MATH 101: "INTEGRAL CALCULUS WITH APPLICATONS TO PHYSICAL SCIENCES AND ENGINEERING", SECTION 205

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ONLINE RESOURCES:

- MAIN WEBPAGE, CONTAINS ALL RELEVANT INFO INCLUDING (IMPORTANT!) PREREQUISITES
- SECTION PAGE; MOSTLY FOR NOTES
- MAIN PAGE LINKS TO SERVICES SUCH AS ACCESS AND DIVERSITY, THRIVE ETC.
- PIAZZA VERY USEFUL FORUM AND BEST PLACE FOR Q&A. USE IT

TIMES:

- 2-3 PM MON-WED-FRI, LSK 200. PLEASE GET TO CLASS INTIME.
- OFFICE HOURS 11-13:45 MON IN MATHX 1220 + ON APPOINTMENT.

TEXTBOOK:

- WE ONLY USE FREE RESOURCES
- CLP CALCULUS TEXTBOOK + PROBLEM BOOK; DOWNLOAD ASAP.

- CREATED SPECIFICALLY FOR THIS COURSE
- FREE SUPPLEMENTARY TEXTS ON WEBSITE
- MY NOTES ARE NO REPLACEMENT FOR CLP
- READ "HOW TO USE THIS TEXT" CAREFULLY

HOMEWORK:

- WEEKLY WEBWORK, LINKS AND INSTRUCTIONS ON WEBPAGE; POSTED THU 8AM, DUE WED 9PM.
- FIRST DUE JAN 10th. NO EXTENSIONS.
- LOWEST SCORE IS DROPPED.
- CAREFULLY WORK THROUGH PROBLEMS WITH PEN AND PAPER; WRITE FULL SOLUTIONS FOR YOUR OWN PRACTICE. OK TO WORK TOGETHER WHEN STUCK.
- BY FAR MOST COMMON REASON FOR DOING BADLY IN FIRST YEAR MATH: NOT ENOUGH HOMEWORK. ALSO WORTH SIGNIFICANT FRACTION OF YOUR GRADE.

ASSESSMENT:

- NO CALCULATORS, FORMULA SHEETS
- 60% FINAL, 20% MIDTERM, 20% WEBWORK
- IF YOU MISS ASSESSMENT DUE TO JUSTIFIED REASONS NOTIFY WITHIN 48 HOURS, DOCUMENTATION WITHIN WEEK
- MORE DETAILS ON WEBPAGE

LAPTOPS AND CELLPHONES:

- DON'T USE CELLPHONES IN CLASS, SILENCE THEM.
- IF YOU REALLY WANT TO TAKE NOTES ON LAPTOP STAY ON THE BACK.
- BRING PEN AND PAPER.

HOW TO SUCCEED:

- BE PROACTIVE
- USE THE TEXTBOOKS!
- TALK TO FELLOW STUDENTS
- ASK QUESTIONS IN CLASS
- USE THE MLC, OFFICE HOURS, PIAZZA...
- DO THESE THINGS EARLY
- COME TO CLASS
- START WEBWORK ASAP
- SELF ASSESS - A LOT. IT'S A FUNDAMENTAL SKILL.

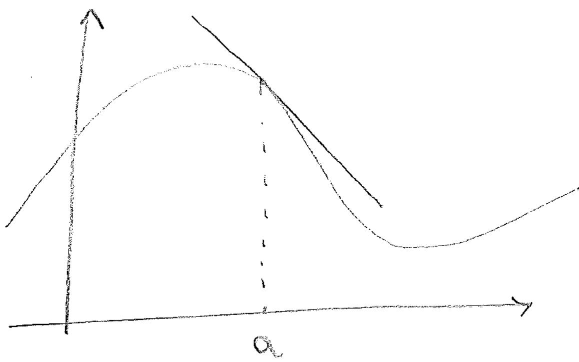
OUTLINE:

- 3 MAIN PARTS:
  - INTEGRATION: BASIC INTEGRATION TECHNIQUES.
  - APPLICATIONS: CENTER OF MASS, WORK, DIFFERENTIAL EQUATIONS.
  - SERIES: "INFINITE SUMS", TAYLOR POLYNOMIALS, INTERACTIONS WITH DIFFERENTIATION AND INTEGRATION.

INTEGRAL CALCULUS

THE TWO BASIC PROBLEMS OF CALCULUS

1. TANGENT PROBLEM



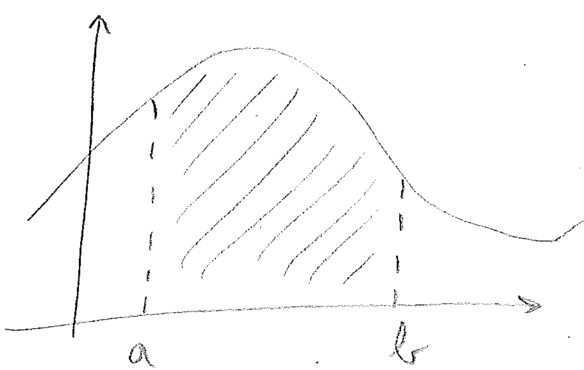
FIND THE SLOPE TO THE TANGENT AT $f(a)$

THIS IS <u>DIFFERENTIAL CALCULUS</u> (SUBJ. OF MATH 100)

