Constrained Colluding Eavesdroppers: An Information-Theoretic Model

Mahtab Mirmohseni and Panagiotis Papadimitratos
KTH Royal Institute of Technology, Stockholm, Sweden
Email: {mahtabmi,papadim}@kth.se

***Abstract*—We study the secrecy capacity in the vicinity of colluding eavesdroppers. Contrary to the *perfect collusion* assumption in previous works, our new information-theoretic model considers *constraints* in collusion. We derive the achievable secure rates (lower bounds on the perfect secrecy capacity), for the discrete memoryless channel and the Gaussian channel. We also compare the proposed rates to the non-colluding and perfect colluding cases.**

I. Introduction

Wyner [1] introduced the information-theoretic model for confidentiality in noisy communications, called *wiretap channel*, where a legitimate transmitter wishes to transmit a confidential message to a legitimate receiver while keeping it hidden from an eavesdropper (wiretapper). The eavesdropper is assumed to have unlimited computation power, to know the coding scheme of the legitimate user, and to only listen to the channel. When the channel to the eavesdropper is a degraded version of the channel to the legitimate receiver, Wyner [1] proposed the secrecy capacity achieving scheme, known also as *Wyner's wiretap channel coding*, which comprises multicoding and randomized encoding [2, Section 22.1.1]. This result is extended to the broadcast channel with confidential message and to the general wiretap channel (not necessarily degraded) by Csiszár and Körner [3].

Recently, different legitimate-wiretapper user combinations were studied [4]–[8]. In this line of works, scenarios with multiple eavesdroppers considered only *non-colluding* ones. This implies that information leakage of a certain message to all eavesdroppers is computed as the maximum of the leakages (to each one). In some applications, this assumption may underestimate the eavesdroppers' power: they can collude, i.e., share their channel outputs (observations), and render the attack more effective [9]. Hence, combating colluding eavesdroppers, especially in wireless networks, has been a significant challenge [9]–[14]. To the best of our knowledge, all previous works modeled k colluding eavesdroppers as one eavesdropper with k antennas; we term these *perfect colluding* eavesdroppers. Using the equivalent Single-Input Multiple-Output (SIMO) Gaussian wiretap channel, the information leakage is determined by the aggregate Signal to Noise Ratio (SNR) of all eavesdroppers; compared to the maximum SNR in the non-colluding case [9]. This assumption significantly overestimates the eavesdropping capability, forcing a legitimate user to increase its power linearly with the number of eavesdroppers to achieve a positive secure rate. However, collusion (esp. in the wireless networks) necessitates communication resources and power consumption. This, in fact, restricts the collusion channel capacity and thus improves the achievable secure rate by the legitimate user. Hence, the problem at hand is to find an appropriate model and to analyze the effect of these constraints on the secrecy capacity.

In this paper, we model *constrained collusion* with an equivalent wiretap channel, called *Wiretap Channel with Constrained Colluding Eavesdroppers* (WTC-CCE). For our *general* WTC-CCE, we assume that colluding eavesdroppers communicate (by defining their channel inputs) over a virtual *collusion channel*, in addition to the main channel. The higher the collusion channel capacity, the more leaked information can be exchanged. Our model captures previously studied models as special cases: non-colluding eavesdroppers with zero collusion rates and perfect colluding ones with infinite collusion rates. We also propose a special case, the *orthogonal* WTC-CCE: the collusion channel is orthogonal to the main one (unlike the general WTC-CCE where the eavesdroppers share the same channel with the legitimate transmitter). First, we derive an achievable secure rate (a lower bound on the perfect secrecy capacity) for the general discrete memoryless WTC-CCE. The idea is to let the eavesdroppers do their best in colluding. Hence, the information leakage rate is derived by considering the outer bound on the capacity region of the collusion channel; this resembles the cut-set upper bound for the relay channel [2]. Next, we extend our result to the general Gaussian WTC-CCE and its orthogonal version. The main difference is that, in the general model, the eavesdroppers may use jamming techniques to confuse the legitimate receiver; but this way they could be exposed (to the legitimate user). In the orthogonal model, beyond the increased required resources, the eavesdroppers may loose some information leakage rate because they cannot send jamming signals. However, the orthogonality may serve eavesdroppers in hiding themselves. We provide numerical examples to analyze the achievable secure rate and evaluate the overestimation amount (by comparing to perfect colluding case) in different scenarios.

