Lecture 6
Properties of laser beams*

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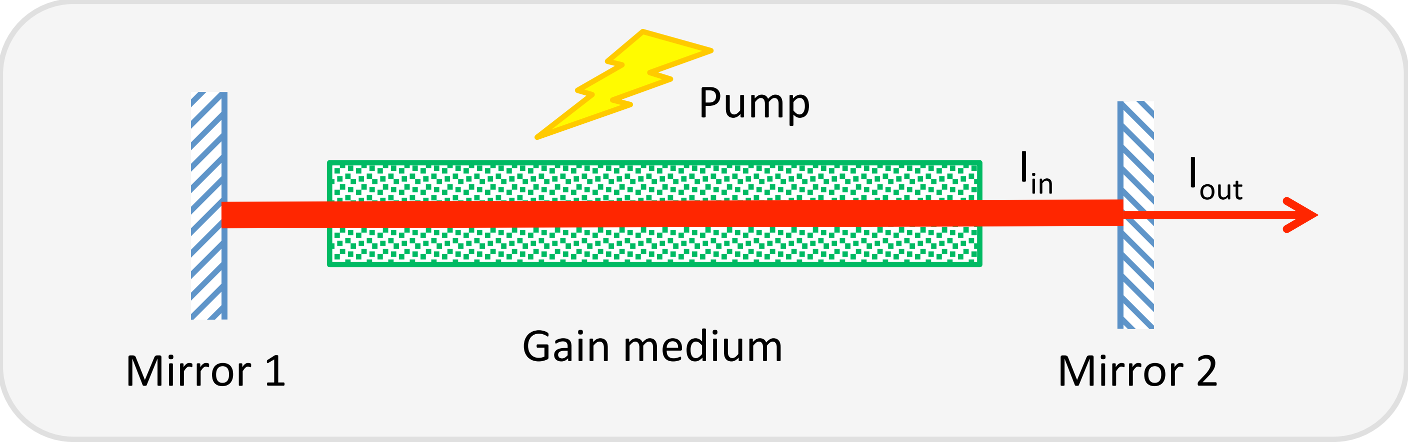
* Some figures and texts belong to: O. Svelto, *Principles of Lasers*, 5th Ed., Springer.

Reading

- *Principles of Lasers* (5th Ed.): Chapter 11.
- Skip: 11.3.4, 11.7.
- Squeeze: 11.3.3, 11.3.5, 11.4.3.

- Web (with video)
  - Mitsubishi CO_2 laser
    http://www.mcmachinery.com/products-and-solutions/ex-plus-series/
  - Mitsubishi fiber laser
    http://www.mcmachinery.com/products-and-solutions/nx-f-series/

Laser



$$E(\mathbf{r},t) = A(\mathbf{r},t)\exp\left[j\left\langle\omega\right\rangle t - \phi(\mathbf{r},t)\right]$$

- Amplitude, frequency, and phase vary w.r.t. time.
- Always more than one frequency exist.

Contents

Content	Time
1. Monochromaticity	10′
2. Coherence - Spatial - Temporal	30′
3. Directionality	20′
4. Brightness	10′
5. Laser speckle	10′
Total:	**80′**

Contents

Monochromaticity

Pure monochromatic light just does not exist.

Causes for frequency fluctuation:
-Amplitude fluctuation (change in pump or cavity loss, <1%)
-Phase fluctuation (zero-point fluctuation limit + vibration and thermal effects)

Active stabilization: 10-50kHz → 0.1Hz

Q-switched or mode-locked laser: $\Delta\nu_L$ can be 100MHz even 50THz

Narrow $\Delta\nu_L$ (10-100kHz): metrology, coherent applications
Broad $\Delta\nu_L$ (1MHz): other common applications (e.g. DWDM)

DWDM: channel spacing 50 GHz (0.4 nm)

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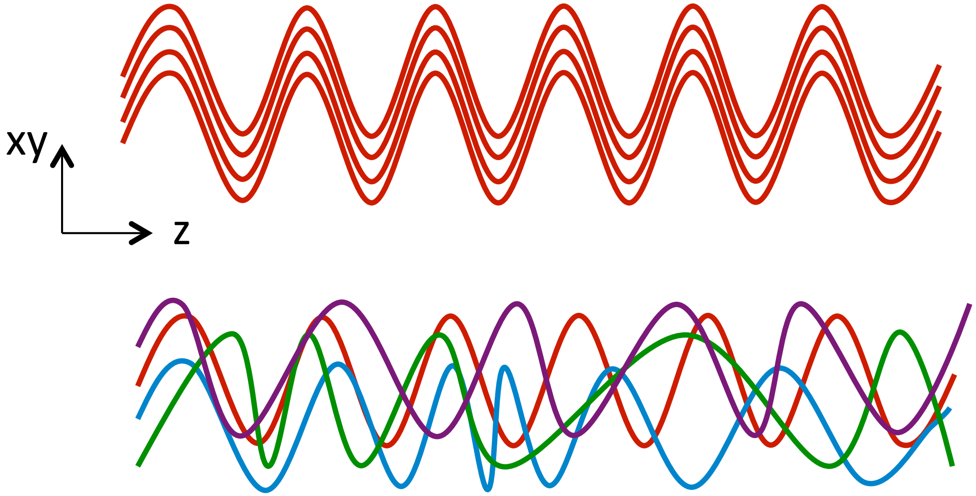
Coherence

