Chaotic Dynamical Systems
5B1490, Fall 2004
Department of Mathematics, KTH
Michael Benedicks

Course overview and reading instructions

The suggested reading material was.

R. Clark Robinsson, *Dynamical Systems: Stability, Symbolic Dynamics and Chaos*, II:nd edition.
D. Robert Devaney, *Chaotic Dynamical Systems*, II:nd edition.

H-S. Hirsh-Smale, *Differential Equations, Dynamical Systems, and Linear Algebra.* Academic Press, New York, 1974

N. Newhouse, Sheldon, webpage http://www.mth.msu.edu/~sen/Math_848-ps/.

I. One dimensional maps

The following material was covered in class following Robinsson's book

R2.2 Periodic points
- R2.2.1 Fixed points for the quadratic family

R2.3 Limit sets and recurrence for maps
R2.4 Invariant Cantor Sets for the quadratic family
- R2.4.1 Middle Cantor Sets
- R2.4.2 Construction of the invariant Cantor Set
- R2.4.3 The invariant Cantor set for $\mu>4$

R2.5 Symbolic dynamics for the Quadratic Map
R2.6 Conjugacy and structural stability
R2.7 Conjugacy and structural stability for the Quadratic Map
R3.1 Sharkovskii's theorem (proof only for period 3)
R3.2 Subshifts of finite type
R3.4 Chaos
R3.6 Lyapunov exponents

This material is also treated in Devaney's book in Chapter 1, sections 1.1-1.19. We did not treat cirle maps (1.14) andd Morse-Smale diffeomorphisms (1.15) and did not treat much of kneading theory (1.18-1.19).

II. Higher dimensional systems

II.1 Linear systems
I expect that you are more or less familiar with the material in Robinsson, Chapter IV and it will not be emphasized in the oral exam. The material in Chapter 4.1 on the Real Jordan Canonical Form is important and perhaps new to some of you. The corresponding material in Devaneys book is in sections 2.1-2.2.

II.2 Analysis near fixed points and periodic points
R5.1-R5.6 is review material. The emphasis is on the Hartman-Grobman theorem treated in R5.6-5.7. 5.7.2 and 5.8 were not treated in class. The Hartman-Grobman theorem is also treated in Devaney, section 1.9. (R5.9, i.e the Poincaré-Bendixon theorem was treated according to Hirsch-Smale (1974) later in the course.)

II.2.1 Stable manifold theorem for a fixed point of a map

For the proof (in the two-dimensional case) we followed Devaney, section 2.6. The presentation in Robinsson is more advanced and treats general dimensions (R5.10).

An important related result is the inclination lemma (R.5.11). This was not covered in class but appeared in an homework assignment.

II.3 Examples of hyperbolic sets and attractors

II.3.1 Horseshoes and shift spaces

R8.1-R8.2 is review material but important. In R8.3 two-sided shift spaces are treated and in R8.4 Smale's horseshoe is treated, in particular how the dynamics on the invariant set is topologically conjugate to a twosided shift. The corresponding material in Devaney is in Section D2.3. I did give an introduction to the material in R8.4.2 and R8.4.3 ``Horseshoe from a Homoclinic Point´´ resp. ``Nontravese Homoclinic Point´´.

II.3.2 Hyperbolic toral automorphisms
This material is in R8.5 and D2.4. A central part here is the construction of Markov Partitions. The material in R8.5.2 and R8.5.3 was not covered.

II.3.3 Attractors
The Solenoid, in Robinsson R8.7, and in Devaney D2.5 were covered --- in particular conjugacy of the Solenoid to an inverse limit space. Review material on Hénon attractors was presented (see R8.10).

III Flows.
In this part of the course we followed Newhouse lecture notes **N**. In particular the Poincaré-Bendixon theorem was proved using the proof in H-S. The index of a vector field was defined and the Umlaufsatz was proved. The relation between index and limit cycles were discussed.

IV Bifurcations
A review of the Saddle Node Bifurcation, the Period Doubling Bifurcation and the Hopf Bifurcation was given. This material is in R7.1-B7.5 and D1.12 and D2.8. You should know about the results but detailed proofs are not required.