II. Channel Model and Preliminaries

Upper-case letters (e.g., X) denote Random Variables (RVs) and lower-case letters (e.g., x) their realizations. The probability mass function (p.m.f) of a RV X with alphabet set $\mathcal{X}$ is denoted by $p_X(x)$; occasionally, the subscript X is omitted. X_i^j indicates a sequence of RVs $(X_i, X_{i+1}, ..., X_j)$; we use

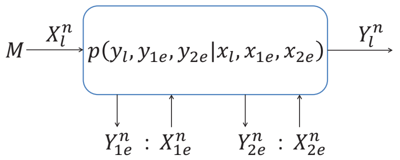


Fig. 1. General Wiretap Channel with Constrained Colluding Eavesdroppers (WTC-CCE).

X^j instead of X_1^j for brevity. $\mathcal{N}(0, \sigma^2)$ denotes a zero-mean Gaussian distribution with variance σ^2.

Consider the WTC-CCE in Fig. 1: a four terminal discrete channel (one transmitter, one legitimate receiver and two eavesdroppers), denoted by $(\mathcal{X}_l \times \mathcal{X}_{1e} \times \mathcal{X}_{2e}, p(y_l^n, y_{1e}^n, y_{2e}^n|x_l^n, x_{1e}^n, x_{2e}^n), \mathcal{Y}_l \times \mathcal{Y}_{1e} \times \mathcal{Y}_{1e})$. $X_l \in \mathcal{X}_l$ and $X_{je} \in \mathcal{X}_{je}$ are the channel inputs of the legitimate transmitter and eavesdropper j and $Y_l \in \mathcal{Y}_l$ and $Y_{je} \in \mathcal{Y}_{je}$ are the channel outputs at the legitimate receiver and eavesdropper j, for $j \in \{1, 2\}$. $p(y_l^n, y_{1e}^n, y_{2e}^n|x_l^n, x_{1e}^n, x_{2e}^n)$ is the channel transition probability distribution. We also assume that the channel is memoryless. In n channel uses, the legitimate transmitter desires to send the message M to the legitimate receiver using the following code.

Definition 1: A $(2^{nR}, n, P_e^{(n)})$ code for WTC-CCE consists of: (i) A message set $\mathcal{M} = [1 : 2^{nR}]$, where m is uniformly distributed over $\mathcal{M}$. (ii) A *randomized* encoding function, f_n, at the legitimate transmitter that maps a message m to a codeword $x_l^n \in \mathcal{X}_l^n$. (iii) Two sets of encoding functions at the eavesdroppers: $\{f_{je,t}\}_{t=1}^n : \mathbb{R}^{t-1} \longrightarrow \mathbb{R}$ such that $x_{je,t} = f_{je,t}(y_{je}^{t-1})$, for $j \in \{1, 2\}$ and $1 \leq t \leq n$. (iv) A decoding function at the legitimate receiver $g : \mathcal{Y}_l^n \mapsto \mathcal{M}$. (v) Probability of error for this code is defined as: $P_e^{(n)} = \frac{1}{2^{nR}} \sum_{m \in \mathcal{M}} Pr(g(y_l^n) \neq m|m \text{ sent})$. (vi) The information leakage rate at eavesdropper $j \in \{1, 2\}$ is defined as:

$$R_{L,j}^{(n)} = \frac{1}{n} I(M; Y_{je}^n). \tag{1}$$

All codewords are revealed to the eavesdroppers. However, the eavesdroppers' mappings are not known to the legitimate user.

Remark 1: The mutual information term in (1) is the same as in the non-colluding case, compared to $I(M; Y_{1e}^n, Y_{2e}^n)$ in the perfect colluding scenario. The difference here comes from the channel distribution and the fact that Y_{2e}^n and Y_{1e}^n given X_l are not independent (due to X_{1e} and X_{2e}).