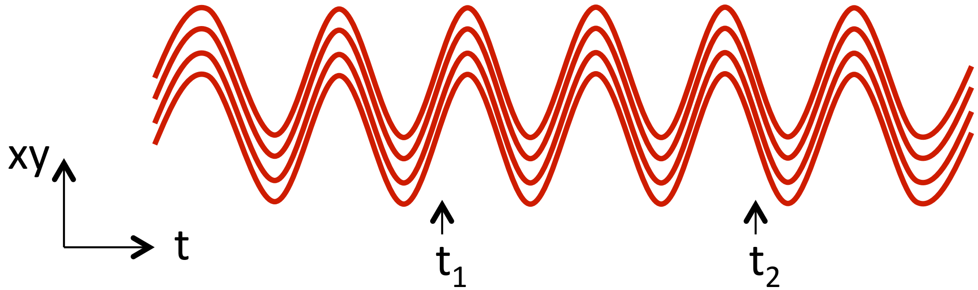
Order, harmony, consistency



Spatial coherence: waves at two lateral points (along x/y direction)
Temporal coherence: waves at two time instances (same position)

Quantification

For stationary beam



$\mathbf{r}=\mathbf{r}_1$

$\tau = t_2 - t_1 < T$

The 1st-order correlation function:

$$\Gamma^{(1)}(\mathbf{r}_1, \mathbf{r}_1, t_1, t_2) = \Gamma^{(1)}(\mathbf{r}_1, \mathbf{r}_1, \tau) = \langle E(\mathbf{r}_1, t+\tau) E^*(\mathbf{r}_1, t)\rangle$$

Complex degree of temporal coherence $\gamma^{(1)}(\mathbf{r}_1,\mathbf{r}_1,\tau)$:

$$\gamma^{(1)} = \frac{\langle E(\mathbf{r}_1,\ t+\tau)\ E^*(\mathbf{r}_1,\ t)\rangle}{\langle E(\mathbf{r}_1,\ t)\ E^*(\mathbf{r}_1,\ t)\rangle^{1/2}\langle E(\mathbf{r}_1,\ t+\tau)\ E^*(\mathbf{r}_1,\ t+\tau)\rangle^{1/2}}$$

Degree of temporal coherence: $|\gamma^{(1)}|$

$\gamma^{(1)}(\mathbf{r}_1,\mathbf{r}_1,\tau)$

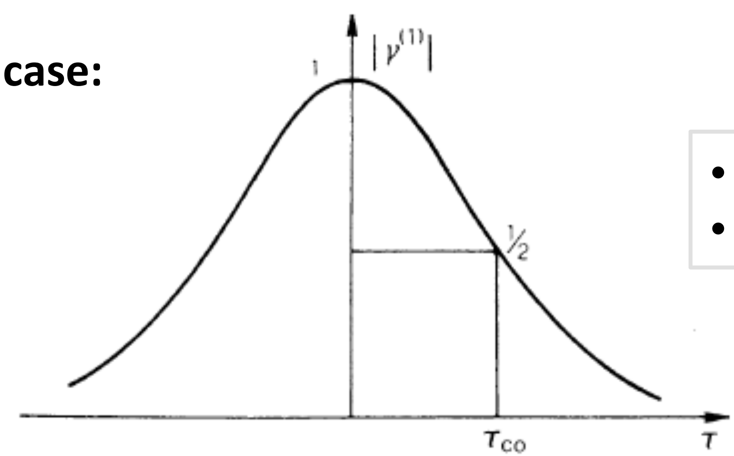
Properties:

1. $\gamma^{(1)}=1$ for $\tau=0$
2. $\gamma^{(1)}(\mathbf{r}_1,\mathbf{r}_1,-\tau) = \gamma^{(1)*}(r_1,r_1,\tau)$
3. $|\gamma^{(1)}(\mathbf{r}_1,\mathbf{r}_1,\tau)|\leq 1$

Two extremities:

- **Perfect temporal coherence**: $|\gamma^{(1)}|=1$ for $\tau \geq 0$
- **Zero temporal coherence**: $|\gamma^{(1)}|=0$ for any $\tau>0$

General case:



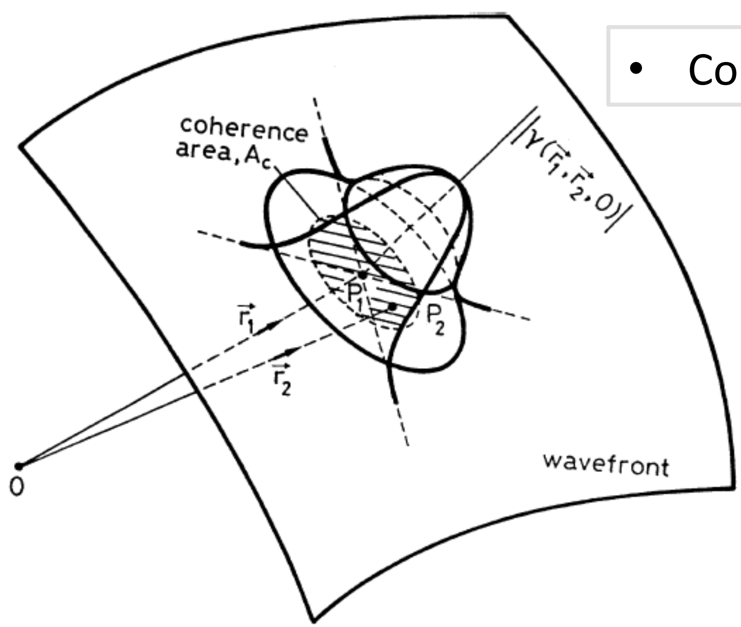
- Coherence time τ_{co}
- Coherence length $L_c=c\tau_{co}$

$\gamma^{(1)}(\mathbf{r}_1,\mathbf{r}_2,0)$

Complex degree of spatial coherence

$$\gamma^{(1)} = \frac{\langle E(\mathbf{r}_1,\ t)\ E^*(\mathbf{r}_2,\ t)\rangle}{\langle E(\mathbf{r}_1,\ t)\ E^*(\mathbf{r}_1,\ t)\rangle^{1/2}\ \langle E(\mathbf{r}_2,\ t)\ E^*(\mathbf{r}_2,\ t)\rangle^{1/2}}$$

Again two extremities exist, and in general: $|\gamma^{(1)}(\mathbf{r}_1,\mathbf{r}_2,0)| \leq 1$ for $|\mathbf{r}_1-\mathbf{r}_2|>0$



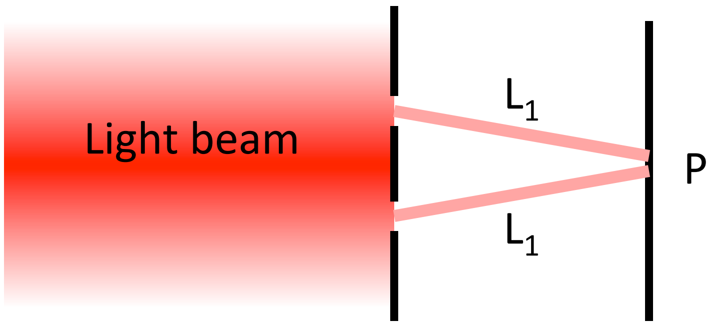
- Coherence area

Complex degree of coherence:
[Temporal + spatial]

$$\gamma^{(1)}(\mathbf{r}_1,\mathbf{r}_2,\ \tau) = \frac{\langle E(\mathbf{r}_1,\ t+\tau)\ E^*(\mathbf{r}_2,\ t)\rangle}{\langle E(\mathbf{r}_1,\ t)\ E^*(\mathbf{r}_1,\ t)\rangle^{1/2}\ \langle E(\mathbf{r}_2,\ t)\ E^*(\mathbf{r}_2,\ t)\rangle^{1/2}}$$

Measurem't: Spatial coherence

Method: Young's interferometer



Fringe visibility at P:

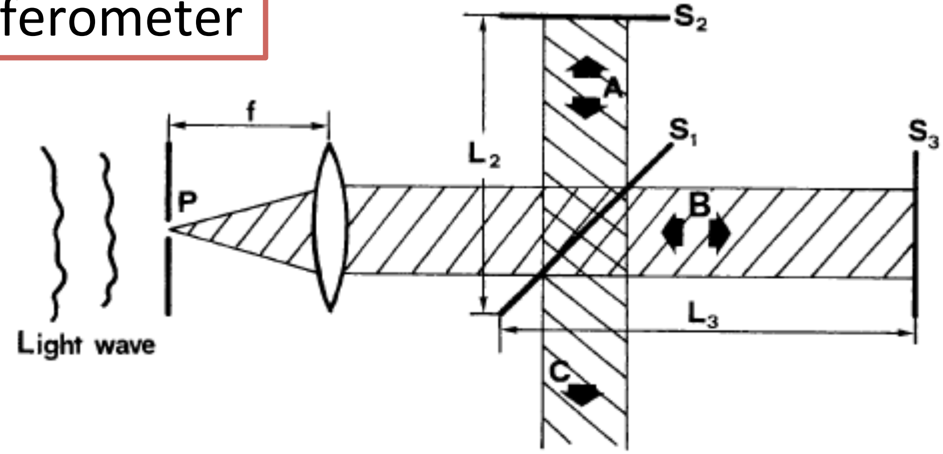
$$V_P = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

