \quad 10786{,}455$

$\overset{\shortparallel}{A} \qquad \overset{\shortparallel}{B}$

(5) PROFIT IS EQUAL TO $R(q)-C(q)$

WE HAVE ALREAD COMPUTED IT

- $P(q) = q\left(300-\frac{q}{50}\right)-(10000+75q) = 225q-\frac{q^2}{50}-10000$



- IT'S AN UPSIDE-DOWN PARABOLA

(6) THE REGION OF PROFIT IS THE REGION BETWEEN THE TWO BREAK-EVEN POINTS

$$465 \lesssim q \lesssim 10785$$

⑦ WHERE IS MAX PROFIT?

AT THE PEAK OF THE PARABOLA

- IN GENERAL WE WOULD MAXIMISE/MINIMISE A FUNCTION WITH CALCULUS (DERIVATIVES) BUT FOR A PARABOLA IT'S EASY

- IF $y = ax^2 + bx + c$ THEN THE PEAK (OR BOTTOM) IS AT $x = \frac{-b}{2a}$

- $P(q) = 225q - \frac{q^2}{50} - 10000$ SO $B = 225$, $A = -\frac{1}{50}$

  MAX PROFIT AT $q = \frac{-225}{-2/50} = \frac{225 \cdot 50}{2} = 5625$

TO DO THIS IN GENERAL WE NEED TO UNDERSTAND "RATE OF CHANGE".

- FIRST STEP: UNDERSTAND <u>LIMITS</u>