Definition 2: A rate-leakage tuple $(R, R_{L,1}, R_{L,2})$ is achievable if there exists a sequence of $(2^{nR}, n, P_e^{(n)})$ codes such that $P_e^{(n)} \to 0$ as $n \to \infty$ and $\limsup_{n \to \infty} R_{L,j}^{(n)} \leq R_{L,j}$ for $j \in \{1, 2\}$. The secrecy capacity $\mathcal{C}_s$ is the supremum of all achievable rates R such that perfect secrecy is achieved, i.e., $R_{L,j} = 0$ for $j \in \{1, 2\}$.

Motivated by the fact that the eavesdroppers prefer to avoid exposure, we also consider a special case of the WTC-CCE. We assume that the collusion channel (used by the eavesdroppers) is decoupled from the main channel and we consider the orthogonal WTC-CCE in Fig. 2. Here, $Y_{je} = (Y_{je}^m, Y_{je}^c)$ for $j \in \{1, 2\}$ and $p(y_l, y_{1e}, y_{2e}|x_l, x_{1e}, x_{2e}) = p(y_l, y_{1e}^m, y_{2e}^m|x_l) p(y_{1e}^c, y_{2e}^c|x_{1e}, x_{2e})$, where the variables relating to the main and the collusion channels are indicated with the superscripts m and c respectively. Substituting $X_{1e} = X_{2e} = \emptyset$ results in the non-colluding case; $Y_{1e}^c = Y_{2e}^m, Y_{2e}^c = Y_{1e}^m$ results in the perfect colluding case. To simplify notation, let $\bar{j}$ be the complement of j in $\{1, 2\}$. Now, consider the general Gaussian WTC-CCE at time $t = 1, \ldots, n$ for $j \in \{1, 2\}$, modeled as:

$$\begin{aligned} Y_{l,t} &= h_l X_{l,t} + h_{1e}^l X_{1e,t} + h_{2e}^l X_{2e,t} + Z_{l,t} \\ Y_{je,t} &= h_l^{je} X_{l,t} + h_{\bar{j}e}^{je} X_{\bar{j}e,t} + Z_{je,t} \end{aligned} \tag{2}$$

where h_i^k is a known channel gain from transmitter i to receiver k. We assume perfect echo cancellation at eavesdroppers ($h_{1e}^{1e} = h_{2e}^{2e} = 0$). $X_{u,t}$ is an input signal with average power constraint $\frac{1}{n} \sum_{t=1}^{n} |x_{u,t}|^2 \leq P_u$ and $Z_{u,t}$ is an independent and identically distributed (i.i.d) zero-mean Gaussian noise component with power N_u, for $u \in \{l, 1e, 2e\}$. In practice, h_{1e}^l and h_{2e}^l may be small. The Gaussian counterpart of the orthogonal WTC-CCE for $j \in \{1, 2\}$ can be shown as:

$$\begin{aligned} Y_{l,t} &= h_l X_{l,t} + Z_{l,t} \\ Y_{je,t}^m &= h_{jm} X_{l,t} + Z_{je,t}^m \quad , \quad Y_{je,t}^c = h_{jc} X_{\bar{j}e,t} + Z_{je,t}^c \end{aligned} \tag{3}$$

where h_{jm} and h_{jc} are known channel gains received at eavesdropper j from the main channel and the collusion channel, respectively; power constraints of P_l, P_{1e}, P_{2e} apply for input signals; $Z_{je,t}^m$ and $Z_{je,t}^c$ are i.i.d zero-mean Gaussian noise components with powers N_{je}^m and N_{je}^c at eavesdropper j from the main channel and the collusion channel, respectively.

III. Discrete Memoryless Channel

Our first result establishes an achievable secure rate for the general discrete memoryless WTC-CCE.

Theorem 1: For the general discrete memoryless WTC-CCE, the secrecy capacity is lower-bounded by:

$$\begin{aligned} \mathcal{R}_s^{DM} &= \sup \inf I(X_l; Y_l) - \min\{I(X_l; Y_{1e}, Y_{2e}|X_{1e}, X_{2e}), \\ &\max\{I(X_l, X_{1e}, X_{2e}; Y_{1e}), I(X_l, X_{1e}, X_{2e}; Y_{2e})\}\} \end{aligned} \tag{4}$$

where the supremum and infimum are taken over all joint p.m.fs of the form $p(x_l|x_{1e}, x_{2e}) p(y_l, y_{1e}, y_{2e}|x_l, x_{1e}, x_{2e})$ and $p(x_{1e}, x_{2e})$ respectively.