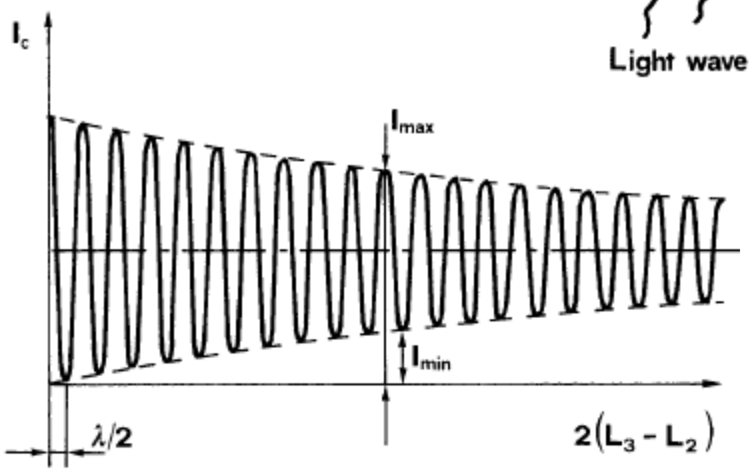
It can be shown that $$V_p = \left|\gamma^{(1)}(\mathbf{r}_1, \mathbf{r}_2, \tau)\right|$$ where $$\tau = \frac{L_2 - L_1}{c}$$

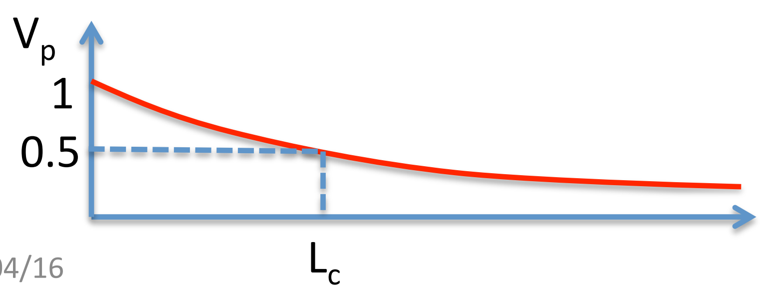
If $L_1 = L_2$ $$V_p = \left|\gamma^{(1)}(\mathbf{r}_1, \mathbf{r}_2, 0)\right|$$

Measurem’t: Temporal coherence

Method: Michelson interferometer







$$V_p(\tau) = \left|\gamma^{(1)}(\mathbf{r}, \mathbf{r}, \tau)\right|$$

where $\tau = \frac{2(L_3 - L_2)}{c}$

Hence $\tau_{co} = \frac{L_c}{c}$

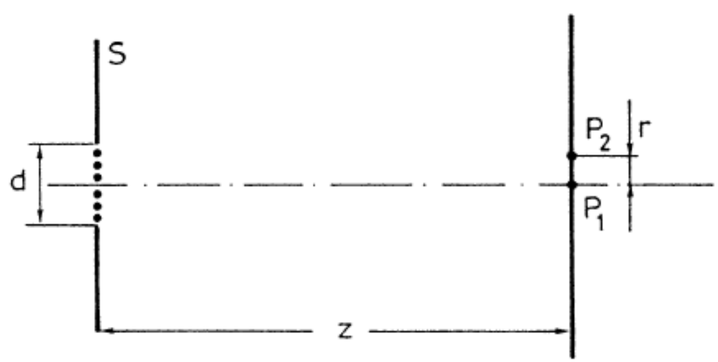
More remarks on coherence

- Temporal coherence and monochromaticity $\tau_{co}\Delta\nu_L \geq \frac{1}{4\pi}$

$\tau_{co}\Delta\nu_L = 1$

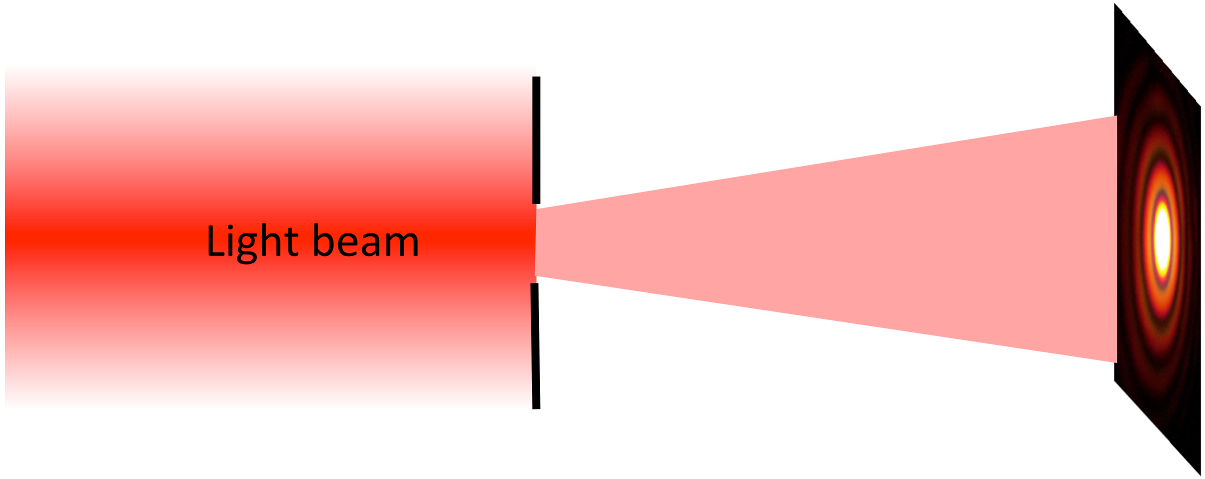
Example:
If $\Delta\upsilon_L$=20kHz,
τ_{co}=? L_c=?

