

Efficient STDP Micro-Architecture for Silicon Spiking Neural Networks

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Abstract— Spiking neural networks (SNNs) are the closest approach to biological neurons in comparison with conventional artificial neural networks (ANN). SNNs are composed of neurons and synapses which are interconnected with a complex pattern. As communication in such massively parallel computational systems is getting critical, the network-on-chip (NoC) becomes a promising solution to provide a scalable and robust interconnection fabric. However, using NoC for large-scale SNNs arises a trade-off between scalability, throughput, neuron/router ratio (cluster size), and area overhead. In this paper, we tackle the trade-off using a clustering approach and try to optimize the synaptic resource utilization. An optimal cluster size can provide the lowest area overhead and power consumption. For the learning purposes, a phenomenon known as spike-timing-dependent plasticity (STDP) is utilized. The micro-architectures of the network, clusters, and the computational neurons are also described. The presented approach suggests a promising solution of integrating NoCs and STDP-based SNNs for the optimal performance based on the underlying application.

Keywords: Spiking Neural Network; Networks-on-Chip; STDP; Neuron Clustering;

I. INTRODUCTION

Brain is the main organ of learning and making decisions. Brain processes the information by taking inputs from the sensory organs (e.g. eyes, ears, and nose), and depending on these inputs it produces an output. Our brain has around 100 billion neurons which are chemically connected to each other. A neuron can be connected to around 10,000 neurons in the circuit. Those connections are called synapses which are usually formed from axons to dendrites. Synapse converts an activity into electrical effects that excites the activity in the connected neurons. Large-scale artificial neural networks (ANNs) have been used to emulate the information processing function of the brain [1]. Spiking neural networks (SNNs) [2] are a type of ANN, which emulate real biological neural networks, conveying information through the communication of short transient pulses (spikes) between neurons via their synaptic connections. Each neuron maintains an internal membrane potential, which is a function of several parameters as input spikes, associated synaptic weights, current membrane potential, and a constant membrane potential leakage coefficient [2][3]. A neuron fires (emits a spike to all connected synapses/neurons) when its membrane potential exceeds the neuron's firing threshold value. Understanding and emulating the behavior of the brain has received much attention not only from neuroscientists but also from engineers and computer scientists. While neuroscientists are interested in biophysical

models, engineers and computer scientists are more interested in utilizing the brain's powerful computing capability. The dramatic developments in brain science and neuroscience over the past few decades, together with the formidable developments in hardware and software technology, have brought us to the edge of building brain-like functioning devices and systems [4]. Developing and applying such devices in real-world information processing has been a dream and vision for quite some time though success has so far been limited. In recent years, computational neuroscience has developed rapidly and researchers are currently designing and studying large-scale complex brain models, thus helping to integrate the vast amounts of information about the brain from different sources and levels of description into working models of the brain. In recent years, supercomputers have become an enabling tool for simulating complex brain models with a scale approaching that of small and medium-sized mammals.

Recently, the network-on-chip (NoC) paradigm has emerged as a promising solution to solve the on-chip communication problems revealed in many-core system-on-chip (MCSoc). NoC architectures are composed of cores, routers, and links which are arranged in a specific topology. In the context of SNNs, the cores refer to the spiking neuron models attached to NoC routers and the NoC topology refers to the way those neurons are interconnected across the network. Most of the current projects have used NoC as an interconnect fabric for SNNs; however, their interconnection strategies (i.e. one-to-one correspondence between neurons) and expensive components (i.e. buffers, handshaking logic, etc.) make it difficult to achieve high scalability with low power consumption. In fact, the interconnection fabric (i.e. routers, channels, etc.) consumes a large portion of the total power in the system [5][14]. In addition, regarding the power and area consumption, neuron blocks consume orders of magnitude less power consumption than the interconnection platform [6]. Therefore, using NoC for large-scale SNNs arises a trade-off between scalability, throughput, neuron/router ratio (cluster size), and power-area overhead.

In this work, we identify possible space for improvements in hardware resources of NoC-based SNN designs. We propose a micro-architecture of the best known learning rule for spiking neurons, called spike-timing-dependent plasticity (STDP) [21], based on resource sharing on a clustered NoC. The XOR application is implemented using the STDP based reinforcement learning technique and used as the benchmark to explore optimal cluster size.