Proof: The proof is based on the random coding scheme, which uses Wyner wiretap coding at the legitimate user. For the eavesdroppers, the idea is to let them do their best in colluding. Hence, the coding strategy of the eavesdroppers is

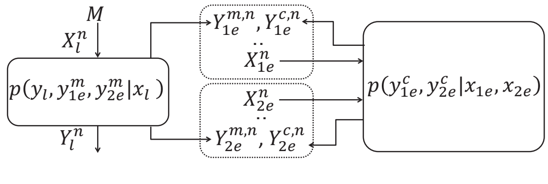


Fig. 2. Orthogonal WTC-CCE.

$$\mathcal{R}_s^{OG}=\theta(\frac{h_l^2P_l}{N_l})-\min\{\theta(P_l(\frac{h_{1m}^2}{N_{1e}^m}+\frac{h_{2m}^2}{N_{2e}^m})),\max\{\theta(\frac{h_{1m}^2P_l}{N_{1e}^m}+\frac{h_{1c}^2P_{2e}}{N_{1e}^c}+\frac{h_{1m}^2h_{1c}^2P_lP_{2e}}{N_{1e}^cN_{1e}^m}),\theta(\frac{h_{2m}^2P_l}{N_{2e}^m}+\frac{h_{2c}^2P_{1e}}{N_{2e}^c}+\frac{h_{2m}^2h_{2c}^2P_lP_{1e}}{N_{2e}^cN_{2e}^m})\}\}. \quad (7)$$

$$\mathcal{R}_s^{G}=\min_{\rho_1,\rho_2,\rho_{12}}\theta(\frac{h_l^2P_l+\rho_1^2(h_{1e}^l)^2P_{1e}+\rho_2^2(h_{2e}^l)^2P_{2e}+2h_lh_{1e}^l\rho_1\sqrt{P_lP_{1e}}+2h_lh_{2e}^l\rho_2\sqrt{P_lP_{2e}}}{(h_{1e}^l)^2P_{1e}(1-\rho_1^2)+(h_{2e}^l)^2P_{2e}(1-\rho_2^2)+2h_{1e}^lh_{2e}^l\rho_{12}\sqrt{P_{1e}P_{2e}}+N_l})$$
$$-\min\{\max\{A(1),A(2)\},\theta(P_l(1-\frac{\rho_1^2P_{1e}^2+\rho_2^2P_{2e}^2+2\rho_1\rho_2\rho_{12}P_{1e}P_{2e}}{P_{1e}P_{2e}(1-\rho_{12}^2)})(\frac{(h_l^{1e})^2}{N_{1e}}+\frac{(h_l^{2e})^2}{N_{2e}}))\}. \quad (8)$$

not determined in the scheme. As a result, the information leakage rate is derived by considering the outer bound on the capacity region of the collusion channel and it looks like the cut-set upper bound for the relay channel [2].

Codebook Generation: Generate $2^{n(R+R_s)}$ i.i.d x_l^n sequences, each with probability $\prod_{t=1}^{n} p(x_{l,t})$. Index them as $x_l^n(m,s)$, where $m \in [1:2^{nR}]$ and $s \in [1:2^{nR_s}]$.

Encoding: To send message $m \in [1:2^{nR}]$, the stochastic encoder at the legitimate transmitter uniformly randomly chooses s and transmits $x_l^n(m,s)$.

Decoding: The decoder at the legitimate receiver wants to correctly recover m,s and seeks a unique message $\tilde{m}$ and some $\tilde{s}$ such that $(x_l^n(\tilde{m},\tilde{s}),y_l^n)$ are jointly typical. Applying the packing lemma [2], with arbitrarily high probability $\tilde{m}=m$ if n is large enough and

$$R+R_s \le I(X_l;Y_l). \quad (5)$$

Analysis of the information leakage rate: To simplify the notation, let $X_e=(X_{1e},X_{2e})$ and $Y_e=(Y_{1e},Y_{2e})$. We derive two bounds for the randomness index rate, R_s. First, we obtain the second term of information leakage rates in the min term in (4), i.e., $R_{L2}=\max\{I(X_l,X_{1e},X_{2e};Y_{1e}),I(X_l,X_{1e},X_{2e};Y_{2e})\}$.

