Application-Oriented Input Design in System Identification

Optimal input design for control

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Model based control design plays a key role in today's industrial practice. Industry demands cutting edge methods for identifying the necessary models; this is where system identification enters the picture. However, additional tools are needed to handle the increasingly stringent conditions on cost and performance related to identifying the models.

The fundamental goal of any kind of framework used to estimate models in industry is to provide the least costly identification experiment. However, the notion of "cost" can be understood in many different ways, and needs in all cases to be defined by the user. Examples of costs are experiment length and power of the input signal being used. Other, more complex, costs are the framework's ability to be used during normal production — that is, the identification experiment is not allowed to noticeably affect the quality of the product. Another crucial cost that needs to be taken into account is of course that the obtained model results in acceptable control performance, when used in the model based control design.

The first purpose of this paper is to present a framework that aims to fulfill these requirements. The framework is related to optimal input design and is referred to as application-oriented input design. Application-oriented input design finds the cheapest identification experiment while fulfilling performance specifications on the application of the model. The core of the method consists of system identification results together with convex optimization.

The second purpose of this article is to present a MATLAB-based toolbox, called MOOSE2, for solving application-oriented input design problems. MOOSE2 allows many different sorts of input design problems to be formulated in a straightforward and uniform way. The toolbox converts the formulation into a convex optimization problem and solves it using YALMIP, a toolbox for modeling and optimization in MATLAB [1]. MOOSE2 requires little knowledge of the underlying identification theory and the necessary MATLAB programming to solve the problem. For more details, see [2].

The article is organized as follows. First, the results from an application-oriented input design experiment performed on a water tanks process are presented. The results serve as motivation for the promotion of application-oriented input design. Second, the basic concepts of optimal input design in an application-oriented setting are described. Third, the section "Input Design Framework" presents a method for implementing application-oriented input design on industrial systems. In the fourth part of the article the capabilities of the framework are illustrated in both simulation and experimental studies. The details are described in sections "Simulation of Model Based Control Design” and “Water Tanks Experiment”. The studies are performed in an open-loop identification set-up. However, as discussed in the subsequent section “Challenges with Industrial Application”, the framework and its key concepts can also be applied in a closed-loop setting. The last part of the article, section “The Grander Scope of Experiment Design”, gives intuitive arguments for how and why application-oriented input design alleviates some of the key issues within system identification. For a short overview of prior and related research in the field of input design for control, the reader is referred to “Related Work”.

Motivating Experiment

The input design framework (IDF) described in this article is evaluated on a water tanks process. The setup is presented in “Water Tanks Process”. The control objective is to regulate the water levels of two tanks in the process, according to a reference trajectory, using model predictive control (MPC). For more details about MPC, see “What is model predictive control?”. The process is nonlinear. However, a linearized and discretized model of the system is used in the control design. The objective of the identification experiment is to estimate the parameters of this model.

Identification experiment

IDF is applied on the water tanks process in twenty different identification experiments. The obtained models are then used in the control design. The resulting outputs of the water tanks process when being controlled by an MPC controller based on the estimated models are displayed in Figure 1. To evaluate the performance of IDF, twenty additional models are estimated using white Gaussian noise as input signal in the identification experiments. Each of the twenty input signals are chosen to have the same power as the input signal found using IDF, and the power is divided equally between the two input channels of the process. That is, in the case of white noise, no properties of the suitable input signal are assumed to be known except the power level which is obtained from IDF. The resulting outputs when applying the estimated models in the control design are also shown in Figure 1.

The control performance is better for models estimated with IDF compared to models estimated with white noise, even though the power of the input signal is the same in all experiments. These properties are a result of the ability to tailor the identification experiment to the application offered by IDF.

In this particular experiment, IDF provides a highly intuitive input design. To see in what way, and to have a more detailed description of the water tanks process and the experiments made, see section “Water Tanks Experiment”.

Dynamic System and Model

A multivariate, linear, time invariant, discrete time, dynamic system is considered. The output response can be expressed as

$$y(t) = \sum_{k=1}^{\infty} g_k u(t-k) + \sum_{k=0}^{\infty} h_k e(t-k), \tag{1}$$

with $y(t) \in \mathbb{R}^p$, $u(t) \in \mathbb{R}^m$ and $e(t) \in \mathbb{R}^p$. The system is asymptotically stable with sequences corresponding to an impulse response $\{g_k\}$ and a noise filter impulse response $\{h_k\}$, where $g_k \in \mathbb{R}^{p\times m}$ and $h_k \in \mathbb{R}^{p\times p}$. The matrix h_0 is equal to the identity matrix. The input signal sequence $\{u(t)\}$ is manipulable and the output signal sequence $\{y(t)\}$ is measurable. The system is affected by an unknown zero mean white Gaussian noise sequence $\{e(t)\}$ with covariance matrix Λ. The variance is denoted λ when the system is univariate.

The dynamic system (1) can be described as

$$y(t) = G(q^{-1})u(t) + H(q^{-1})e(t), \tag{2}$$

where $G(q^{-1})$ and $H(q^{-1})$ are transfer function matrices with dimensions commensurate with the signal dimensions. The symbol q^{-1} denotes the backward-shift operator defined by $q^{-1}u(t) = u(t-1)$.

The structure of system (2) can be expressed using the parametrized model

$$\mathcal{M}(\theta): \quad y(t) = G(q,\theta)u(t) + H(q,\theta)e(t), \tag{3}$$

where G and H are parameterized by an unknown vector $\theta \in \Theta \in \mathbb{R}^n$. The true system (2) is obtained when θ is equal to the true parameter vector θ^0 and $\{e(t)\}$ has the true covariance matrix Λ_0. That is,

$$\mathcal{S}: \quad y(t) = G(q,\theta^0)u(t) + H(q,\theta^0)e_0(t), \tag{4}$$

where $\{e_0(t)\}$ denotes a realization of the noise that has the true covariance matrix Λ_0.