II. RELATED WORK

NoC is used in a number of works as an interconnection fabric for SNNs [10]–[13].

The Neurogrid project [15], [28], [28] is an analog-based neuron system. Ion-channel activity and STDP are simulated by an analog circuit. Synaptic weights are stored in digital SRAM. The overall structure consists of the main building block (neurocore) which contains 65,536 neuron models placed on 16 layers. Neurogrid uses FPGA and SRAM for connecting neurocores together. The power consumption of the platform is promising as it requires 5W for simulating activity of one million neurons. However, because of the limitation on the size of the layer it is not capable of offering biological real-time (1ms inter-spike interval).

The Fast Analog Computing with Emergent Transient States project (FACETS) is based on mixed (analog-digital) approach [16]. The HICANN (High Input Count Analog Neural Network) is a building block of a system incorporating 512 analog neurons and more than 131,072 synapses with 4-bit SRAMs to store weights. Up to 384 HICANN chips are placed on the wafer and connected through hierarchical busses. Each chip has access to 256 2-bit bus lanes, 8-bit packets with neuron addresses are transmitted serially through them. The wafers can be connected in a 2D torus topology through FPGAs that handle the Ethernet protocol. The FACETS can work beyond biological real-time, but at the cost of power consumption, which goes up to 1kW for a single wafer.

The SpiNNaker project (Spiking Neural Network Architecture) [12][13] is based on utilizing Multiprocessor-based approaches. The building block of the system comprises 18 ARM968 processor cores. Each building block requires about 1W and can emulate 16,000 *Izhikevich* neurons with STDP learning. The learning implementation is optimized to minimize the number of computations [27]. The interconnection between each node is handled by a NoC using six links, which is wrapped in a triangular lattice; this lattice is then folded onto a surface of a toroid [17]. SpiNNaker can compute 10^9 neurons in a biological real-time.

EMBRACE (Emulating Biologically InspiRed ArChitectures in hardwarE) utilizes hierarchical (H-NOC) approach, which gives a good trade-off between scalability and power consumption [18]. Like FACETS, EMBRACE is capable of working beyond biological real-time. The H-NOC approach offers a high throughput of spikes per second along with the low power consumption of 13mW for a single cluster facility. However, each module contains a fixed amount of neurons but not an optimized amount. On top of that, EMBRACE has not integrated any learning technique in the network.

III. THE HIERARCHICAL REPRESENTATION OF THE PROPOSED APPROACH

A. Networks-on-Chip and Spiking Neural Networks

An NoC consists of an interconnection of many routers to enable a large number of processing elements (PE) to

communicate with each other. PEs communicate with each other by propagating packets through routers in the network. Each router is connected to the local core and to its neighbors in the north, south, east, and west directions through bidirectional links.

Obviously, hardware implementation of a large-scale SNN has superior speed advantages over its software implementation counterpart. However, one of the main challenges to implement SNNs in hardware is the complexity of inter-neuron connectivity and its significant wiring overhead. The packet-switching mechanism in NoCs will simplify the complexity of inter-neuron connectivity while reducing the area overhead. In addition, NoC is inherently a reconfigurable and scalable platform which is essentially suitable for large-scale SNNs.

When combining NoC and SNN, a spiking neuron refers to the processing element. The inter-neuron connectivity is implemented in the forms of transmitting spike packets through the interconnection network.

B. A Neuron Block

A neuron block is connected to the local port of the router. It consists of a neuron and a synaptic block (Fig. 1). Synaptic block comprises a number of synapses, which are modeled as synaptic weights, and learning data for each synapse. In this work, we take advantage of the STDP method to adapt the synaptic weights.

1) Neuron

Different mathematical models of SNNs have been explored over time with different levels of computational efficiency and biological plausibility [19]. *Leaky integrate and fire* (L&F) and *Izhikevich* models [20] are two neuron models of SNNs which are used in this work. The L&F model is one of the most widely used models due to its simplicity, but it is limited to a single type of response and not considered for biologically plausible simulations. The *Izhikevich* model is more complex, but it allows to achieve a large variety of spiking patterns [20].