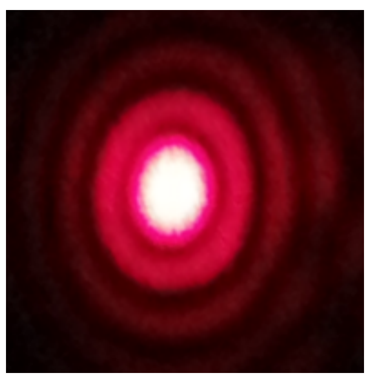
- **Single-mode laser**: spatially coherent, τ_{co} limited by $\Delta\upsilon_L$
- **Single-transverse-multi-longitudinal-mode laser**:
  - Spatially coherent, τ_{co} limited by $\Delta\upsilon_0$
  - (if mode-locked: spatially coherent, τ_{co} limited by $\Delta\upsilon_L$)
- **Multi-transverse-multi-longitudinal mode laser**:
  - partial spatial coherence, τ_{co} limited by $\Delta\upsilon_0$
- **Thermal light**: τ_{co}<1ps, spatial correlation ↑ as distance ↑



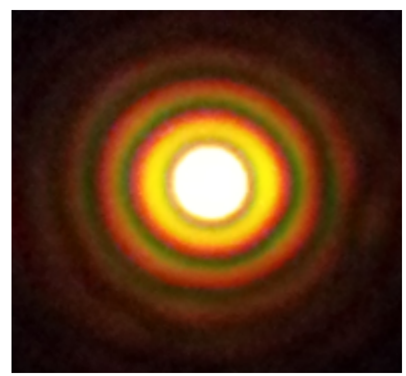
$$|\gamma^{(1)}| = 0.88 \text{ if } r \approx 0.16\frac{\lambda z}{d}$$

Coherence length: single-pinhole test





Laser pointer



Supercontinuum source

Pinhole diameter: 600µm

Coherence length of Sun

Photo courtesy of Gunnar Björk

Photo taken with a Thorlabs 50μm-diameter pinhole mounted about 50 mm from the CMOS-detector of a Sony α300 digital camera mounted on a tripod. Exposure time ¼ s at ISO 200.

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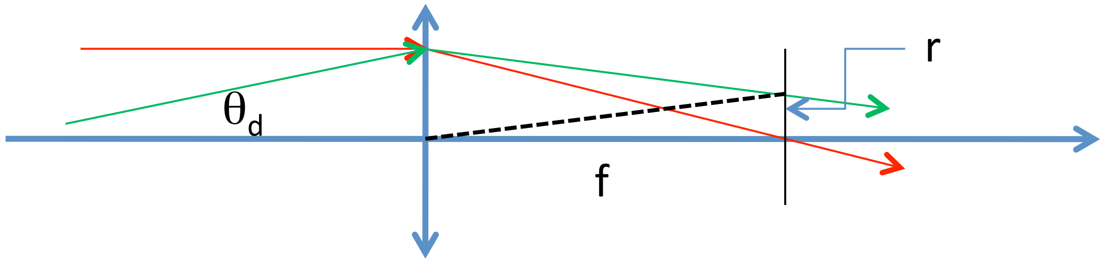
Directionality (divergence)

$\propto$ spatial coherence

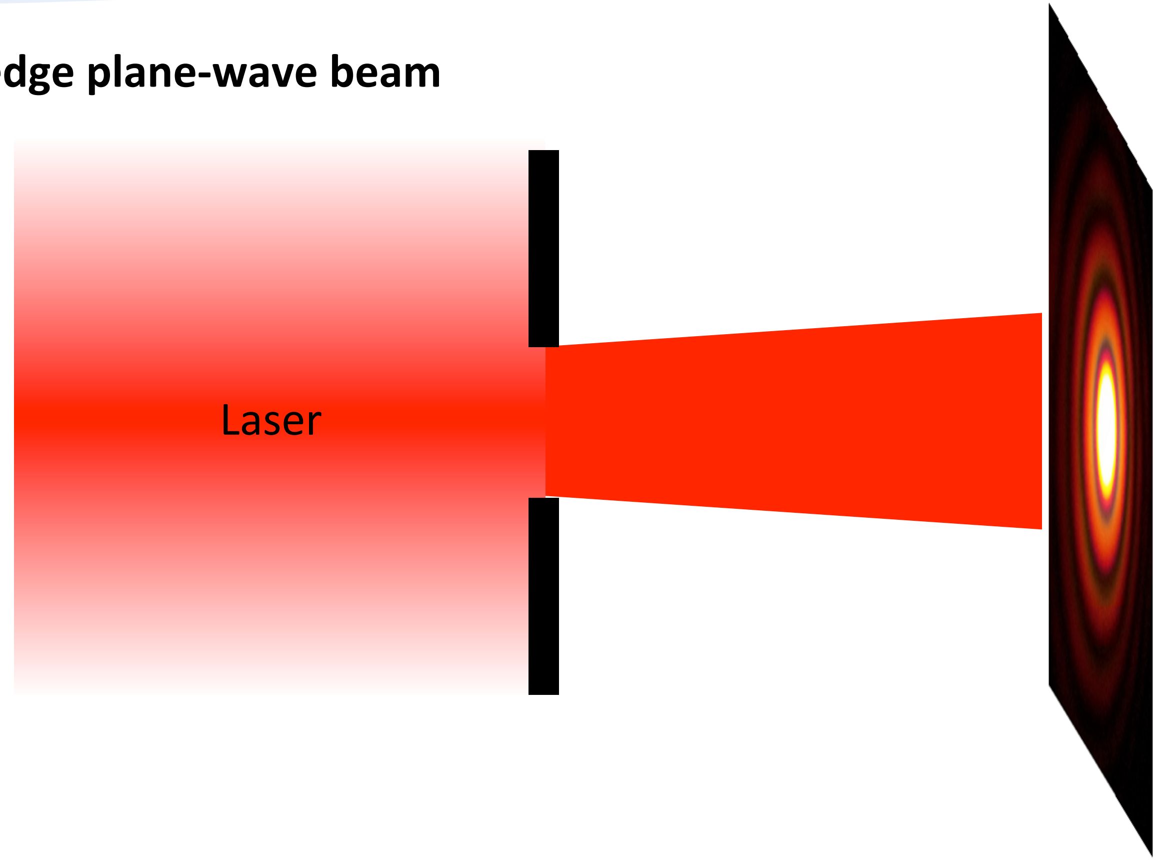
Measurement methods:

- Measure spot size at a very large distance: $\theta_d = W/z$
- Measure beam spot at a lens' focus: $\theta_d = r/f$



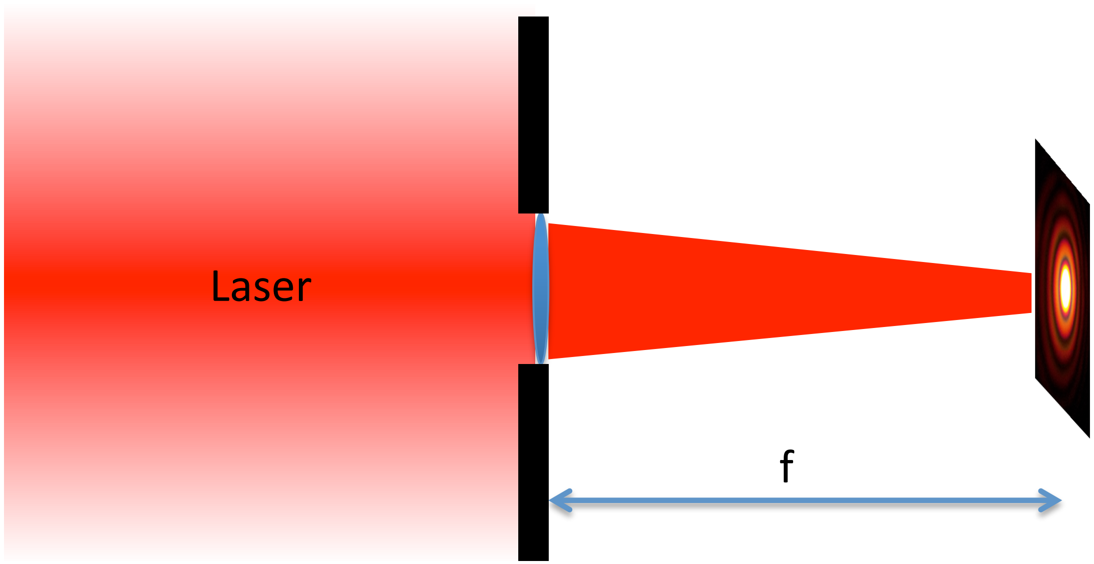
Perfect spatial coherence

- **Hard-edge plane-wave beam**



Perfect spatial coherence

- **Hard-edge plane-wave beam**



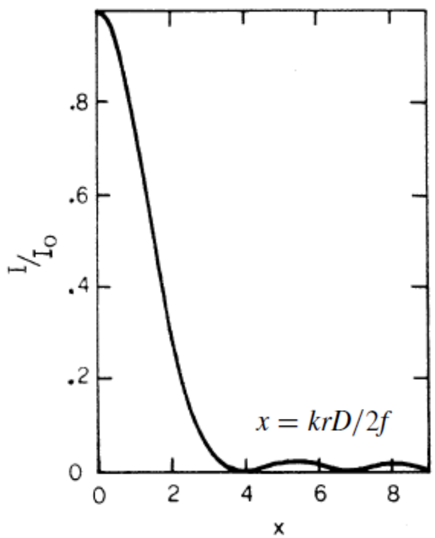
Perfect spatial coherence

Diffraction limited beam: $\theta_d = \beta \dfrac{\lambda}{D}$

$\beta \approx 1$

- **Hard-edge plane-wave beam**

At lens focus



Airy beam: $I = \left[\dfrac{2 J_0\left(\frac{krD}{2f}\right)}{\frac{krD}{2f}} \right]^2 I_0$

$$\theta_d = 1.22 \frac{\lambda}{D}$$

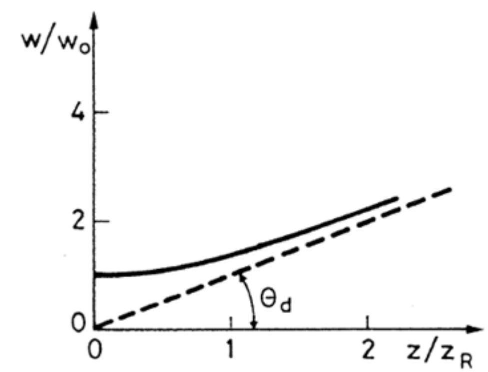
D: aperture (lens) diameter

- **Gaussian beam**

$$w^2(z) = w_0^2 \left[1 + (z/z_R)^2\right]$$
$$R(z) = z \left[1 + (z_R/z)^2\right]$$
$$\phi(z) = \tan^{-1}(z/z_R)$$