The focus of this article is on how to estimate the parameter vector θ in an application-oriented setting. The true covariance matrix of the noise is assumed to be known.

The estimated parameters are denoted $\hat{\theta}_N$, where N is the number of observations of the input and output signals used in the identification experiment.

Objective of Application-Oriented Input Design

The main objective of application-oriented input design is to, while minimizing the cost of the identification experiment, deliver a model that gives acceptable application performance. The requirements on the model are given by its intended use. For instance, consider two applications of a model of a car engine. In cruise control, a simple model from the position of the accelerator to the vehicle speed may suffice. However, a much more complex model of the chemical reactions inside the engine is required if the goal is to study emissions in a simulation of the engine. Even though the latter model is a much more accurate description of the engine, it is unnecessarily complex for the cruise control application.

Application-oriented input design designs the input signal applied in the identification experiment such that an acceptable application performance is guaranteed, with high probability, when the obtained model is used in the application. The applied input signal directly influences the estimates, consequently, its design can affect the achievable application performance. The following example illustrates how the input signal can influence the estimates.

Example 1: Influence of input signal on estimates

A finite impulse response (FIR) system is modeled by

$$\mathcal{M}(\theta): \quad y(t) = \sum_{k=1}^{n} \theta_k u(t-k) + e(t),$$

where the unknown parameters $\theta = [\theta_1 \dots \theta_n]^{\mathrm{T}}$ are the impulse response coefficients. The true system is $\mathcal{S} = \mathcal{M}(\theta^0)$. The model is required to have the same static gain as the true system. That is, $\sum_{k=1}^{n} \theta_k^0$ must be estimated with high accuracy. With a constant input signal, for instance $u(t) = 1$ for all t, the output of the system becomes

$$y(t) = \sum_{k=1}^{n} \theta_k^0 + e(t). \tag{5}$$

The static gain can be approximated by the averaged value of the obtained output (5). A constant input signal can in fact be proven to give the lowest possible variance of the static gain estimate [3]. The constant input signal hides the influence of each individual impulse coefficient θ_k and highlights the static gain of the system. If instead a particular impulse coefficient is sought after, a constant input signal is useless. This example shows that it is important to take the application of the model into account when designing the input signal used in the identification experiment.

Parameter Sets in Application-Oriented Input Design

Application-oriented input design is formulated in terms of two parameter sets, one related to the application and the other to the identification method, which are illustrated graphically in Figure 2. The plane represents the space of all possible parameter values for a given model structure. In this space, there is a set of parameters that give a model that results in acceptable application performance. This set is denoted Θ_{app}. The goal of the modeling is to deliver parameter estimates that lie inside this set, that is, $\hat{\theta}_N \in \Theta_{\mathrm{app}}$. Typically, a certain point estimate cannot be ensured beforehand. Instead, a set of parameters that contains the estimate with high probability is considered. This set is denoted $\mathcal{E}_{\mathrm{SI}}$. The purpose of the input design is then to reshape and rotate $\mathcal{E}_{\mathrm{SI}}$ in such a way that the set is completely inside Θ_{app}.

The parameter sets $\mathcal{E}_{\mathrm{SI}}$ and Θ_{app} both depend on the true parameter values θ^0 which are unknown. However, as explained in sections "Input Design Framework" and "The Grander Scope of Experiment Design"; and illustrated in sections "Simulation of Model Based Control Design" and "Water Tanks Experiment", θ^0 can be exchanged by an initial estimate thereof in the calculation of the sets.

The details of the construction of these sets and how input design can be used to give acceptable application performance are given in what follows. First, a simple example of the set Θ_{app} is presented.

Example 2: Set of parameters with acceptable application performance

A stable output-error system is modeled by

$$\mathcal{M}(\theta): \quad x(t+1) = \theta_1 x(t) + \theta_2 u(t),$$

$$y(t) = x(t) + e(t),$$

where $\theta = [\theta_1 \ \theta_2]^{\mathrm{T}}$. The true system is $\mathcal{S} = \mathcal{M}(\theta^0)$. A dead-beat controller is designed to drive the system from a nonzero initial state to a state equal to zero. The optimal controller is a proportional state-feedback of the form

$$u(t) = -fx(t),$$

where $f = \theta_1^0/\theta_2^0$. It is important to estimate f with high accuracy, not the individual parameters θ_1 and θ_2, to achieve acceptable control performance. Thus, the set Θ_{app} is designed such that only small deviations from f are allowed. An example of such a set is an ellipse centered at θ^0 with an infinitely long semi-axis in the direction of $[f \ \ 1]^{\mathrm{T}}$ and a finite semi-axis in the orthogonal direction. The length of the finite semi-axis corresponds to the acceptable level of degradation in control performance.

Figure 3 shows a realization of an input signal designed according to IDF with the settings in Table V. For comparison, a realization of a white Gaussian input signal is also displayed. The variance of the white Gaussian input signal is chosen such that the same accuracy of the estimated parameters is obtained with the same probability as for the optimal input signal. In this example, the required variance is approximately seven times the variance of the optimal input signal. Consequently, seven times less power of the input signal can be used with IDF than with white Gaussian noise. The example illustrates the enormous possibilities of reducing the cost of identification experiments when using IDF, where the power of the input signal is related to the economical cost of the experiment.

