Data Compression Conference 2007

Half-Pel Accurate Motion-Compensated Orthogonal Video Transforms

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Motivation

- Motion-compensated lifted Haar wavelet deviates sub-stantially from orthonormality due to motion compensation
- Why orthogonal transforms?
  - Optimal for certain transform coding schemes at high rates
  - Provide highly robust video representations
- Motion-adaptive transform that strictly maintains orthonormality while permitting flexible
  - Integer-pel accurate motion compensation **and**
  - Sub-pel accurate motion compensation

Outline

- Motion-Compensated Orthogonal Transform (MCOT)
- Single MC incremental transform
  - Energy concentration constraint
  - Example for a dyadic decomposition of a group of pictures
- Double MC incremental transform
  - Euler rotations
  - Energy concentration constraint
- P-hypothesis MC incremental transform
- Example: Half-pel MC with averaging filter
- Experimental results

Orthogonal Video Transform

- Orthogonal transform for **pairs of input images**:

$$\begin{array}{r} \text{low band image} \longrightarrow \\ \text{high band image} \longrightarrow \end{array} \begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{pmatrix} = T \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix}$$

- Factor *T* into a sequence of *k* **incremental transforms**:

$$T = T_k T_{k-1} \cdots T_\kappa \cdots T_2 T_1$$

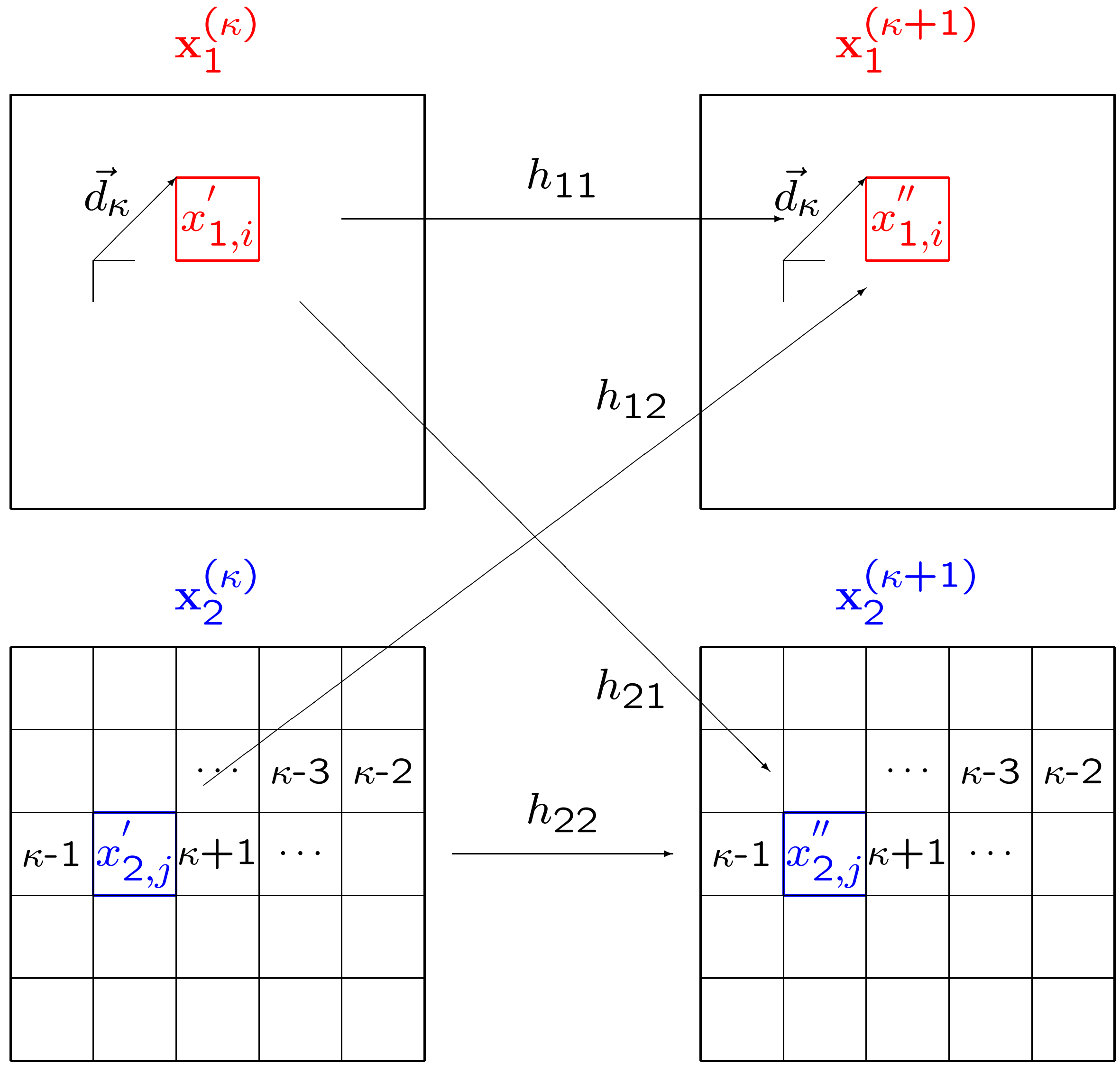
- Each incremental transform is orthogonal: $T_\kappa T_\kappa^T = I$