2) Synapse

Biological synapses connect neurons to each other, providing a mechanism to transmit a signal between neural cells. Variable synaptic strength - the possibility to cause different excitation amplitude at the receiving neuron, is an important property known as synaptic plasticity. Plasticity is modeled in ANN as variable synaptic weight values assigned to the inputs of the neuron model. Learning of neural networks, both biological and artificial, is based on the changing of the synaptic weights. STDP changes weights based on a time difference between incoming and outgoing spikes of a neuron. These spikes are commonly referred to as pre- and post-synaptic, respectively. A popular exponential form of STDP function [8] is as:

$$\Delta w = \begin{cases} -A_+ \exp(-\Delta t / \tau_+), & \text{if } \Delta t > 0 \\ A_- \exp(\Delta t / \tau_-), & \text{if } \Delta t < 0 \end{cases} \quad (1)$$

The reaction of a synaptic block on the spike is shown in Fig. 3. When a new spike arrives at the synapse, the corresponding weight is injected to the neuron cell. In parallel, the STDP process starts, first the time stamp is stored into the corresponding synaptic table and a spike event triggers the STDP control block. Then, the control block fetches post-synaptic time stamps to apply the STDP rule to the newly received pre-synaptic spike with every stored post-synaptic spike, falling into the time window. Sequential calculation is possible due to the sufficient period between spikes and the fact that updated weight is required only by the next spike event on that particular synapse. A similar procedure happens at the post-synaptic spike event. This time, the current post-synaptic time stamp should be saved and compared with every pre-synaptic spike from each synapse. A pre-synaptic spike event causes the recalculation of the single synaptic weight and a post-synaptic spike leads to the recalculation of all the weights. This is the main reason of using an event buffer in the controller.

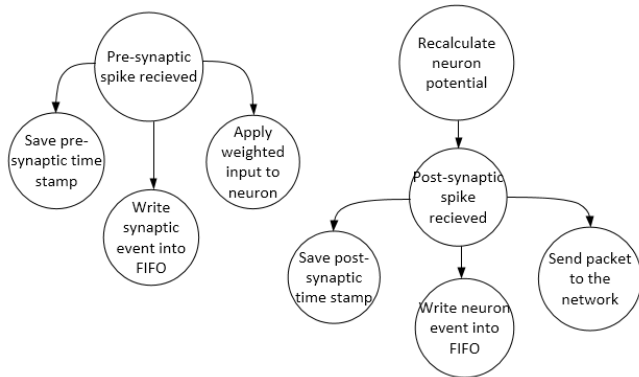


Fig. 3. Block diagram of reaction on receiving a spike

Our analysis shows that the XOR implementation with STDP block shared among the synapses of one neuron provides 1000 STDP operations per second for each neuron. It is less than required, but we assume that a digital implementation allows the synaptic block to work at a higher clock rate than a Neuron Block. For example, the clock rate of the network should be higher to provide a lower spiking delay. With the assumption of having more space for improvements due to higher STDP calculation speed or lower simulation time step, we have modeled a block diagram of STDP synaptic block shared among neurons as shown in Fig. 4. In addition, a possibility to share time stamps for synaptic connections coming from one pre-synaptic neuron is also considered. In the case of a topology with fully connected layers, such an approach allows to reduce the amount of time stamps to S/N , where N is the number of neurons in a cluster. When a spike comes to a shared synapse, only one time stamp is stored and the weighted input is applied to all neurons. When other types of NN topologies are used, additional tables with the connectivity information are required. The behavior of the controller slightly changes after adding sharing modules for pre-synaptic time stamps. A pre-synaptic event then also triggers multiple STDP calculations. The sequence of operations for both synaptic events is similar as depicted in Fig. 5. In case of fully connected layers, the controller can read all the pre- and post-synaptic time stamps sequentially omitting the communication to connectivity tables.

C. A clustering Block

As assumed in different works [13], a neuron block is connected to one router. This implies that the area overhead will increase considerably as the number of neurons increases. This is the simplest way of implementing a neural network