System Identification Set

The influence of the input signal on the estimated parameters can be described using system identification theory. The estimates are obtained by means of the prediction error method (PEM), see “Prediction Error Method”. PEM guarantees, under weak assumptions and asymptotically in the number of observations N, with probability α that the obtained estimates lie inside a particular ellipsoid centered around the true parameters. The ellipsoid is called the system identification set and is defined as

$$\mathcal{E}_{\mathrm{SI}}(\alpha) = \left\{ \theta \;\middle|\; (\theta - \theta^0)^{\mathrm{T}} \bar{\mathbf{I}}_{\mathrm{F}} (\theta - \theta^0) \leq \frac{\chi^2_\alpha(n)}{N} \right\}, \tag{6}$$

where $\chi^2_\alpha(n)$ is the α-percentile of the χ^2-distribution with n degrees of freedom. The symbol $\bar{\mathbf{I}}_{\mathrm{F}}$ denotes the averaged Fisher information matrix defined in (S10) in “Fisher Information”. The matrix is an affine function of the input spectrum. Thus, the spectrum of the input signal affects the shape of the system identification set. Consequently, the input spectrum can be used to influence the estimates.

Example 3: System identification set

An FIR system is modeled by

$$\mathcal{M}(\theta): \quad y(t) = \theta_1 u(t-1) + \theta_2 u(t-2) + e(t),$$

where $\theta = [\theta_1 \ \theta_2]^T$. The true system is $\mathcal{S} = \mathcal{M}(\theta^0)$. The averaged Fisher information matrix is

$$\bar{\mathbf{I}}_{\mathrm{F}} = \frac{1}{\lambda} \begin{bmatrix} r_0 & r_1 \\ r_1 & r_0 \end{bmatrix},$$

where $r_k = \mathrm{E}\left\{u(t)u(t-k)\right\}$. The corresponding system identification set is

$$\mathcal{E}_{\mathrm{SI}}(\alpha) = \left\{ \theta \;\middle|\; (\theta-\theta^0)^{\mathrm{T}} \frac{1}{\lambda} \begin{bmatrix} r_0 & r_1 \\ r_1 & r_0 \end{bmatrix} (\theta-\theta^0) \leq \frac{\chi^2_\alpha(2)}{N} \right\}.$$

Application Set

The input signal is designed with the intended application of the model in mind. The degradation in application performance due to a mismatch between the model and system is specified by an application cost function. The cost emphasizes an important application quality of the system. Examples of such qualities in control are the sensitivity function and closed-loop output response.

The cost function is denoted $V_{\mathrm{app}}(\theta)$. The minimal value of V_{app} is defined as zero and is achieved when the true parameters are used in the function. When $V_{\mathrm{app}}(\theta)$ is twice differentiable in a neighborhood of θ^0, these conditions imply the constraints $V'_{\mathrm{app}}(\theta^0) = 0$ and $V''_{\mathrm{app}}(\theta^0) \succeq 0$.

An increased value of the application cost reflects an increased degradation in application performance. The maximal allowed degradation is defined by

$$V_{\mathrm{app}}(\theta) \leq \frac{1}{\gamma}, \tag{7}$$

where γ is a positive scalar. The parameters satisfying inequality (7) are called acceptable parameters and they belong to the application set. The set is defined as

$$\Theta_{\text{app}}(\gamma) = \left\{ \theta \;\middle|\; V_{\text{app}}(\theta) \leq \frac{1}{\gamma} \right\}.$$

Example 3 revisited: Application set

Example 3 is revisited to analytically exemplify the notion of an application set. The considered system has $G(q,\theta) = \theta_1 q^{-1} + \theta_2 q^{-2}$ and $H(q,\theta) = 1$. The objective is to design a proportional controller that can reject a constant disturbance applied to the system. By using the controller gain

$$K(\beta,\theta) = \frac{\beta^2}{\beta\theta_1 - \theta_2},$$

for some $0 < \beta \leq 1$, the true ($\theta = \theta^0$) closed-loop poles of the system become $-\beta$ and $-\beta\theta_2^0/(\beta\theta_1^0 - \theta_2^0)$. The true closed-loop system is stable by assumption. A possible choice of the application cost is the squared error in static gain of the sensitivity function,

$$V_{\text{app}}(\theta) = \left(\lim_{q\to 1} \frac{1}{1 + K(\beta,\theta^0)G(q,\theta^0)} - \lim_{q\to 1} \frac{1}{1 + K(\beta,\theta)G(q,\theta^0)} \right)^2.$$

The application set then becomes

$$\Theta_{\text{app}}(\gamma) = \left\{ \theta \;\middle|\; \left(\frac{\beta\theta_1^0 - \theta_2^0}{\beta\theta_1^0 - \theta_2^0 + \beta^2(\theta_1^0 + \theta_2^0)} - \frac{\beta\theta_1 - \theta_2}{\beta\theta_1 - \theta_2 + \beta^2(\theta_1^0 + \theta_2^0)} \right)^2 \leq \frac{1}{\gamma} \right\}.$$

Application-Oriented Input Design Problem

The aim here is to formally combine the results from system identification with the application requirements on the model. The design of the input signal is considered in terms of the design of the input signal spectrum. The quality of the estimates relates to the averaged Fisher information matrix, which is affine in the input spectrum. Therefore, the solution to the application-oriented input design problem is an optimal input spectrum.

Application-oriented input design has two objectives. First, the estimated parameters must give a model with acceptable application performance. Thus, the estimated parameters must be acceptable parameters with high probability. Second, the cost related to the experiment must be minimized. Thus, the application-oriented input design problem can be formulated as

$$\begin{aligned} \underset{\Phi_u}{\text{minimize}} \quad & \text{experimental cost}, \\ \text{subject to} \quad & P\left\{\hat{\theta}_N \in \Theta_{\text{app}}(\gamma)\right\} \geq \alpha, \end{aligned} \tag{8}$$

where Φ_u denotes the input spectral density (also referred to as the input spectrum), $\alpha < 1$ is a positive number close to one and $P\{A\}$ is the probability of A being true. The estimates $\hat{\theta}_N$ are random asymptotically Gaussian variables in the PEM framework. Thus, the constraint $\hat{\theta}_N \in \Theta_{\text{app}}(\gamma)$ can only be ensured with some probability.