Now, consider the leaked information to Y_{1e}^n averaged over the random codebook $\mathcal{C}$.

$$\begin{aligned}
&I(M;Y_{1e}^n|\mathcal{C})=H(M|\mathcal{C})-H(M|Y_{1e}^n,\mathcal{C})\\
&=nR-H(M,Y_{1e}^n,X_l^n,X_e^n|\mathcal{C})+H(X_l^n,X_e^n|M,Y_{1e}^n,\mathcal{C})+H(Y_{1e}^n|\mathcal{C})\\
&=nR-H(X_l^n,X_e^n|\mathcal{C})-H(M,Y_{1e}^n|X_l^n,X_e^n,\mathcal{C})\\
&+H(X_l^n,X_e^n|M,Y_{1e}^n,\mathcal{C})+H(Y_{1e}^n|\mathcal{C})\\
&\le nR-H(X_l^n|\mathcal{C})-H(Y_{1e}^n|X_l^n,X_e^n,\mathcal{C})\\
&+H(X_l^n,X_e^n|M,Y_{1e}^n,\mathcal{C})+H(Y_{1e}^n|\mathcal{C})\\
&=nR-n(R+R_s)+I(X_l^n,X_e^n;Y_{1e}^n|\mathcal{C})+H(X_l^n,X_e^n|M,Y_{1e}^n,\mathcal{C})\\
&\overset{(a)}{\le}-nR_s+nI(X_l,X_e;Y_{1e})+H(X_l^n,X_e^n|M,Y_{1e}^n,\mathcal{C})\overset{(b)}{\le}n\delta_1
\end{aligned}$$

(a) holds because the channel is memoryless; (b) follows from [2, Lemma 22.1]: if $R_s \ge I(X_l,X_{1e},X_{2e};Y_{1e})$, then $H(X_l^n,X_{1e}^n,X_{2e}^n|M,Y_{1e}^n,\mathcal{C}) \le nR_s-nI(X_l,X_{1e},X_{2e};Y_{1e})+n\delta_1$. Following similar steps, one can show that if $R_s \ge I(X_l,X_{1e},X_{2e};Y_{2e})$, then $I(M;Y_{2e}^n|\mathcal{C}) \le \delta_2$. Considering (1), combining (5) and these constraints on R_s gives $\mathcal{R}_s^{DM}$ with R_{L2}.

Now, to derive the first term of information leakage rates in min in (4), i.e., $R_{L1}=I(X_l;Y_{1e},Y_{2e}|X_{1e},X_{2e})$, we evaluate the leaked information to both Y_{1e}^n and Y_{2e}^n, averaged over the random codebook $\mathcal{C}$.

$$\begin{aligned}
&I(M;Y_e^n|\mathcal{C})=H(M|\mathcal{C})-H(M|Y_e^n,\mathcal{C})\\
&=nR-H(M,Y_e^n,X_l^n|\mathcal{C})+H(X_l^n|M,Y_e^n,\mathcal{C})+H(Y_e^n|\mathcal{C})\\
&\overset{(a)}{=}nR-H(X_l^n|\mathcal{C})-H(M,Y_e^n|X_l^n,\mathcal{C})\\
&+H(X_l^n|M,Y_e^n,X_e^n,\mathcal{C})+H(Y_e^n|\mathcal{C})\\
&\overset{(b)}{\le}nR-n(R+R_s)+I(X_l^n;Y_e^n|\mathcal{C})+H(X_l^n|M,Y_e^n,X_e^n,\mathcal{C})\\
&\overset{(c)}{=}-nR_s+\sum_{i=1}^{n}I(X_l^n;Y_{e,i}|Y_e^{i-1},X_{e,i},\mathcal{C})+H(X_l^n|M,Y_e^n,X_e^n,\mathcal{C})\\
&\overset{(d)}{\le}-nR_s+nI(X_l;Y_e|X_e)+H(X_l^n|M,Y_e^n,X_e^n,\mathcal{C})\overset{(e)}{\le}n\delta_3 \quad (6)
\end{aligned}$$

