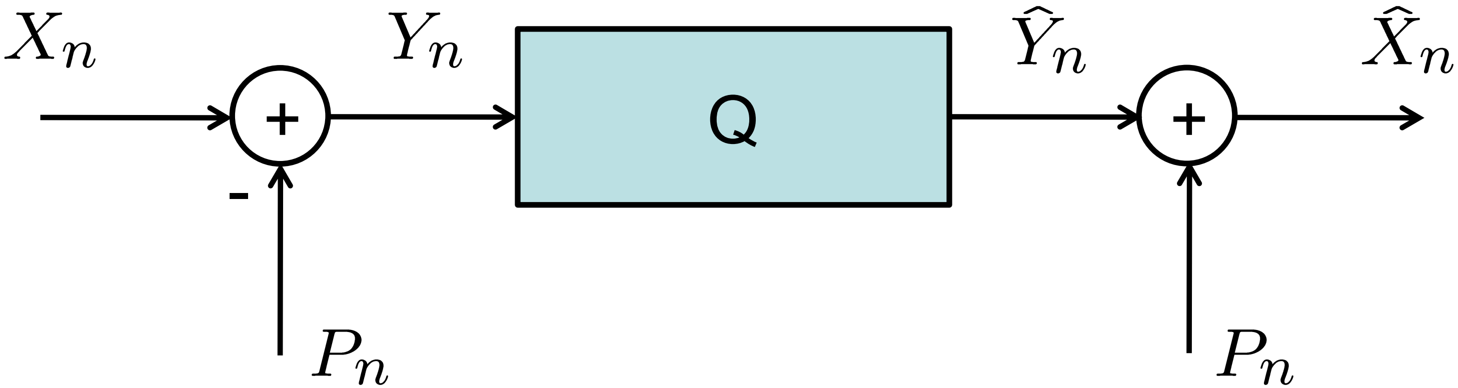
Predictive Coding

- Coding of the prediction error
- MSE-optimal linear prediction
- Linear prediction gain

Coding of the Prediction Error



$$Y_n = X_n - P_n$$

$$\hat{X}_n = \hat{Y}_n + P_n$$

Coding of the Prediction Error

- Coding of the prediction error

$$Y_n = X_n - P_n$$

$$\hat{Y}_n = Q(Y_n)$$

$$\hat{X}_n = \hat{Y}_n + P_n$$

- Distortion

$$D = E\{(X_n - \hat{X}_n)^2\} = E\{(Y_n - \hat{Y}_n)^2\}$$

MSE-Optimal Linear Prediction

- Linear prediction

$$Y_n = X_n - \sum_{k=1}^{\infty} h_k X_{n-k}$$

- Conditions for MSE-optimal linear prediction

$$\min_{h_k} D$$

$$E\{Y_n X_{n-k}\} = 0 \quad \forall \quad k \geq 1$$

- Yule-Walker equations
- Extended Wiener-Hopf equations

Linear Prediction Gain

- Stationary Gauss-Markov sources
- Asymptotic linear prediction gain

$$G_P = \frac{\frac{1}{2\pi}\int_{-\pi}^{\pi}\Phi_{XX}(\omega)d\omega}{2^{\frac{1}{2\pi}\int_{-\pi}^{\pi}\log_2\Phi_{XX}(\omega)d\omega}}$$

- Reminder: Szegö's Theorem

$$\lim_{N\to\infty}\log_2\left[\prod_{n=1}^{N}\Lambda_{nn}\right]^{\frac{1}{N}} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\log_2\Phi_{XX}(\omega)d\omega$$

- Coding gain for DPCM coding at high rates

$$G_{DPCM} = G_P$$