Problem (8) is a chance constrained optimization problem and typically not convex. Furthermore, evaluating the probability in the constraint is far from trivial. However, an approximate problem can be considered instead. The chance constraint can be exchanged with $\mathcal{E}_{\text{SI}}(\alpha) \subseteq \Theta_{\text{app}}(\gamma)$, for specific values of α and γ, since the estimates, at least asymptotically in N, lie in the ellipsoid $\mathcal{E}_{\text{SI}}(\alpha)$ with probability α. The complete objective of application-oriented input design can then be stated as the optimization problem

$$\underset{\Phi_u}{\text{minimize}}\ f_{cost}(\Phi_u), \tag{9a}$$

$$\text{subject to } \mathcal{E}_{\text{SI}}(\alpha) \subseteq \Theta_{\text{app}}(\gamma), \tag{9b}$$

$$\Phi_u(\omega) \succeq 0, \text{ for all } \omega, \tag{9c}$$

where f_{cost} denotes the experimental cost assumed to be a convex function. The second constraint (9c) is a technical constraint needed to ensure that Φ_u is a spectrum, see "Constraint on input spectrum". The use of the approximation with $\mathcal{E}_{\text{SI}}$ is just one of many ways of dealing with the chance constraint. Other approaches are evaluated in an input design setting in [4].

The optimization problem (9) can, in turn, be approximated by a convex formulation, which can be solved accurately and efficiently [5]. The different steps of the approximation are described in the following sections.

Constraint on system identification and application set

Two methods of approximating the first constraint (9b) with a convex constraint are the scenario approach and ellipsoidal approximation.

Scenario approach

The scenario approach [6] relaxes (9b) by considering a finite number of parameters to satisfy the constraint. These parameters are chosen from Θ_{app} according to a given probability distribution, for example a uniform distribution. When M such parameters are chosen, constraint

(9b) is exchanged with

$$(\theta_i - \theta^0)^{\mathrm{T}} \bar{\mathbf{I}}_{\mathrm{F}} (\theta_i - \theta^0) \geq \frac{\chi^2_\alpha(n)\gamma}{N} V_{\mathrm{app}}(\theta_i) \text{ for } i = 1 \ldots M < \infty,$$

where $\theta_i \in \Theta_{\mathrm{app}}$. The set $\mathcal{E}_{\mathrm{SI}}$ lies inside Θ_{app} with high probability when M is large enough. See [6] and [7] for a theoretical background of the scenario approach.

Ellipsoidal approximation

The application cost is assumed to be twice differentiable at the true parameters. Meaning, $V'_{\mathrm{app}}(\theta^0) = 0$ and $V''_{\mathrm{app}}(\theta^0) \succeq 0$. The application cost is approximated by its second order Taylor expansion centered around the true parameters. The corresponding application set becomes an ellipsoidal region. Thus, $\Theta_{\mathrm{app}} \approx \mathcal{E}_{\mathrm{app}}$ for θ close to θ^0, where

$$\mathcal{E}_{\mathrm{app}}(\gamma) = \left\{ \theta \;\middle|\; \frac{1}{2}(\theta - \theta^0)^{\mathrm{T}} V''_{\mathrm{app}}(\theta^0)(\theta - \theta^0) \leq \frac{1}{\gamma} \right\}. \tag{10}$$

It is shown in [8] that $\mathcal{E}_{\mathrm{SI}}$ lies inside $\mathcal{E}_{\mathrm{app}}$ if and only if $\bar{\mathbf{I}}_{\mathrm{F}} \succeq \chi^2_\alpha(n)\gamma V''_{\mathrm{app}}(\theta^0)/(2N)$.

Constraint on input spectrum

The second constraint (9c) is due to the fact that the spectral density of a stationary process is a non-negative entity. The spectral density of a stationary signal $u(t)$ can be written as

$$\Phi_u(\omega) = \sum_{k=-\infty}^{\infty} r_k e^{i\omega k}, \tag{11}$$

where $r_k = \mathrm{E}\{u(t)u(t-k)\}$. Therefore, application-oriented input design problems can be formulated as convex optimization problems with decision variables r_k. The design is then a matter of finding the coefficients r_k. Two approaches for choosing the coefficients r_k are the partial correlation parameterization and the finite dimensional parameterization.

Partial correlation parameterization

In partial correlation parametrization the coefficients of a truncated sequence $\{r_k\}_{k=0}^{m-1}$ are chosen such that its extension (11) defines a spectrum. The extension problem is a trigonometric moment problem. Thus, the necessary and sufficient condition for (9c) to hold for the extended sequence is

$$\begin{bmatrix} r_0 & r_1 & \cdots & r_{m-1} \\ r_{-1} & r_0 & \cdots & r_{m-2} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m-1} & r_{m-2} & \cdots & r_0 \end{bmatrix} \succ 0, \tag{12}$$

see [9], [10]. Equation (12) is a linear matrix inequality (LMI) in r_k. For extensions of the partial correlation approach in closed-loop, see [11].

Finite dimensional parameterization

To make the problem of finding the coefficients r_k computationally tractable, a finite expansion

$$\Phi_u(\omega) = \sum_{k=-(m-1)}^{m-1} r_k e^{i\omega k}, \tag{13}$$

is considered instead of (11). The finite dimensional parameterization requires that $\{r_k\}_{k=0}^{m-1}$ is chosen such that (13) is a spectrum. This means that condition (9c) must hold for the truncated sum (13), which can be achieved in various ways. The most frequently used technique is an application of the positive real lemma, which springs from the Kalman-Yakubovich-Popov lemma [12], see Lemma 1.

