Energy Compaction on Graphs for Motion-Adaptive Transforms

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Given a graph, various signal processing techniques can be engaged based on the eigenbasis of the Laplacian matrix or the adjacency matrix, e.g., [1]. As our interest is in energy compaction of motion-compensated image sequences that employ underlying graphs to represent the motion, we focus on covariance matrices based on graphs. In our previous work [2], instead of directly using the eigenbasis of the Laplacian, we used the eigenbasis of a vertex-weighted Laplacian and showed good energy compaction for temporal decompositions of videos. The limitations of [2] encourage us to analyze the relation between a graph-based covariance matrix and the Laplacian of the given graph.

In this work, we consider a covariance model based on graphs. The covariance matrix based on the classic AR(1) model is given by the relative distance of two variables representing pixel values and a correlation factor ρ. For two variables at time i and j, the covariance is $\mathrm{cov}(i,j) = \sigma^2\rho^{|i-j|}$, where σ^2 is the variance of the signal. Using a similar concept, we consider a covariance matrix M based on a graph as $M(i,j) = \sigma^2\rho^{D(i,j)}$, where D is the distance matrix of the graph. Since our graph is inherited from motion information, the graphs are trees. It can be shown that such covariance matrices of trees are positive definite.

The relation between this covariance model and the Laplacian is studied. For tree graphs, we obtain an explicit expression between the inverse covariance matrix and the Laplacian as $M^{-1} = \frac{1}{1-\rho^2}\left[\rho L - (\rho-\rho^2)\mathrm{diag}(\mathbf{d}) + (1-\rho^2)I\right]$, where L is the unweighted Laplacian matrix, $\mathrm{diag}(\mathbf{d})$ the degree matrix, and I the identity matrix. For simplicity, σ^2 is set to 1. Based on this relation, the diagonalizing property of the Laplacian eigenbasis on M can be analyzed.

To conclude, the proposed graph-based covariance is a good model for motion-compensated image sequences where the motion is represented by graphs. Our work allows us to study the relation between our covariance model and the classical Laplacian of a given graph. In terms of energy compaction, our graph-based covariance model has the potential to outperform the classical Laplacian-based signal analysis.

[1] D. I. Shuman, S. K. Narang, P. Frossard, A. Ortega, and P. Vandergheynst, "The emerging field of signal processing on graphs," *IEEE Signal Processing Magazine*, vol. 30, no. 3, pp. 83–98, 2013.

[2] D. Liu and M. Flierl, "Motion-adaptive transforms based on the Laplacian of vertex-weighted graphs," in *Proc. IEEE Data Compression Conference*, 2014.

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