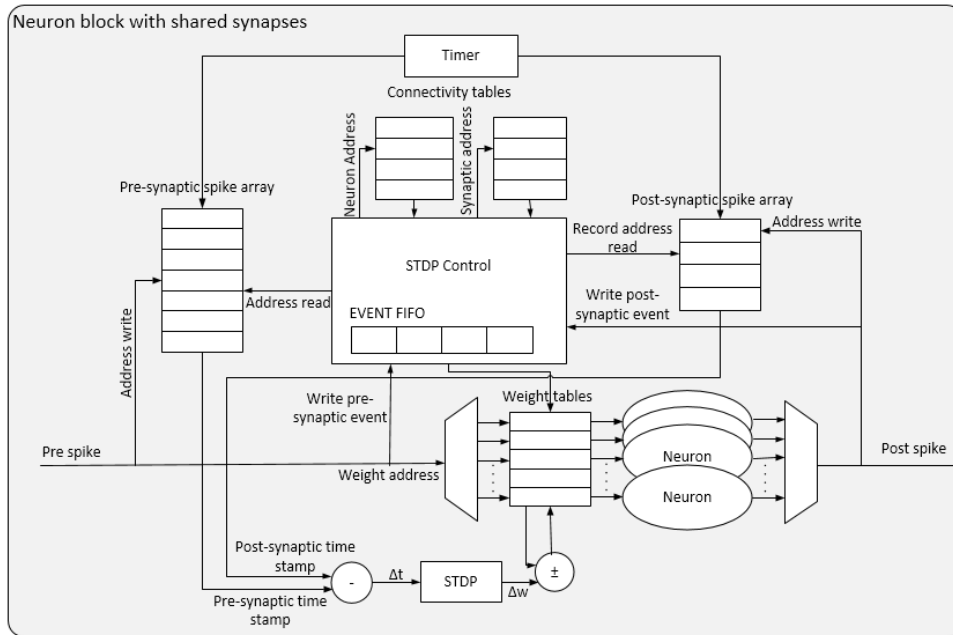


Fig. 4. A neuron block with shared synapses

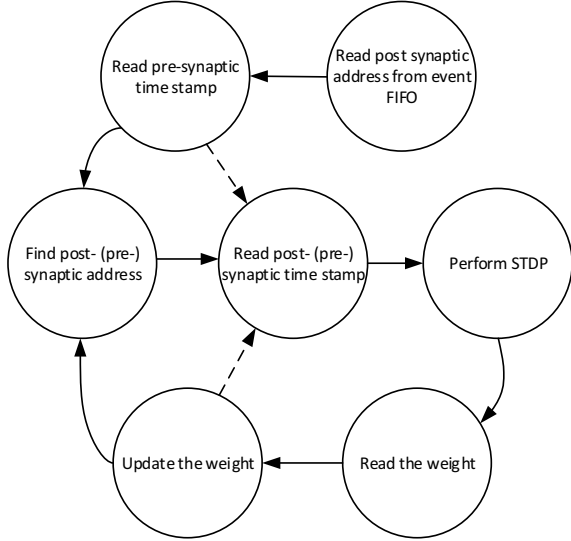


Fig. 5. Sequence of STDP operations

platform in NoCs but at the cost of considerable area overhead and power consumption. These overheads are paid whereas most of the times the network resources (e.g. buffers and links) are underutilized. In the clustered approach, we connect as many neurons as possible to a router such that to reach the optimal point of resource utilization before approaching the saturation point of the network. This guarantees the maximum throughput and performance with the lowest area overhead and power consumption in this particular aspect. Fig. 6 shows the structure of integrating four neuron blocks.

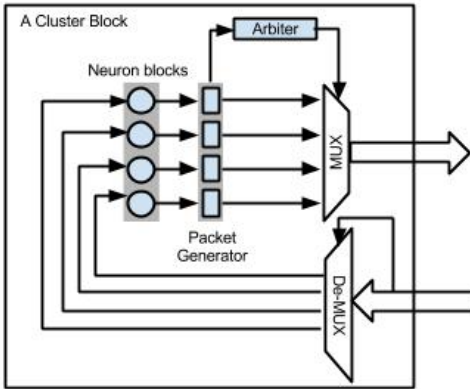


Fig. 6. A cluster block

IV. EXPERIMENTAL RESULTS

To evaluate the proposed approach, a simulation platform of combining SNN and mesh-based NoC is developed. RTL-level of all major components of SNN including cluster blocks (neurons, synapses, and the STDP learning) and NoC modules (routers and links) have been implemented. Each cluster has an additional area overhead of a multiplexer and a de-multiplexer. We have measured latency, throughput, area overhead, and power consumption of the proposed approach for different

cluster sizes and spiking injection rates. A spiking injection rate refers to the average of spike events that a neuron produces within a simulation time. The simulations are carried out for an XOR problem. The XOR application is a classic benchmark for learning algorithms of artificial neural network training. We use it as an example of classical layered neural network architecture.