Lemma 1: If $\{A,\ B,\ C,\ D\}$ is a controllable state-space realization of

$$\Phi_u^+(\omega) = \frac{1}{2}r_0 + \sum_{k=1}^{m-1} r_k e^{i\omega k}.$$

Then there exists a matrix $Q = Q^T$ such that

$$K(Q, \{A, B, C, D\}) \triangleq \begin{bmatrix} Q - A^T Q A & -A^T Q B \\ -B^T Q A & -B^T Q B \end{bmatrix} + \begin{bmatrix} 0 & C^T \\ C & D + D^T \end{bmatrix} \succeq 0, \tag{14}$$

if and only if $\Phi_u(\omega) = \sum_{k=-(m-1)}^{m-1} r_k e^{i\omega k} \geq 0,$ for all ω.

Thus, the necessary and sufficient condition for (9c) to hold for the truncated sequence (13) is to find a matrix Q such that the matrix inequality (14) holds. The matrix inequality is an LMI in r_k and Q when the only matrices that are linearly dependent on the coefficients r_k are C and D. For an example of a controllable state-space realization of $\Phi_u^+(\omega)$, see “Example 4: Finite dimensional parameterization”.

Example 4: Finite dimensional parameterization

A controllable state-space realization of $\Phi_u^+(\omega)$ is

$$A = \begin{bmatrix} O_{1\times m-2} & 0 \\ I_{m-2} & O_{m-2\times 1} \end{bmatrix},\ B = [1\ 0\ldots 0]^T,\ C = [r_1 \ldots r_{m-1}],\ D = \frac{1}{2}r_0,$$

where $O_{n\times m}$ is an n-by-m zero matrix and I_n is an n-by-n identity matrix. Hence constraint (14) is an LMI in r_k and Q.

Approximate optimization problem

Based on the previous discussion, optimization problem (9) can be approximated by either

$$\underset{r_0,\dots,r_{m-1}}{\text{minimize}}\ f_{cost}(r_0,\dots,r_{m-1}), \tag{15a}$$

$$\text{subject to } (\theta_i-\theta^0)^{\mathrm{T}}\bar{\mathbf{I}}_{\mathrm{F}}(\theta_i-\theta^0)\geq\frac{\chi^2_\alpha(n)\gamma}{N}V_{\mathrm{app}}(\theta_i),\ i=1\dots M, \tag{15b}$$

$$K(Q,\{A,B,C,D\})\succeq 0, \tag{15c}$$

using the scenario approach or

$$\underset{r_0,\dots,r_{m-1}}{\text{minimize}}\ f_{cost}(r_0,\dots,r_{m-1}), \tag{16a}$$

$$\text{subject to } \bar{\mathbf{I}}_{\mathrm{F}}\succeq\chi^2_\alpha(n)\gamma V''_{\mathrm{app}}(\theta^0)/(2N), \tag{16b}$$

$$K(Q,\{A,B,C,D\})\succeq 0, \tag{16c}$$

using the ellipsoidal approximation. The finite dimensional parameterization is used to handle the spectrum constraint in both problem formulations. In addition, the experiment cost is chosen as a convex function with respect to the coefficients in the spectral density function of the input signal. Consequently, the approximate optimization problems (15) and (16) are convex, and there exist efficient and reliable methods for solving such problems [5].

Example 3 revisited: Ellipsoidal approximation

The objective function is set to be the power of the input signal, that is $f_{cost} = r_0$. The Hessian of the application cost is

$$V''_{\text{app}}(\theta^0) = \frac{2\beta^4(b_1^0 + b_2^0)^2}{(\beta b_1^0 - b_2^0 + \beta^2(b_1^0 + b_2^0))^4} \begin{bmatrix} \beta^2 & -\beta \\ -\beta & 1 \end{bmatrix}.$$

Thus, the optimization problem using the ellipsoidal approximation and partial correlation parametrization is

$$\begin{aligned} &\underset{r_0,\ r_1}{\text{minimize}}\ r_0, \\ &\text{subject to } \frac{1}{\lambda}\begin{bmatrix} r_0 & r_1 \\ r_1 & r_0 \end{bmatrix} \succeq \frac{\chi^2_\alpha(n)\gamma}{N} \frac{\beta^4(b_1^0 + b_2^0)^2}{(\beta b_1^0 - b_2^0 + \beta^2(b_1^0 + b_2^0))^4} \begin{bmatrix} \beta^2 & -\beta \\ -\beta & 1 \end{bmatrix}, \\ &\qquad\qquad \begin{bmatrix} r_0 & r_1 \\ r_1 & r_0 \end{bmatrix} \succeq 0. \end{aligned}$$

The problem can be solved analytically since it has 2-by-2 LMIs. The solution is

$$\begin{aligned} r_0^* &= \frac{\lambda\chi^2_\alpha(n)\gamma}{N} \frac{\beta^4(b_1^0 + b_2^0)^2}{(\beta b_1^0 - b_2^0 + \beta^2(b_1^0 + b_2^0))^4}, \\ r_1^* &= -\beta\frac{\lambda\chi^2_\alpha(n)\gamma}{N} \frac{\beta^4(b_1^0 + b_2^0)^2}{(\beta b_1^0 - b_2^0 + \beta^2(b_1^0 + b_2^0))^4}, \end{aligned}$$

see [13]. The corresponding input signal can be generated by the autoregressive process

$$u(t) = -\beta u(t-1) + e(t),$$

where the variance of $\{e(t)\}$ is $((r_0^*)^2 - (r_1^*)^2)/r_0^*$.

This example shows how experiment design relates to the application of the model. For instance, the pole of the autoregressive process, $-\beta$, is the same as the true closed-loop pole of the FIR system.