- Incremental transforms generate a sequence of transformed image pairs:

$$\begin{pmatrix} \mathbf{x}_1^{(\kappa+1)} \\ \mathbf{x}_2^{(\kappa+1)} \end{pmatrix} = T_\kappa \begin{pmatrix} \mathbf{x}_1^{(\kappa)} \\ \mathbf{x}_2^{(\kappa)} \end{pmatrix}$$

Single MC Incremental Transform

low band
image to-be



high band
image to-be

Single MC Incremental Transform

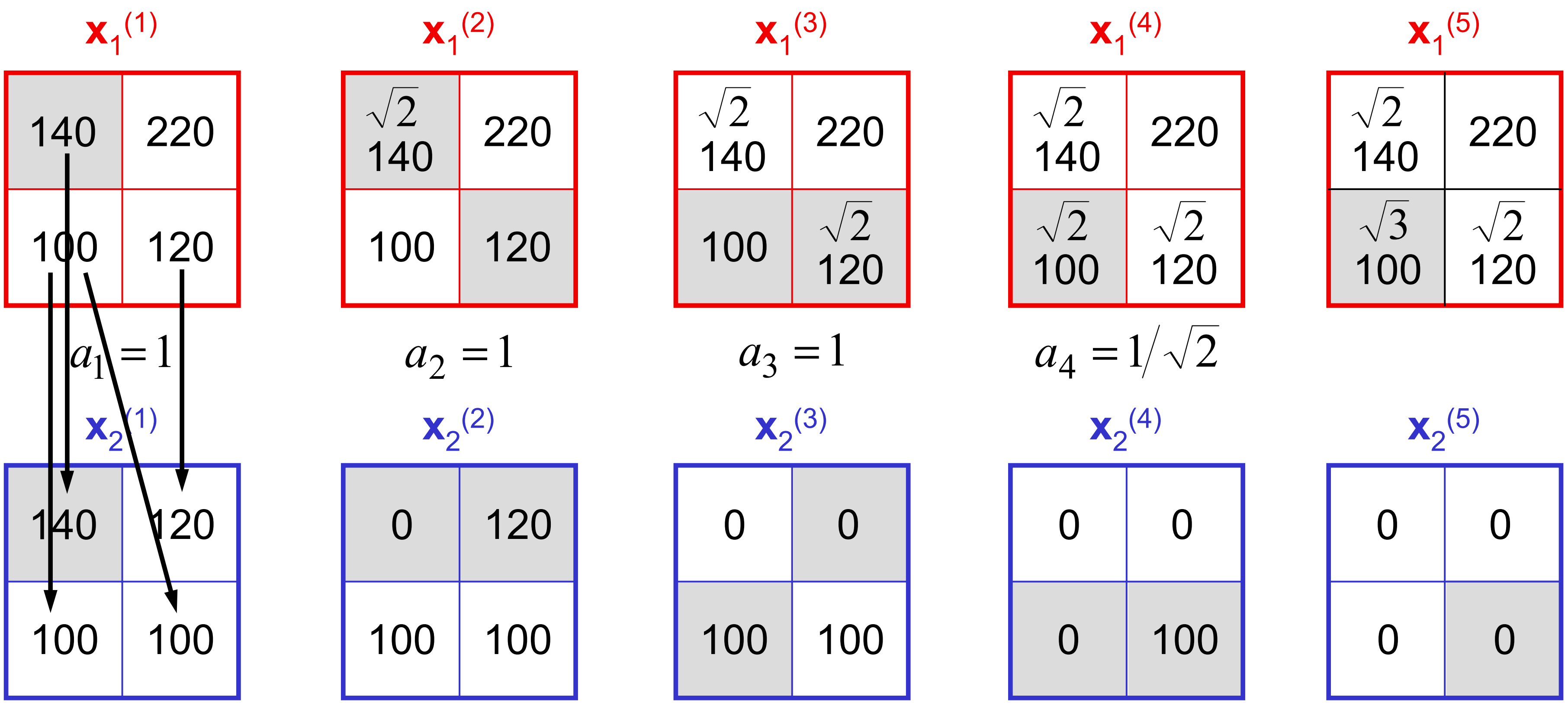
$$
T_\kappa = \begin{pmatrix}
\ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\cdots & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots \\
\cdots & 0 & {\color{red}h_{11}} & 0 & \cdots & 0 & h_{12} & 0 & \cdots \\
\cdots & 0 & 0 & 1 & \cdots & 0 & 0 & 0 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
\cdots & 0 & 0 & 0 & \cdots & 1 & 0 & 0 & \cdots \\
\cdots & 0 & h_{21} & 0 & \cdots & 0 & {\color{blue}h_{22}} & 0 & \cdots \\
\cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 1 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\begin{array}{l}
\\ \\ \text{i-th pixel in } \mathbf{x_1} \\ \\ \\ \\ \text{j-th pixel in } \mathbf{x_2} \\ \\ \\
\end{array}
$$