A. XOR Application

In the XOR problem, the output 1 should be resulted when the inputs are as $\{0, 1\}$ or $\{1, 0\}$, otherwise the output 0 should be produced [24]–[26]. Similar to [7], [24] for solving this problem, 60 neurons in the input layer, 60 neurons in the hidden layer, and 1 neuron in the output layer are used. As shown in Fig. 7, each layer is fully connected to the next layer meaning that all neurons in the input layer are connected to all neurons in the hidden layer and all neurons in the hidden layer are connected to the single neuron in the output layer. For solving the XOR problem, 121 neurons are needed. For mapping the neurons to an NoC, the first layer of neurons is placed in borderline routers while the hidden layer neurons along with the output neuron are mapped in the central part of the network. We assume that such placement of a first layer should be caused by the presence of continuous external input (e.g. pixel data for video processing tasks). Fig. 8(a) and (b) show the mapping of neurons in 5×5 and 5×4 NoC when 5 and 6 neurons are integrated into one cluster, respectively. The character 0, 1, 2, and 3 stands for non-used, first-layer, hidden-layer, and output layer, respectively. Out of 60 input neurons, 30 neurons represent logic 0 and other 30 represent logic 1. The input 1 was represented by a Poisson spike train at different frequencies (ranges from 25Hz to 50Hz) and the input 0 was represented by no spiking. Each output spike was rewarded when network was supposed to produce logic 1 output or punished for each spike if desired output was logic 0. Upon iterating this process for few hundred cycles, the network will be capable of producing correct outputs. Fig. 9 shows an output of a SNN trained to perform XOR application. As can be seen in this figure, when the inputs are $\{0,1\}$ or $\{1,0\}$, the output neuron spikes frequently compared with the input $\{1,1\}$ and $\{0,0\}$: no spike}.

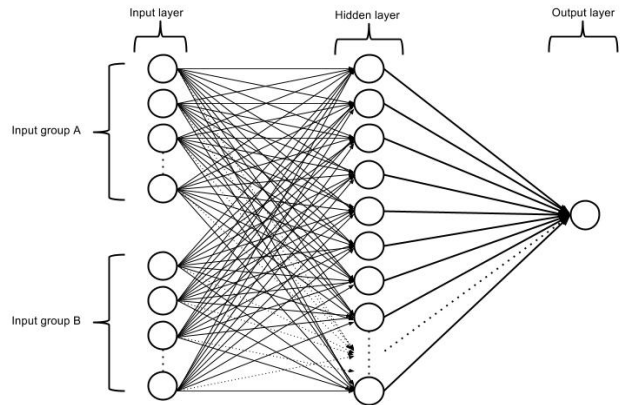


Fig. 7. An XOR implementation with 60 neurons in the first layer, 60 neurons in the hidden layer and one neuron in the output layer

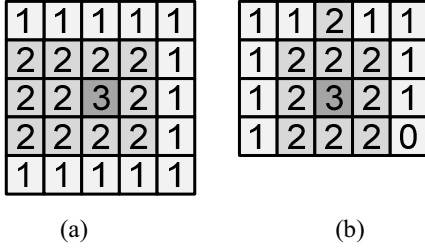


Fig. 8. (a) mapping of neurons in a 5×5 mesh-based NoC with $5/N/C$
 (b) mapping of neurons in a 5×4 mesh-based NoC with $6/N/C$

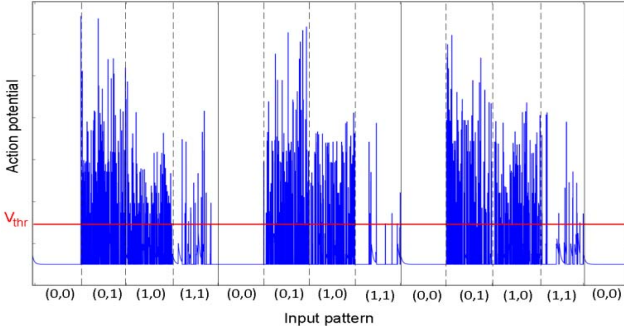


Fig. 9. An XOR application in SNN

SNN has two phases: Exploration and Exploitation. During the Exploration phase, the network is trained for the particular application; while during the Exploitation phase, the execution is performed according to the learned patterns.