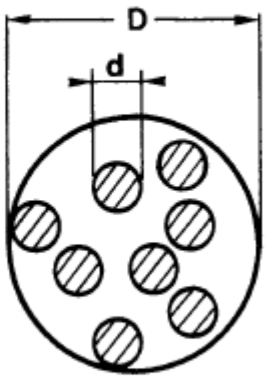
with $z_R = \pi\, w_0^2 / \lambda$

$$\theta_d = \frac{w}{z} = \frac{\lambda}{\pi w_0}$$



50% of hard-edge case

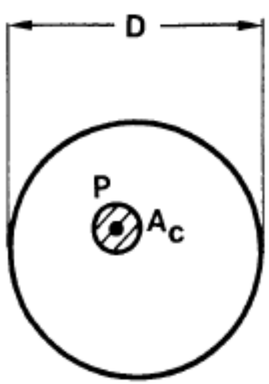
Partial spatial coherence



Un-correlated small beams: $\theta_d=\beta\lambda/d$

Correlated small beams: $\theta_d=\beta\lambda/D$

General case:



Ac: coherence area

$\theta_d=\beta\lambda/D_c$

M^2 factor

Eliminates ambiguity of beam-diameter definition

Simplified definition: Quality of a general beam compared to a Gaussian beam

$M^2 \geq 1$, being 1 for TEM_{00} Gaussian mode.

A **multimode laser** beam propagating along z axis, its beam waist (across x)

$$W^2(z) = W_0^2 + M^4 \frac{\lambda^2}{\pi^2 W_0^2}(z - z_0)^2$$

As $z \to \infty$

$$W(z) = M^2 \frac{\lambda}{\pi W_0}(z - z_0)$$

Hence

$$\theta_d = \frac{W(z)}{z - z_0} = M^2 \frac{\lambda}{\pi W_0}$$

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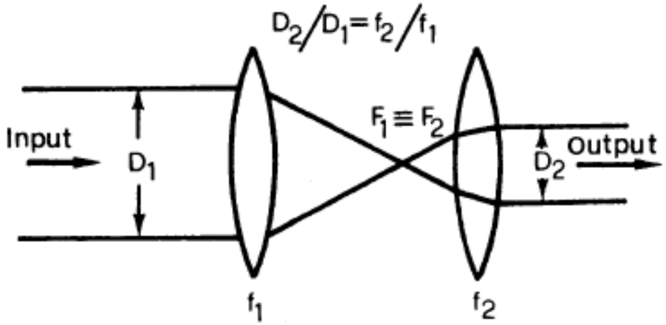
Brightness

- Brightness (power per unit area per solid angle) ≠ Irradiance (power per unit area)
- Proportional to the max peak intensity achievable by focusing a beam

$$B = \frac{P}{A\Omega}$$

P: power
A: area
Ω: Solid angle



- **Cause:** Coherence → directionality → brightness

Example:
- Ar laser (TEM_{00}, 1W, λ=514nm): $B = 4P/\lambda^2$ = **1.6×10^9**W/cm^2·sr
- Lamp (10W output, examined at λ=546nm): B = **95**W/cm^2·sr