Input generation

When the input spectrum is found, a corresponding time realization of the signal has to be generated. The realization is then used to excite the system in the identification experiment.

One possible input generation is to let the input signal be white Gaussian noise filtered through a transfer function matrix. The matrix is chosen such that the filtered signal has the required spectrum. The matrix design is a problem of minimum phase spectral factorization and, as such, it has several known solutions. Results applicable to the finite dimensional parameterization are discussed in detail in [14]. For partial correlation parameterizations, an autoregressive filter can be used to generate a signal with the desired properties, as in "Example 3 revisited: Ellipsoidal approximation". The filter is found through the Yule-Walker equations, see [12] and [11].

Another possibility is to use periodic inputs since any partial correlation sequence can be realized with a discrete spectrum. For details, see [15].

Input Design Framework

The attentive reader has noticed that application-oriented input design requires knowledge of the true parameters, but these values are not known a priori. In practice, an initial estimate of the parameters can be used instead of the true values. An estimate can be obtained through, for example, an identification experiment or physical insight of the process. The idea is that the initial estimate does not have to be accurate in comparison to the accuracy necessary to fulfill the application requirements. Thus, the cost of obtaining the initial estimate is only a small part of the cost of the entire procedure of finding a model.

An initial estimate can also be used instead of the true noise covariance matrix Λ_0.

A framework for application-oriented input design is described by IDF in Table III. IDF can be iterated so that, after implementing the framework once, the initial estimate in Step 0 is set to the obtained estimate in Step 4. As more data are used in the identification step and when there exist parameters θ^0 such that $\mathcal{S} = \mathcal{M}(\theta^0)$, the estimate converges to the true values. Therefore, the input design based on an initial estimate is expected to converge to the design based on the true values as IDF is iterated.

To apply the IDF in a sequential manner, as described above, is referred to as *iterative input design*. A related strategy for dealing with the unknown true parameters in the input design is *adaptive input design*.

In adaptive input design, the input signal is adjusted with respect to the current estimate of the unknown true parameters while the system identification experiment is performed. Also here, the input design based on an initial estimate is expected to converge to the design based on the true values as the experiment is running. A discussion on this and a formal proof for autoregressive systems with exogenous inputs are found in [16]. It is shown that the design converges to what would be obtained had the true system been known. Furthermore, it is also shown that the statistical properties of the estimates are asymptotically the same as if the true optimal design had been known.

An alternative approach to the initial estimate is to find input designs that are robust with respect to parameters in a known set. The approach is known as *robust input design* and is described in [17] and [18].

Simulation of Model Based Control Design

A simulation study is made to exemplify and evaluate IDF. The IDF is used to design an MPC for reference tracking of a system. The simulations are performed using the application-oriented input design toolbox MOOSE2.

Linear model

The model of the system is given in state space form as

$$\mathcal{M}(\theta): \quad x(t+1) = \theta_2 x(t) + u(t),$$
$$y(t) = \theta_1 x(t) + e(t).$$

Here $x(t) \in \mathbb{R}$ is the state, $u(t) \in \mathbb{R}$ is the input, $y(t) \in \mathbb{R}$ is the output and $e(t) \in \mathbb{R}$ is zero mean white Gaussian noise with variance λ.

The parameters to be estimated are $\theta = [\theta_1 \ \theta_2]^{\mathrm{T}}$ and their true values are $\theta^0 = [0.6 \ 0.9]^{\mathrm{T}}$. The true system is $\mathcal{S} = \mathcal{M}(\theta^0)$ with λ_0 = 1.

Control strategy

The system is controlled using the MPC defined in (S3). The MPC does not have integral action incorporated. The cost function in (S3a) is set to

$$J(t) = \sum_{i=0}^{5} \|\hat{y}(t+i|t) - r(t+i)\|^2,$$

where both the prediction and control horizons are set to five. The predicted output $\hat{y}(t+i|t)$ is dependent of the model used in MPC. Thus, the quality of the control design is related to the quality of the parameter estimates. Two different constraint sets are considered in the MPC. First, both the input and output are unconstrained. That is,

$$\mathcal{Y} = \mathbb{R}, \ \mathcal{U} = \mathbb{R}, \tag{17}$$

in conditions (S3b) and (S3c), respectively. Second, the amplitude of the input is constrained to be strictly smaller than one. In other words,

$$\mathcal{Y} = \mathbb{R},\ \mathcal{U} = \left\{ u(t) \mid |u(t)| < 1 \right\}. \tag{18}$$

Experiments

IDF is applied to the system both with and without constraints on the input signal.

Initial model

The initial estimate of the parameters are obtained from an identification experiment using PEM and a white Gaussian input signal. The initial estimate is $[0.62\ 0.88]^{\mathrm{T}}$ in the unconstrained case and $[0.69\ 0.89]^{\mathrm{T}}$ in the constrained case. The true variance of the noise is assumed to be known.

Application cost

The objective of the controller is reference tracking. Consequently, the performance of the controller improves when the model is able to predict the true output of the system with higher accuracy. Thus, an application cost that punishes the output error is chosen. That is,

$$V_{\mathrm{app}}(\theta) = \frac{1}{N_{\mathrm{app}}} \sum_{t=1}^{N_{\mathrm{app}}} \| y(t,\theta) - y(t,\theta^0) \|^2, \tag{19}$$

with $N_{\mathrm{app}} = 10$. Here $y(t,\theta)$ and $y(t,\theta^0)$ are the closed-loop output signals using an MPC with a model based on θ and θ^0, respectively.