B. System Configuration

Each router includes input buffers, a routing unit, an allocator, and a crossbar unit. A router has five input/output ports without using virtual channels. Buffer depth is chosen to be 4. For routing packets, the simple deterministic XY routing algorithm is utilized. It is worth mentioning that performance can be improved by employing adaptive routing algorithms as discussed in different works but this efficiency is out of the concept of this paper. Link width is equal to the packet length that is 16 bits. The packet consists of two fields as source address and destination address. The round robin arbitration is used which provide fairness among requests from different input buffers. As shown in [13] prioritized round-robin arbitration outperforms the FCFS round-robin mechanism. All these optimizations can be done in the network without affecting the general idea of this paper as it is independent of the network configuration such as the underlying routing algorithm and arbitration mechanism. The network size is initially set to 11×11 with 1 neuron per cluster ($1/N/C$) which is reduced to 3×4 with $10/N/C$ using the clustering approach.

C. Latency Evaluation

Communication latency should be kept as low as possible like in most other applications. One network clock cycle equals to $1/400\text{ms}$ of simulated biological time. The supposed maximum average latency should be 3ms or 1200 cycles. The

presented results can be improved by applying multicasting routing scheme, but out of the scope of this work.

Fig. 10(a) shows the latency results for different spiking injection rates under the XOR benchmark. As demonstrated in this figure, several neurons can be attached to one router with negligible impact on the latency value. With the spike injection rate of 35Hz, the latency value increases from 310 clock cycles with $1/N/C$ to 757 clock cycles (unit) with $8/N/C$. This figure indicates that under higher frequencies, a lower number of neurons can be included into one cluster. For an instance, 6 neurons can be integrated into a cluster when the frequency is 50Hz while 10 neurons can be integrated in a cluster when the frequency is 25Hz without even reaching the saturation point.

Fig. 10(b) shows the tradeoff between latency, frequency and cluster size. According to this figure, for the frequencies smaller than 35Hz, 10 neurons is the optimal number of neurons per cluster. For the frequencies between 35Hz to 40 Hz, 8 neurons is the best option. The number of neurons per cluster decreases to 6 neurons for the frequencies between 40Hz and 50Hz. This figure can easily show a stable level of latency for different cluster sizes and frequencies. Thereby, according to the application and desired frequency, the optimal cluster size can be found.

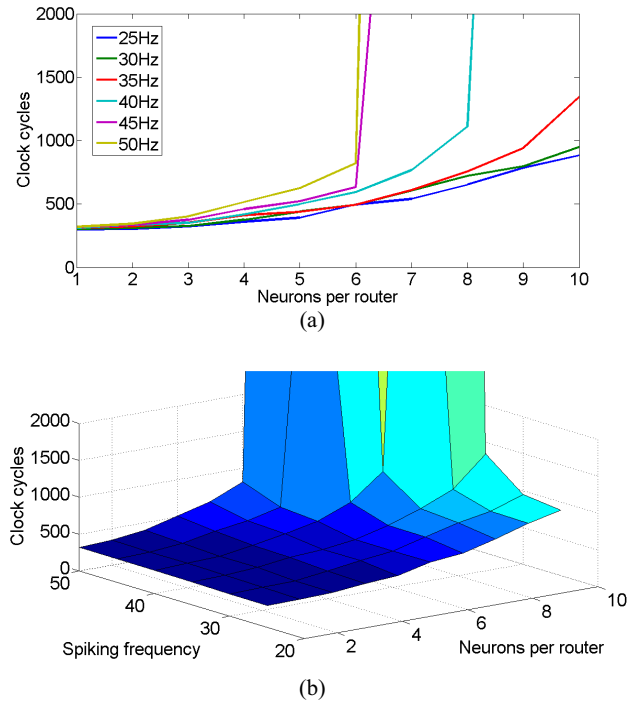


Fig. 10. Latency measurement under different frequencies

D. Throughput Evaluation

Throughput is also measured for different frequencies and cluster sizes. As shown in Fig. 11(a), throughput increases by integrating more number of neurons per cluster. However, for each frequency there is a breaking point from which throughput starts degrading. This point shows the optimal number of neurons per cluster for that frequency. The tradeoff between

throughput, frequency and number of neurons per cluster is shown in Fig. 11(b). As can be seen from this figure, throughput can be improved by integrating more neurons, but there is a limit for this integration and beyond this limit the network resources overflow and thus throughput decreases. The improvements in throughput mean that network resources are underutilized.