- **Use:** Material processing

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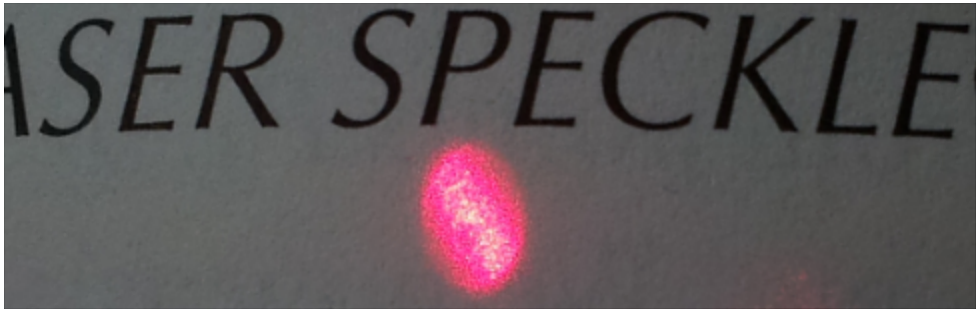
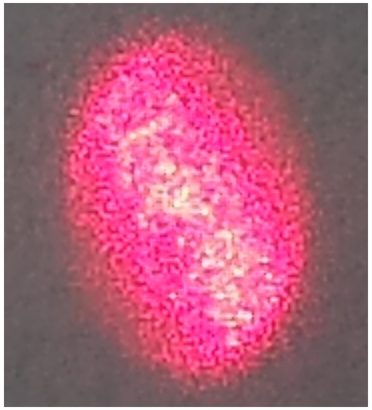
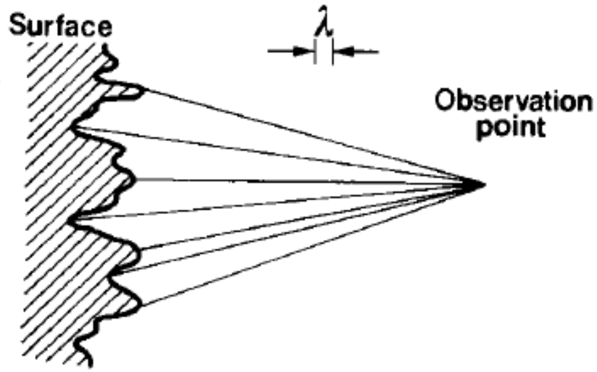
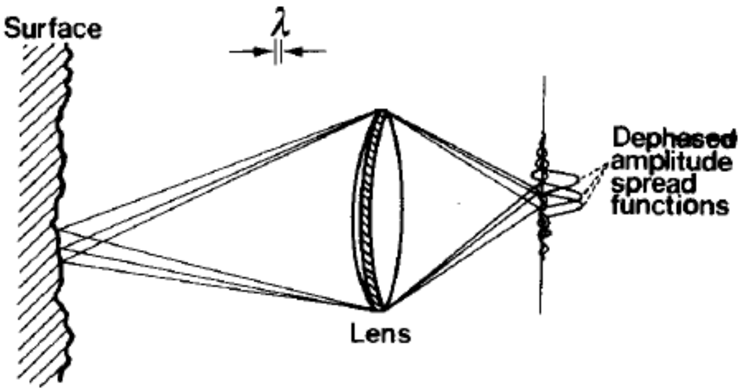
Laser speckles





Cause: high degree of laser light coherence

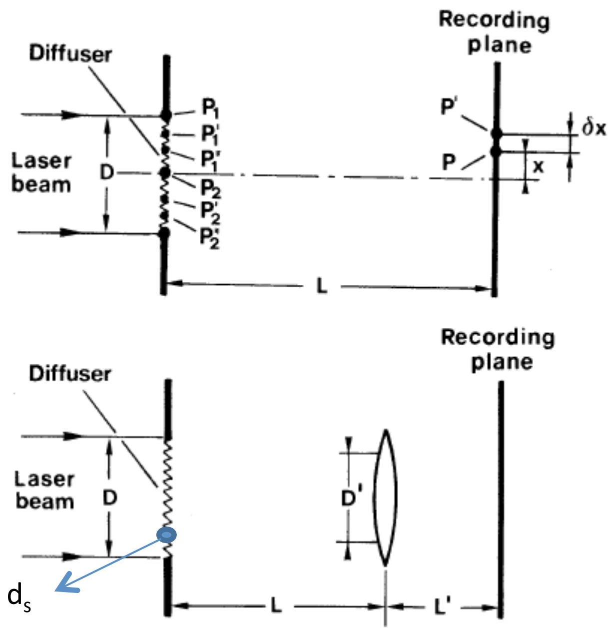




Calculation and impact

- **Grain size d_g**

D<<L



Scatterer size d_s

$d_g = 2\lambda L/D$

if $\frac{2\lambda L}{d_s} \geq D'$

$d_g = 2\lambda L'/D'$

- **Impact:** Limits the image resolution of an object illuminated with laser (speckle noise rather than diffraction limit)

- **Use:** Sensors (stress, vibrations)

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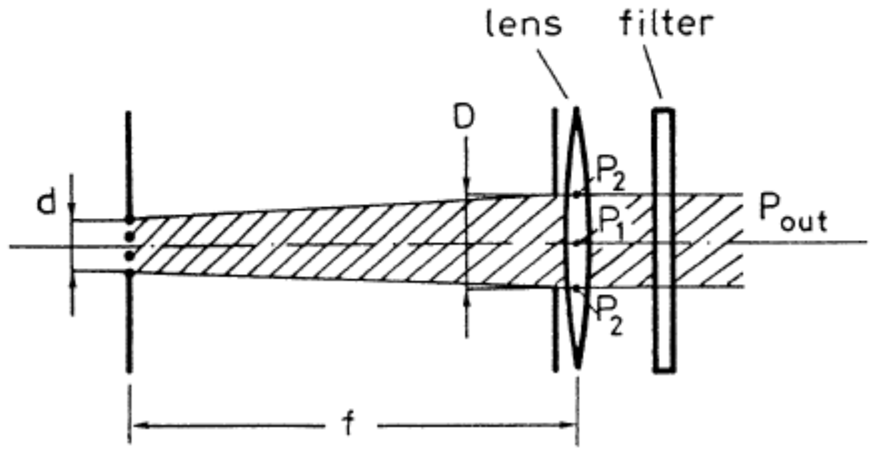
One more slide

- **Laser v.s. Thermal light**



10W → 10^{-18}W