(a) and (c) follow because $x_{je,t}=f_{je,t}(y_{je}^{t-1})$, for $j \in \{1,2\}$ and $1 \le t \le n$; (b) is due to the fact that conditioning does not increase the entropy; (d) holds due to the memoryless property of the channel; (e) follows from [2, Lemma 22.1]: if $R_s \ge I(X_l;Y_{1e},Y_{2e}|X_{1e},X_{2e})$, then $H(X_l^n|M,Y_{1e}^n,Y_{2e}^n,X_{1e}^n,X_{2e}^n,\mathcal{C}) \le nR_s-nI(X_l;Y_{1e},Y_{2e}|X_{1e},X_{2e})+n\delta_3$. Note that (6) implies $I(M;Y_{je}^n|\mathcal{C}) \le n\delta_3$ for $j \in \{1,2\}$ (for the individual leakage rates). Now, combining (5) and this constraint on R_s gives $\mathcal{R}_s^{DM}$ with R_{L1}. This completes the proof. ■

Remark 2: Substituting $Y_{je}=(Y_{je}^m,Y_{je}^c)$ for $j \in \{1,2\}$ in (4) results in an achievable secure rate ($\mathcal{R}_s^{ODM}$) for the orthogonal discrete memoryless WTC-CCE, where the supremum is taken over all joint p.m.fs of the form $p(x_l|x_{1e},x_{2e})p(y_l,y_{1e}^m,y_{2e}^m|x_l)p(y_{1e}^c,y_{2e}^c|x_{1e},x_{2e})$.

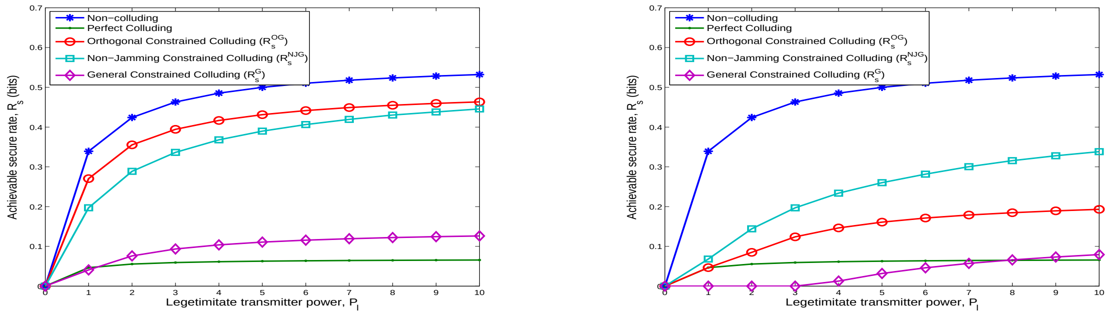
Remark 3: By setting $X_{1e}=X_{2e}=\emptyset$ in (4), $\mathcal{R}_s^{DM}$ reduces to $\sup I(X_l;Y_l)-\max\{I(X_l;Y_{1e}),I(X_l;Y_{2e})\}$ for the non-colluding case. Furthermore, redefining $Y_{1e}^c=Y_{2e}^m, Y_{2e}^c=Y_{1e}^m$ in $\mathcal{R}_s^{ODM}$ results in the achievable secure rate for the perfect colluding case, i.e., $\sup I(X_l;Y_l)-I(X_l;Y_{1e},Y_{2e})$.

IV. Gaussian Channel

We study the Gaussian WTC-CCE. First, we consider the orthogonal Gaussian WTC-CCE. Let $\theta(x) \doteq \frac{1}{2}\log(1+x)$.

Theorem 2: $\mathcal{R}_s^{OG}$ in (7), shown at the top of the page, is an achievable secure rate for orthogonal Gaussian WTC-CCE (defined in (3)).