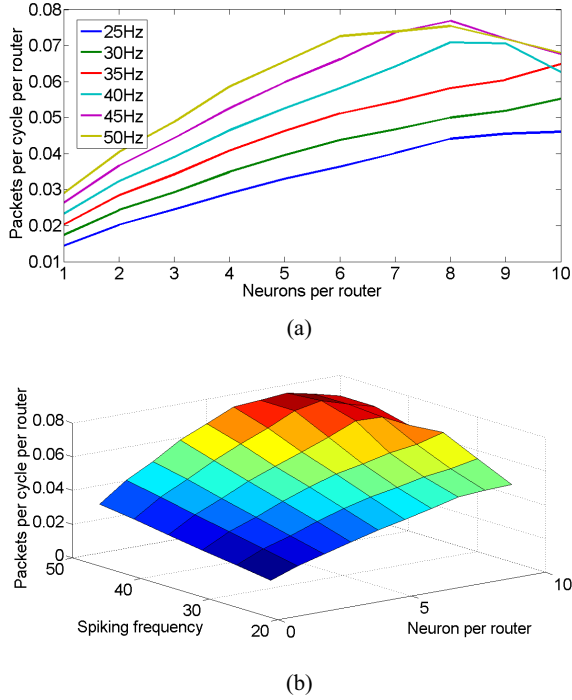


Fig. 11. Throughput measurement under different frequencies

E. Hardware Analysis

In this subsection, the hardware overhead is evaluated under all configurations. The major components of the system as NoC routers, neuron and STDP blocks are synthesized with Synopsys Design Compiler using the UMC 90nm technology with a timing constraint of 500MHz for the system clock and supply voltage of 1.2V.

Two models of neurons have been implemented: L&F and *Izhikevich* models. In the first configuration, the L&F model of neurons is implemented. The area overhead and power consumption of each individual component is listed in Table 2. As can be seen in this table, the area overhead of the L&F neuron is around six times smaller than a router. The area overhead of the whole platform is listed in Table 1. Results show that by integrating 10 neurons per cluster with a single synaptic block per router, the area overhead of the whole platform decreases by 36% when employing L&F neurons while it decreases by 52% when using *Izhikevich* neurons. The introduction of shared synaptic resources among neurons provides further improvements of 46% for L&F neurons and 24% for *Izhikevich* neurons. These results should be considered as raw and not suitable for comparison with other projects

although they allow presenting a possible value of the proposed approach.

Table 1. Area overhead of the whole platform in μm^2 using two models of neuron as L&F and *Izhikevich*

Neuron per Cluster	Area (μm^2) L&F		Savings %	Area (μm^2) <i>Izhikevich</i>		Savings %
	Single	Shared		Single	Shared	
1	3,703	3,703	0,0	5,336	5,336	0,0
2	2,739	2,251	17,8	4,458	3,970	10,9
3	2,291	1,666	27,2	3,991	3,367	15,6
4	2,035	1,343	34,0	3,654	2,963	18,9
5	2,016	1,287	36,1	3,702	2,974	19,7
6	1,872	1,116	40,4	3,492	2,737	21,6
7	2,129	1,352	36,5	4,018	3,242	19,3
8	1,899	1,106	41,7	3,626	2,834	21,8
9	2,104	1,301	38,2	4,047	3,244	19,8
10	1,746	0,935	46,4	3,366	2,555	24,1

Table 2. Area overhead (A: nm^2) and power consumption (P: μW) of individual components

	Router	L&F neuron	<i>Izhikevich</i> neuron	Synaptic	Cluster Overhead (10N/R)
A	13,012	2,647	16,140	10,203	2,055
P	223	102	1,892	375	85

V. CONCLUSION

In this paper, we presented micro-architectures of Spike-Timing Dependent Plasticity (STDP) based Spiking Neural Network (SNN). A model of sharing synaptic resources for clustered network-on-chip (NoC) is introduced. It improves hardware utilization and cost of STDP computational block. The XOR application is programmed through reinforcement learning technique. An operational regime that guarantees the maximum throughput with the lowest cost is found. Area consumption of the main components of the system is extracted. The results show a significant area reduction.

VI. ACKNOWLEDGEMENT

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