$$
H = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} = \frac{1}{\sqrt{1+a^2}} \begin{pmatrix} 1 & a \\ -a & 1 \end{pmatrix}
$$

$$
HH^T = I
$$

decorrelation factor

Example: Single MC Orthogonal Transform



$$\begin{bmatrix} x''_{1,i} \\ x''_{2,j} \end{bmatrix} = \frac{1}{\sqrt{1+a_\kappa^2}} \begin{bmatrix} 1 & a_\kappa \\ -a_\kappa & 1 \end{bmatrix} \begin{bmatrix} x'_{1,i} \\ x'_{2,j} \end{bmatrix}$$

SMCOT: Energy Concentration Constraint

- Choose decorrelation factor for each incremental transform such that the energy in the high band to-be is removed
- Assume that pixel $x_{2,j}$ is connected to pixel $x_{1,i}$, i.e., $x_{2,j} = x_{1,i}$
- Note that pixel $x_{1,i}$ may have been processed previously!
- Therefore, let v_1 be the scale factor for pixel $x_{1,i}$
- After processing, let u_1 be the scale factor for pixel $x_{1,i}$
- For higher levels of temporal decomposition, $x_{2,j}$ is a low band coefficient that carries a scale factor
- Therefore, let v_2 be the scale factor for pixel $x_{2,j}$
- Now, resulting high band pixel to-be $x''_{2,j}$ shall be zero:

$$\begin{pmatrix} u_1 x_{1,i} \\ 0 \end{pmatrix} = H \begin{pmatrix} v_1 x_{1,i} \\ v_2 x_{1,i} \end{pmatrix}$$