The application set is

$$\Theta_{\text{app}}(200) = \left\{ \theta \;\middle|\; V_{\text{app}}(\theta) \leq \frac{1}{200} \right\},$$

with $\gamma = 200$. The ellipsoidal approximation of the application set is

$$\mathcal{E}_{\text{app}}(200) = \left\{ \theta \;\middle|\; (\theta - \theta^0)^{\mathrm{T}} V''_{\text{app}}(\theta^0)(\theta - \theta^0) \leq \frac{1}{100} \right\},$$

in accordance to definition (10). The constrained optimization problem (S3) in MPC lacks a closed-form solution. Consequently, there is no closed-form expression for the application cost (19) and a numerical evaluation of its Hessian is necessary.

The level curves together with the ellipsoidal approximation of the application set and uniformly distributed scenarios over $\Theta_{\text{app}}(200)$ are shown in Figure 4 and 5 for the constraint sets (17) and (18), respectively. The level curve of $V_{\text{app}}(\theta)$ equal to $1/200$ is closer to the ellipsoidal approximation in the unconstrained case than in the constrained case. Imposing constraints on the input enlarges the application set, reflecting the fact that an input-constrained MPC is more robust to model errors than an unconstrained MPC. Note that, in these figures, the application set and its approximations are based on θ^0 and not the initial estimate.

Optimal spectrum

The optimal input spectrum is calculated with the probability α set to 0.99, the experiment length equal to 400 samples and the experimental cost equal to input power. The optimal input spectra based on both the initial estimate and θ^0 are shown in Figure

6 for the unconstrained system and in Figure 7 for the constrained system. The spectrum based on the initial estimate is approximately the same as the spectrum based on θ^0 for both cases.

Optimal input signal

An optimal input signal with the desired spectrum is generated by filtering white Gaussian noise through a transfer function matrix as briefly discussed in section "Input generation".

Estimate

To evaluate IDF, one hundred identification experiments are performed. Each experiment uses a new realization of the input signal. Only input signals corresponding to the design based on the initial estimate are used. For comparison, one hundred identification experiments are performed using white Gaussian noise of equal power to the optimal input signal for both the unconstrained and constrained case. The resulting one hundred estimates of the true parameters using IDF and white Gaussian noise respectively, system identification set, and application ellipsoid are presented in Figure 8 for constraint set (17) and in Figure 9 for constraint set (18). In Figure 8, the application ellipsoid from the initial estimate is similar in shape and size to the true application ellipsoid. In Figure 9, the shape and size are no longer similar. However, the limiting direction of the application ellipsoid from the initial estimate coincides and has the same length as in the true application ellipsoid. The direction is limiting in the sense that the application set prevents the system identification set to grow any further in that direction. Consequently, even though the application ellipsoids are based on the initial estimate they succeed to capture the important directions of the true application ellipsoids and can therefore be used in the input design.

The estimates using IDF are less spread out than the estimates using white Gaussian noise in both the unconstrained and constrained case. All of the estimates using IDF lie inside the true application ellipsoid in both cases. Only 79 and 80 of the estimates using white Gaussian noise do so for the unconstrained and constrained case, respectively.

The resulting application costs when using an optimal input signal and a white Gaussian input signal in the identification experiments are shown in Figure 10 for the unconstrained system and in Figure 11 for the constrained system. Also, the resulting output signals when evaluating the application costs are shown in Figure 12 and 13. The models obtained using IDF outperform the models found using white Gaussian noise. The application cost requirement is fulfilled for 100 % of the models from IDF in both the unconstrained and constrained case. The percentages for models based on white Gaussian noise are 78 % for the unconstrained case and 74 % for the constrained case.

Summary

The use of an initial estimate instead of the true parameter values do not affect the input designs notably. The limiting directions coincide and have similar lengths, see Figure 8 and 9. Consequently, the input spectra based on the initial estimates are approximately the same as the input spectra based on the true parameter values.

The scenario approach and ellipsoidal approximation manage to approximate the application sets with high accuracy. Naturally, the former is better than the latter at approximating non-ellipsoidal application sets, see Figure 9.

The IDF outperformed the use of white Gaussian noise in terms of fulfilling the requirement on the application cost, see Figures 10-13. In fact, 28 % more models based on IDF than white noise fulfilled the application requirement in the unconstrained case, and 35 % more in the constrained case.

Water Tanks Experiment

The identification experiments on the water tanks process in section “Motivating Experiment” are revisited in more detail.

Linear model

A nonlinear, continuous time model of the process can be derived from Torricelli’s principle. However, standard MPC uses a linear, discrete time model. Thus, the nonlinear model is linearized around a working point and then discretized using zero order hold sampling at a sampling rate of 1 Hz. The obtained model to be used in MPC is

$$\mathcal{M}(\theta): \quad x(t+1) = A(\theta)x(t) + B(\theta)u(t) + Ke(t), \tag{20a}$$

$$y(t) = C(\theta)x(t) + e(t), \tag{20b}$$

with

$$A = e^{A_c}, \ B = \int_0^1 e^{A_c(1-t)} B_c dt,$$

and

$$A_c = \begin{bmatrix} -\tau_1 & 0 & \tau_3 & 0 \\ 0 & -\tau_2 & 0 & \tau_4 \\ 0 & 0 & -\tau_3 & 0 \\ 0 & 0 & 0 & -\tau_4 \end{bmatrix}, \ B_c = \begin{bmatrix} \frac{k_1\gamma_1}{A} & 0 \\ 0 & \frac{k_2\gamma_2}{A} \\ 0 & \frac{k_2(1-\gamma_2)}{A} \\ \frac{k_1(1-\gamma_1)}{A} & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} l_1 & 0 & 0 & 0 \\ 0 & l_2 & 0 & 0 \end{bmatrix}, \ \tau_i = \frac{a_i}{A}\sqrt{\frac{g}{2{x_i}^o}}.$$

The model has innovations form. The state vector is $x(t) = [x_1(t) \ x_2(t) \ x_3(t) \ x_4(t)]^{\mathrm{T}}$. The component x_i is the deviation of the water level of tank i from the working point x_i^o expressed in centimeters. The input signal is $u(t) = [u_1(t) \ u_2(t)]^{\mathrm{T}}$. The component u_j is the deviation of the voltage of pump j from the working point expressed in volts. The output signal is $y(t) = [y_1(t) \ y_2(t)]^{\mathrm{T}}$, where y_i is the pressure in volts of tank i. The measurement noise $e(t) = [e_1(t) \ e_2(t)]^{\mathrm{T}}$ is assumed to be white, zero mean Gaussian with covariance matrix Λ. The states at time t can be estimated from the measurement $y(t)$ using a Kalman filter.