Proof: We can extend the achievable secrecy rate in Theorem 1 (after applying Remark 2) to the Gaussian case with continuous alphabets using standard arguments [15]. As we do not know the optimal distribution $p(x_l|x_{1e},x_{2e})$ that maximizes $\mathcal{R}_s^{ODM}$, we use a Gaussian input distribution (at the legitimate transmitter) to achieve a lower bound. Let $X_l \sim \mathcal{N}(0,P_l)$. Note that the leakage rates in $\mathcal{R}_s^{ODM}$ (i.e., R_{L1} and R_{L2}) are Multiple Access Channel (MAC) type bounds. From the maximum-entropy theorem [15] (or [2, P. 21]), these bounds are largest (or equivalently $\mathcal{R}_s^{ODM}$ is minimized over $p(x_{1e},x_{2e})$) for Gaussian inputs at the eavesdroppers. Hence, set $X_{je} \sim \mathcal{N}(0,P_{je})$ for $j \in \{1,2\}$ and define $-1 \le \rho_j \le 1$ as the correlation coefficient between



(a) $h^{1e}_{2e} = h^{2e}_{1e} = h_{jc} = \sqrt{0.1}, j \in \{1, 2\}$.   (b) $h^{1e}_{2e} = h^{2e}_{1e} = h_{jc} = \sqrt{0.6}, j \in \{1, 2\}$.

Fig. 3. Achievable secure rates $\mathcal{R}_s$ for $P_{je} = 1$, $h^l_{je} = \sqrt{0.2}$, $h^{je}_l = h_{jm} = 1$, $N_l = N_{je} = N^m_{je} = N^c_{je} = 1, j \in \{1, 2\}$.

X_{je} and X_l, i.e., $E(X_{je}X_l) = \rho_j\sqrt{P_{je}P_l}$ for $j \in \{1, 2\}$ and $\rho_{12} = \frac{E(X_{1e}X_{2e})}{\sqrt{P_{1e}P_{2e}}}$. After calculating the mutual information terms in (4), one can easily show that the leakage rate is maximized (or secure rate in minimized) for $\rho_{12} = \rho_1 = \rho_2 = 0$. This means that in the orthogonal setup the best strategy for the eavesdroppers is to use independent codewords. This achieves $\mathcal{R}^{OG}_s$ in (7). ■

Remark 4: To achieve the non-colluding rate, i.e., $\theta(\frac{h_l^2 P_l}{N_l}) - \max\{\theta(\frac{h^2_{1m}P_l}{N^m_{1e}}), \theta(\frac{h^2_{2m}P_l}{N^m_{2e}}\}$, set $P_{1e} = P_{2e} = 0$ in $\mathcal{R}^{OG}_s$. Moreover, it is enough to set $P_{1e}, P_{2e} \to \infty$ in $\mathcal{R}^{OG}_s$ to derive the perfect colluding rate: $\theta(\frac{h_l^2 P_l}{N_l}) - \theta(P_l(\frac{h^2_{1m}}{N^m_{1e}} + \frac{h^2_{2m}}{N^m_{2e}}))$.

Next, we obtain the secure rate for the general Gaussian WTC-CCE. The proof is similar to Theorem 2.

Theorem 3: $\mathcal{R}^G_s$ in (8), shown at top of the previous page, is an achievable secure rate for Gaussian WTC-CCE (in (2)). For $j \in \{1, 2\}$: $A(j) = \theta(\frac{(h^{je}_l)^2 P_l + (h^{je}_{je})^2 P_{je} + 2h^{je}_l h^{je}_{je}\rho_2\sqrt{P_l P_{je}}}{N_{je}})$.

Remark 5: Channel gains h^l_{1e} and h^l_{2e} make jamming possible for the eavesdroppers. However, they also increase the probability of exposure. In order to compare the two strategies (through numerical examples), we define the non-jamming rate $\mathcal{R}^{NJG}_s$ by setting $h^l_{1e} = h^l_{2e} = 0$ in $\mathcal{R}^G_s$. In addition, by setting $P_{1e}, P_{2e} \to \infty$ in $\mathcal{R}^G_s$, the secure rate is zero, which is less than (or equal to) the perfect colluding rate. This is due to jamming and it is achieved by $\rho_{12} = \rho_1 = \rho_2 = 0$.