Definition of Scale Counters

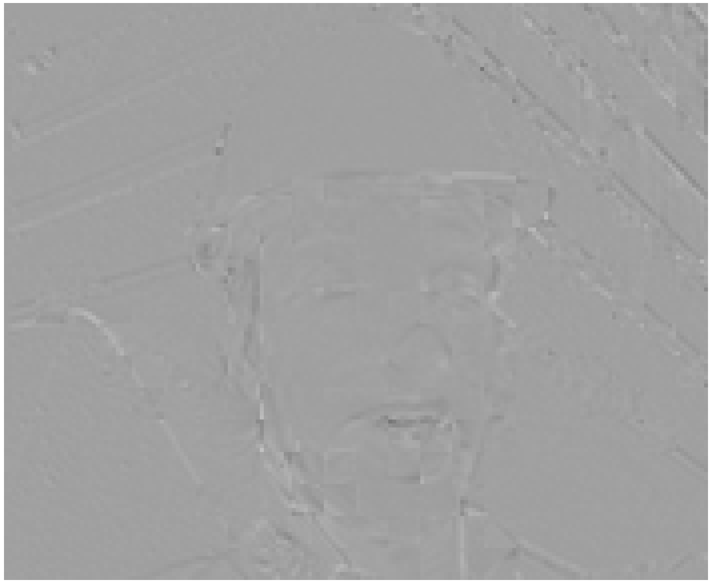
- Let n_1, n_2 be the scale counters for pixel $x_{1,i}$, $x_{2,j}$
- n_1, n_2 simply count how often the pixel $x_{1,i}$, $x_{2,j}$ are used as reference for motion compensation
- In the beginning, the scale counter is $n = 0$ and the scale factor is $v = 1$
- Let m_1 be the scale counters for pixel $x_{1,i}$ after being processed by the incremental transform
- For arbitrary scale counter m and n, the scale factors are

$$u = \sqrt{m+1} \quad \text{and} \quad v = \sqrt{n+1}$$

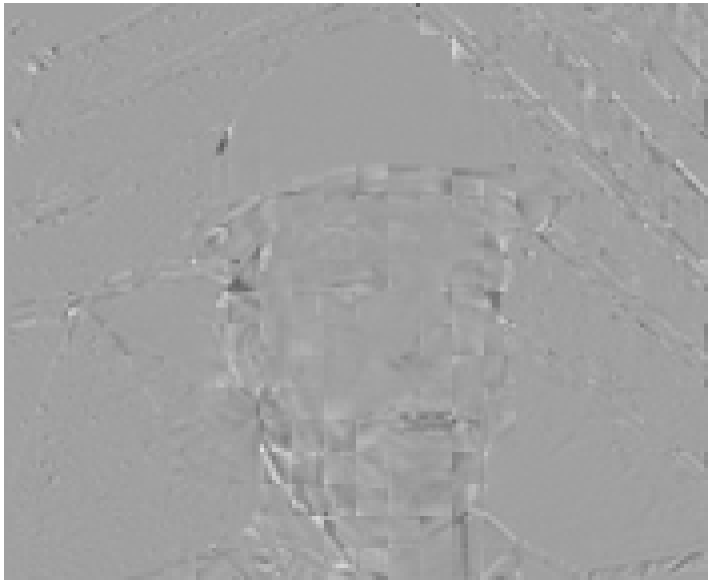
- Example: Scale counter update rule for SMCOT:

$$m_1 = n_1 + n_2 + 1$$

IP-MCOT Experimental Results



temporal high band
first decomposition level



temporal high band
second decomposition level

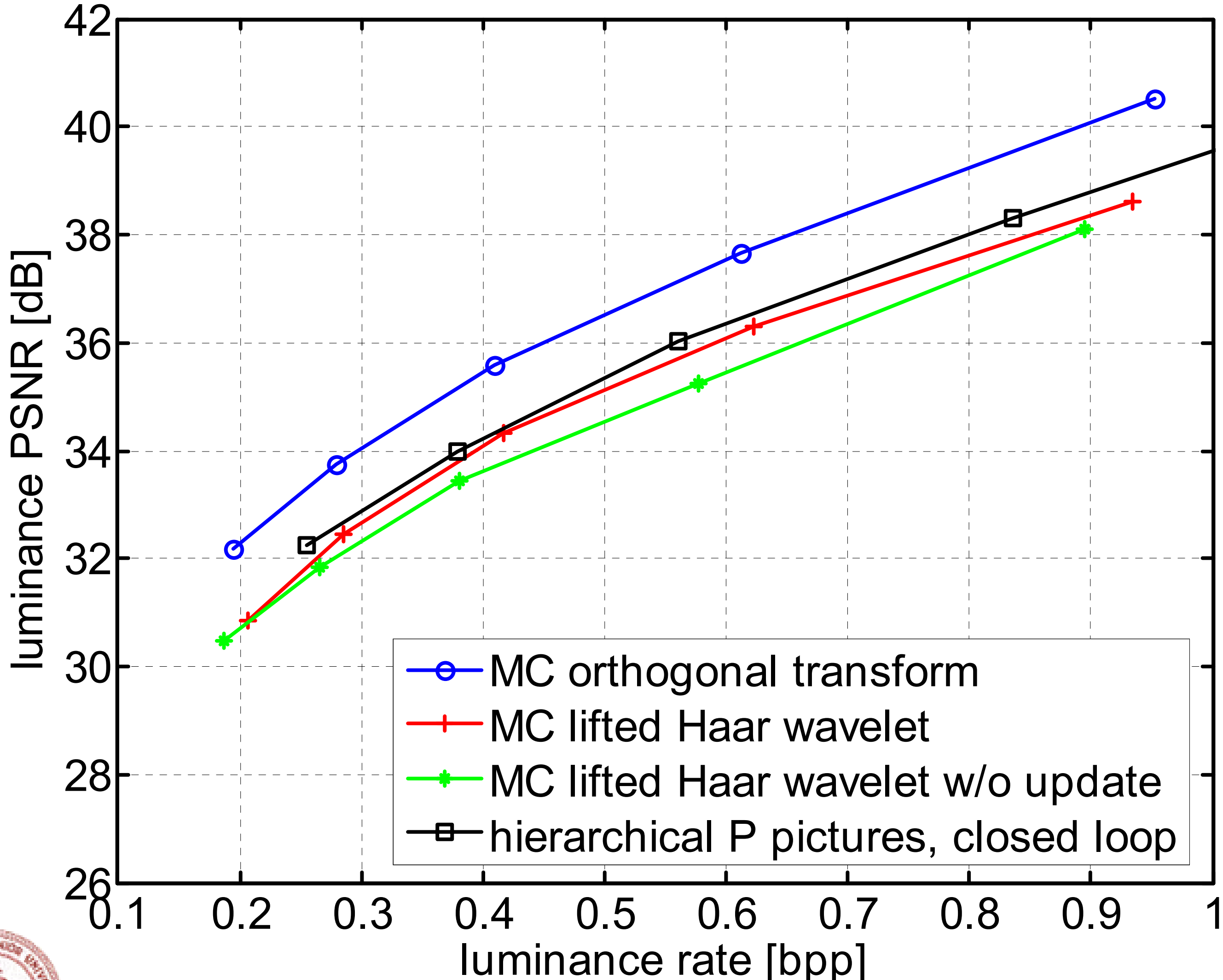
IP-MCOT Experimental Results

temporal low band
second decomposition level

rescaled temporal low band
second decomposition level

$$v = \sqrt{n+1}$$

IP-MCOT Experimental Results



Foreman

QCIF

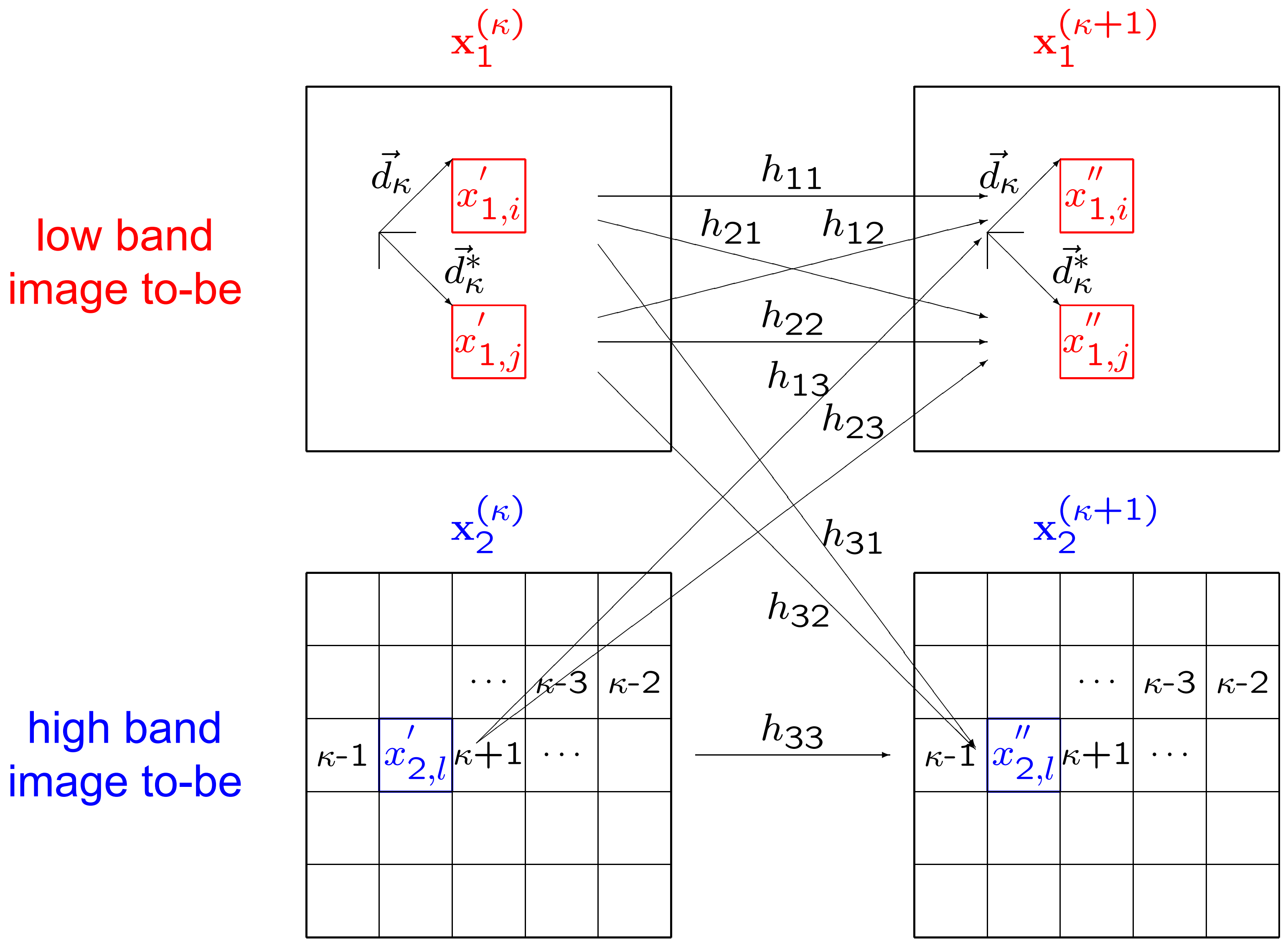
30 fps

288 frames

GOP size K=16

8x8 block motion

Double MC Incremental Transform



Double MC Incremental Transform

$$T_\kappa = \begin{pmatrix}
\ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\cdots & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots \\
\cdots & 0 & \color{red}{h_{11}} & 0 & \cdots & 0 & h_{12} & 0 & \cdots & 0 & h_{13} & 0 & \cdots \\
\cdots & 0 & 0 & 1 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\cdots & 0 & 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots \\
\cdots & 0 & h_{21} & 0 & \cdots & 0 & \color{red}{h_{22}} & 0 & \cdots & 0 & h_{23} & 0 & \cdots \\
\cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 1 & \cdots & 0 & 0 & 0 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
\cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 1 & 0 & 0 & \cdots \\
\cdots & 0 & h_{31} & 0 & \cdots & 0 & h_{32} & 0 & \cdots & 0 & \color{blue}{h_{33}} & 0 & \cdots \\
\cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 1 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}$$

i-th pixel in $\mathbf{x}_1$

j-th pixel in $\mathbf{x}_1$

l-th pixel in $\mathbf{x}_2$

$$H = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \quad \text{with} \quad HH^T = I$$

Euler's Rotation Theorem

- Any rotation in 3D can be given as a composition of rotations about three axes, i.e., $H = H_3 H_2 H_1$
- We choose the following composition:

$$H = \begin{pmatrix} \cos(\psi) & 0 & \sin(\psi) \\ 0 & 1 & 0 \\ -\sin(\psi) & 0 & \cos(\psi) \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{pmatrix} \cdot \begin{pmatrix} \cos(\phi) & 0 & \sin(\phi) \\ 0 & 1 & 0 \\ -\sin(\phi) & 0 & \cos(\phi) \end{pmatrix}$$

