Distributed Attitude Control of Multi-Agent Formations

L. Wang$^{1,2}$, J. Markdahl$^1$, and X. Hu$^1$

$^1$Division of Optimization and Systems Theory
Royal Institute of Technology (Sweden)

$^2$Department of Automation
Shanghai Jiao Tong University (China)

30 August, 2011 · IFAC World Congress · Milano
Cooperative manipulation

- Perform a manipulation task using multiple manipulators.
- Carry heavier loads, use several tools simultaneously.
- Centralized or decentralized control.
### Multi-Agent Model

Every manipulator corresponds to an agent. The agent state is given by the end-effector position.

**Manipulator model**
- Manipulator kinematics, $\dot{\mathbf{p}}_i = J_i(q_i)\dot{q}_i$, $\dot{q}_i = u_i$
- Bilateral constraints at the grasp points

**Multi-agent model**
- Single integrator kinematics, $\dot{\mathbf{p}}_i = u_i$
- Formation of agents should be maintained, *i.e.* distances between agents should be constant
Geometry of the Model

Aim: rotate the object to a desired orientation.

Let

\[ p_{ij} = p_i - p_j, \]
\[ p_c = \frac{1}{3} \sum_{i=1}^{3} p_i, \]
\[ n = p_{12} \times p_{23}. \]
Commonly used parametrizations of orientation:

- Rotation matrices, $SO(3)$
- Unit quaternions
- Euler angles

We use three vectors: $p_{12}$, $p_{23}$, and $n = p_{12} \times p_{23}$.

- Gram-Schmidt mapping onto $SO(3)$

\[
(p_{12}, p_{23}, n) \rightarrow \left[ \frac{p_{12}}{\|p_{12}\|} \frac{p_{12} \cdot p_{23}}{\|p_{12}\| \|n\|} - \frac{(p_{12} \cdot p_{23})p_{12}}{\|p_{12}\| \|n\|}, \frac{n}{\|n\|} \right].
\]

- Formalize goal as $\lim_{t \to \infty} n = n_d$. This leaves one degree of rotational freedom.
Theory of rigid graphs

Total number of constraints $n(n - 1)/2$, requires a complete communications graph?

Theory of rigid graphs

- If there is no set of three collinear points, then $3n - 6$ constraints of the type $\|p_{ij}\| = c_{ij}$ ensure rigidity.
- Rigidity is preserved if $\dot{p}_{ij}$ belongs to the nullspace of a $|E| \times 3n$ matrix $R(G)$. $E$ is the edge set in a constraint graph $G$. 
The control law

\[ u_i = v + w \times p_{ic}, \]
\[ w = \alpha n \times n_d, \]

and requires agent \( i \) to know \( p_{ic}, \) \( n, \) and \( n_d. \) For example, all agents know \( n_d \) (global information) and are neighbors of the special agents 1, 2, and 3.
We prove . . .

- the equilibrium \( n = n_d \) is almost global asymptotically stable, \( n = -n_d \) is unstable,

- the convergence rate is locally exponential.

The proof is by Lyapunov theory methods.
An example

Set up

- Six agents forming an equilateral triangle.
- All agents can sense their relative position with respect to the three special agents 1, 2, and 3.

Communications graph

Simulation
A distributed control law for rotating a multi-agent formation to any desired orientation.

Future work

- Include manipulator kinematics, $\dot{p}_i = J_i(q_i)\dot{q}_i$, with rank deficient Jacobian matrix.
- The 2D case.
- Control all three degrees of rotational freedom.
Questions?

Thank you