Fig. 3 compares the secure rates for the Gaussian WTC-CCE, i.e., $\mathcal{R}^G_s, \mathcal{R}^{OG}_s, \mathcal{R}^{NJG}_s$, to the non-colluding and perfect colluding scenarios in two different collusion channel conditions. It can be seen that the perfect collusion assumption significantly overestimates the eavesdroppers. Recall that the WTC-CCE rates consider the best possible strategy for the eavesdroppers; which may not be achievable for them. In Fig. 3a (a weak collusion channel), using the orthogonal collusion channel for eavesdroppers is worse than using the non-orthogonal one (because $\mathcal{R}^{OG}_s \geq \mathcal{R}^{NJG}_s$). In fact, with weak direct collusion links, eavesdroppers may benefit from the main channel by relaying (transmitting correlated codewords). Hence, the optimal ρ_1, ρ_2 for $\mathcal{R}^{NJG}_s$ are not zero; but they are zero for $\mathcal{R}^{OG}_s$. However, for an improved collusion channel (in Fig. 3b), using an orthogonal collusion channel is better if one cannot use jamming (or does not want to use jamming, to avoid exposure). To evaluate $\mathcal{R}^G_s$, one should note the effect of jamming in addition to collusion, which enables the eavesdroppers (or, now, jammers) to make the secure rate zero.

V. Conclusion

We proposed WTC-CCE, a wiretap-based channel model to capture collusion constraints and derived the achievable secure rates. Our results showed that, indeed, the perfect collusion model overestimates the eavesdroppers if they choose to be unexposed. With no exposure constraint, they can jam to further reduce the secure rate in some cases.

References

[1] A. Wyner, “The wire-tap channel,” *Bell Syst. Tech. J.*, vol. 54, no. 8, Oct. 1975.
[2] A. El Gamal and Y.-H. Kim, *Network information theory*. Cambridge Univ. Press, 2011.
[3] I. Csiszar and J. Korner, “Broadcast channels with confidential messages,” *IEEE Trans. Inf. Theory*, vol. 24, no. 3, May 1978.
[4] R. Liu, I. Maric, P. Spasojevic, and R. D. Yates, “Discrete memoryless interference and broadcast channels with confidential messages: secrecy rate regions,” *IEEE Trans. Inf. Theory*, vol. 54, no. 6, Jun. 2008.
[5] E. Ekrem and S. Ulukus, “Multi-receiver wiretap channel with public and confidential messages,” *IEEE Trans. Inf. Theory*, vol. 59, no. 4, April 2013.
[6] Y. K. Chia and A. El Gamal, “Three-receiver broadcast channels with common and confidential messages,” *IEEE Trans. Inf. Theory*, vol. 58, no. 5, May 2012.
[7] Y. Oohama, “Capacity theorems for relay channels with confidential messages,” *Proc. IEEE ISIT*, Nice, France, June 2007
[8] L. Lai and H. El Gamal, “The relay-eavesdropper channel: cooperation for secrecy,” *IEEE Trans. Inf. Theory*, vol. 54, no. 9, Sep. 2008.
[9] P. C. Pinto, J. Barros, and M. Z. Win, “Wireless physical-layer security: the case of colluding eavesdroppers,” *Proc. IEEE ISIT*, Jun. 2009
[10] O. O. Koyluoglu, C. E. Koksal, and H. A. El Gamal, “On Secrecy Capacity Scaling in Wireless Networks,” *IEEE Trans. Inf. Theory*, vol. 58, no. 5, May 2012.
[11] J. Zhang, L. Pu, X. Wang, “Impact of secrecy on capacity in large-scale wireless networks,” *Proc. IEEE INFOCOM, Mini-Conference*, 2012.
[12] P. C. Pinto, J. Barros, and M. Z. Win, “Secure communication in stochastic wireless networks part II: maximum rate and collusion,” *IEEE Trans. Inf. Forensics and Security*, vol. 7, no. 1, Feb. 2012.
[13] S. Goel and R. Negi, “Secret communication in presence of colluding eavesdroppers,” *Proc. IEEE MILCOM*, Oct. 2005.
[14] J. Wang, P. Huang, and X. Wang, “Cross-layer scheduling in multi-user system with delay and secrecy constraints,” http://arxiv.org/abs/1210.1139v2, Aug. 2013.
[15] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, 2nd ed., Wiley, 2006.