- Euler angles ψ, θ, ϕ are determined by the energy concentration constraint

DMCOT: Energy Concentration Constraint

- Choose 3 Euler angles for each incremental transform
- Assume that pixel $x_{2,l}$ is connected to pixel $x_{1,i}$, i.e., $x_{2,l} = x_{1,i}$
- Assume that pixel $x_{2,l}$ is connected to pixel $x_{1,j}$, i.e., $x_{2,l} = x_{1,j}$
- State zero-energy constraint for the high band pixel

$$\begin{pmatrix} u_1 x_{1,i} \\ u_2 x_{1,i} \\ 0 \end{pmatrix} = H_3 H_2 H_1 \begin{pmatrix} v_1 x_{1,i} \\ v_2 x_{1,i} \\ v_3 x_{1,i} \end{pmatrix}$$

- Obtain Euler angles for averaging the 2 hypotheses
- Use definition of scale counters
- Choose scale counter update rule for double MCOT:

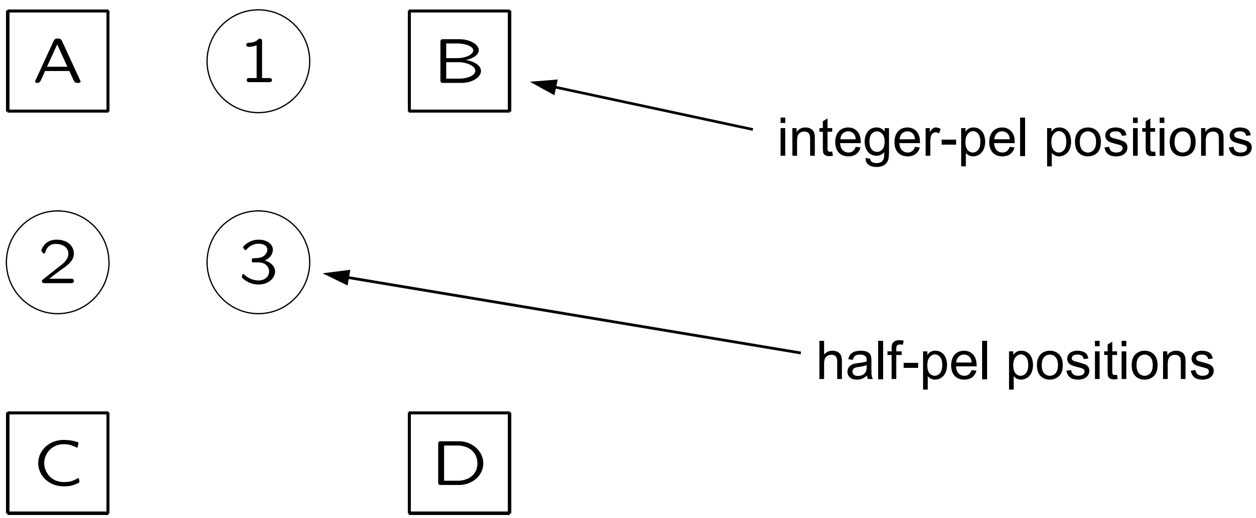
$$m_1 = n_1 + \frac{n_3 + 1}{2} \quad \text{and} \quad m_2 = n_2 + \frac{n_3 + 1}{2}$$

P-Hypothesis MC Incremental Transform

- Number of hypotheses P is a power of 2
- Assume that high-band pixel to-be $x_{2,l}$ is connected to all $P = 2^r$ hypotheses pixel, where $r = 0, 1, 2, \ldots$
- Incremental transform is given as a composition of Euler rotations in $P+1$ dimensions
- Obtain Euler angles for dyadic averaging of pairs of hypotheses
- Hence, each of the P hypotheses is weighted by $1/P$
- Choose scale counter update rule for P-MCOT:

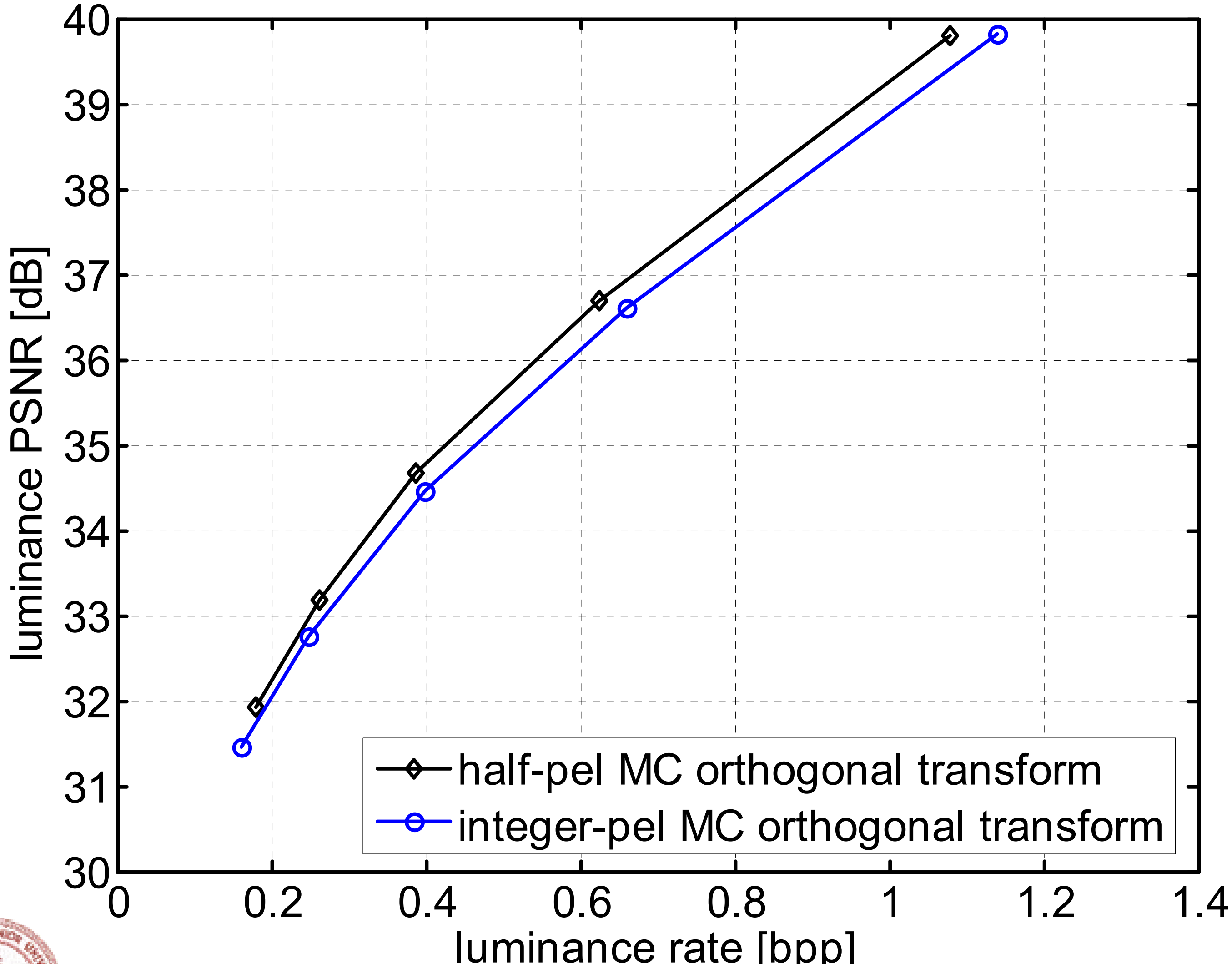
$$m_p = n_p + \frac{n_{P+1} + 1}{P} \quad \text{for} \quad p = 1, 2, \ldots, P$$

Half-Pel MC with Averaging Filter



- IP position via 1-hypothesis MC incremental transform
- HP positions 1 and 2 via 2-hypothesis MC incremental transform averaging IP positions A, B and A, C, respectively
- HP position 3 via 4-hypothesis MC incremental transform
- Type of incremental transform can be chosen on block level

HP-MCOT Experimental Results



City

CIF

30 fps

64 frames

GOP size K=16

8x8 block motion

Conclusions

- Class of motion-compensated orthogonal video transforms
- Highly flexible incremental transforms
- Energy concentration constraint
- Permit sub-pel accurate motion compensation
- Bidirectionally MC orthogonal transform to be presented at ICASSP 2007

Further Reading

http://www.orthogonalvideo.org