• SLOPE OF TANGENT $\longleftrightarrow$ RATE OF CHANGE AT a

ALLOWS TO COMPUTE MAXIMA, MINIMA, OPTIMIZE THINGS

2. AREA PROBLEM



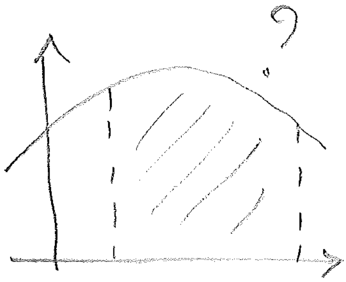
FIND THE AREA BETWEEN $f(x)$ AND THE x AXIS (RUNNING FROM a TO b)

• AREA UNDER $f(x)$ $\longleftrightarrow$ "TOTAL EFFECT OF $f(x)$"

POSSIBLY MOST IMPORTANT OPERATION IN MATHEMATICS, ALLOWS TO COMPUTE ACTION OF FORCES, SOLVE DIFFERENTIAL EQUATIONS, ETC...

THESE TWO PROBLEMS ARE "INVERSE" TO EACH OTHER, AS WE WILL SEE.

HOW DO WE FIND AREAS ?



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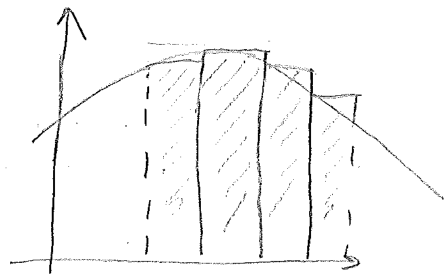

APPROXIMATE WITH RECTANGLES

AREA OF RECTANGLE = BASE × WIDTH

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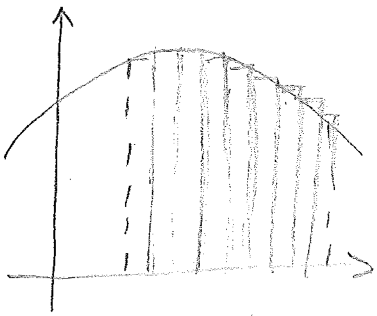

TAKE SMALLER AND SMALLER RECTANGLES ⇝ THE SUM OF THE AREAS OF RECTANGLES CONVERGES TO AREA UNDER FUNCTION

(UNDER SUITABLE CONDITIONS)

WE NEED TO DEVELOP A BIT OF NOTATION

SUMMATION

THE $\sum$ SYMBOL DENOTES SUMMATION. SOME EXAMPLES:

- THE SUM OF THE FIRST 20 INTEGERS

$$1+2+3+\dots+17+18+19+20=\sum_{k=1}^{20} k = \sum_{i=1}^{20} i$$

i AND k ARE "DUMMY VARIABLES" THAT ONLY EXIST INSIDE THE SUM; WE COULD WRITE

$$\sum_{☺=1}^{20} ☺$$ AND IT WOULD BE THE SAME.

NOTE: i, k, ☺ ONLY EXIST AS INDEXES; EXPRESSIONS SUCH AS $i\sum_{i=1}^{20} i$ ARE <u>GIBBERISH</u>!

- A SUM OF CUBES

$$\sum_{K=4}^{7} K^3 = 4^3 + 5^3 + 6^3 + 7^3$$

- A SUM OF A FUNCTION EVALUATED AT INTEGER POINTS

$$\sum_{K=0}^{3} f(K) = f(0) + f(1) + f(2) + f(3)$$

- A SUM OF CONSTANTS $\sum_{i=1}^{m} C = \underbrace{C + \ldots + C}_{m \text{ TIMES}} = mC$

- A FORMAL SUM $\sum_{i=1}^{m} a_i = a_1 + a_2 + \ldots + a_m$

NOTE: THERE ARE MANY WAYS TO WRITE THE SAME SUM!

$$\sum_{i=1}^{5} \frac{1}{i} = \sum_{K=5}^{9} \frac{1}{K-4} = \sum_{J=-1}^{3} \frac{1}{J+2} = \frac{137}{60} = 2.28\overline{3}$$

THE SUM SYMBOL IS A "LINEAR OPERATOR"

- $\sum_{i=1}^{m} c \cdot a_i = c \cdot \sum_{i=1}^{m} a_i$

- $\sum_{i=1}^{m} (a_i + b_i) = \sum_{i=1}^{M} a_i + \sum_{i=1}^{m} b_i$

- $\sum_{i=1}^{m} (a_i - b_i) = \sum_{i=1}^{m} a_i - \sum_{i=1}^{m} b_i$

SOME SUMS YOU SHOULD KNOW:

- FIRST m INTEGERS

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$

WHY?

$$S = 1 + 2 + \dots + m-1 + m$$

$$S = m + m-1 + \quad + 2 + 1$$

$$2S = (m+1) + (m+1) + (m+1) + (m+1) = m \cdot (m+1)$$

- SUM OF POWERS:

$$\sum_{k=0}^{m} r^k = \frac{r^{m+1} - 1}{r-1} \quad \text{FOR ALL } r \neq 1$$

WHY?

$$r^{m+1} - 1 = (r-1)(r^m + r^{m-1} + \dots + 1)$$

- FIRST m SQUARES:

$$\sum_{k=1}^{m} k^2 = \frac{m(m+1)(2m+1)}{6}$$

- FIRST m CUBES:

$$\sum_{k=1}^{m} k^3 = \frac{m^2(m+1)^2}{4}$$

(SQUARES AND CUBES:) PROVEN BY INDUCTION

LET'S TRY COMPUTING A SIMPLE AREA