The physical meaning of the parameters in model (20) is described in Table IV. Due to the illustrative options available when only estimating two parameters, the parameters to be estimated in the first set of identification experiments (Experiment 1) are $\theta = [\gamma_1 \ k_1]^{\mathrm{T}}$. All other parameters are assumed to be known, including the matrices K and Λ_0. However, in Section “Additional experiments”, eight parameters are estimated instead of only two.

Control Strategy

The objective of the controller is to perform reference tracking of the water levels in the two lower tanks. The controller implemented is the MPC defined in (S3). The MPC has incorporated integral action. The cost function is

$$J(t) = \sum_{i=0}^{10} \|\hat{y}(t+i|t) - r(t+i)\|_Q^2 + \sum_{i=0}^{10} \|\hat{u}(t+i+2|t) - \hat{u}(t+i+1|t)\|_R^2,$$

where both the prediction and control horizon are set to 10, $Q = 100I_2$ and $R = 20I_2$. The system outputs are constrained by the physical limitations of the tanks. That is, the tanks are not allowed to overflow. The inputs are constrained to never go below 0.5 volts, to ensure that there is water in all connecting tubes. If a tube is drained, an additional time delay is present in the system, which is not captured by the chosen model. The output signals are low-pass filtered before sent to the MPC.

Experiments

IDF is applied to the water tanks process to estimate $\theta = [\gamma_1 \ k_1]^{\mathrm{T}}$.

Initial model

The initial estimates are taken from the water tanks process specification, $\theta^{\text{init}} = [0.7\ 1.6]^{\mathrm{T}}$. A check is made to ensure that the initial estimates do not already fulfill the application requirement.

To evaluate how the strategy of using initial estimates performs, the resulting input design is compared with the result obtained when the true values are used. In real experiments, the true parameters are unknown. Thus, a long identification experiment is performed on the process to obtain high quality estimates. These estimates are then used as an approximation of the true values. That is, θ^0 is replaced by high quality estimates, $\theta^0 \approx [0.72\ 2.32]^{\mathrm{T}}$.

The matrix K and covariance matrix Λ_0 are also given by high quality estimates from the long identification experiment.

The true values of all parameters, the matrix K and the covariance matrix Λ_0 are given in Table VI. Two sets of values are given, one when K is estimated and one when K is assumed to be zero. The latter is used in Section “Additional experiments”.

Application cost

Both the scenario approach and the ellipsoidal approximation require evaluations of the application cost. However, it is unlikely that the application cost can be evaluated using outputs from a process as required in the scenario approach or ellipsoidal approximation. Such an evaluation means controlling the process based on models with more or less arbitrary parameter values. Instead, the evaluation is performed in simulation where the true process is represented by a linear model. For this purpose, an approximate application cost is used. The approximate cost is denoted $\tilde{V}_{\text{app}}(\theta)$. The approximation is evaluated in simulation using outputs from the linear model of the process, where the parameters of the linear model are set to the initial estimate. To obtain an application-oriented input design using the approximate application cost, $V_{\text{app}}(\theta)$ is replaced by $\tilde{V}_{\text{app}}(\theta)$ in the relevant expressions. For examples of approximate application costs and their evaluations, see [19].

The application cost chosen here is the same as in (19) but with $N_{\text{app}} = 150$. However, as discussed, the application cost is evaluated in simulation instead of on the real process.

The value of γ is chosen such that a degradation of 1 % of $y(\theta)$ with respect to $y(\theta^0)$ is acceptable. That is,

$$\gamma = \frac{100}{V(\theta_0)},$$

with

$$V(\theta_0) = \frac{1}{150}\sum_{t=1}^{150} \|y(t,\theta^0) - r(t)\|^2.$$

Also here, $V(\theta_0)$ is evaluated in simulation using the linear initial model instead of the real process. The resulting γ is 21468.

The ellipsoidal approximation is used to approximate the application set in the input design. As in section "Simulation of Model Based Control Design", the Hessian necessary for the ellipsoidal approximation is evaluated numerically.

Optimal spectrum

The optimal input spectrum is calculated with the probability α set to 0.95, the experimental length set to 300 samples and the experimental cost set to the power of the input signal used in the identification experiments.

The optimal input spectra based on both the initial estimate and θ^0 are shown in Figure 14. The spectra have similar shape and attenuate high frequency content. The spectra turn out to give an optimal u_2 that remains at its steady state level. The design is highly intuitive, since the only way for the output signals to be influenced by γ_1 and k_1 are by varying u_1. Also, by not varying u_2, the optimal input signal effectively hides parts of the system that are unimportant for the identified model to fulfill the application requirement.

As a comparison, a check is made of how long the experiment length must be to fulfill the application requirement with probability α when using white noise of equal power to the optimal input signal. (The power is divided equally between the two input channels of the process.) The identification experiment must then be 7244 samples long. That is, approximately 24 times longer than the experiments using IDF.

Optimal input signal

An optimal input signal with the desired spectrum is generated the same way as in section "Simulation of Model Based Control Design".

Estimate

To evaluate the performance of IDF, twenty identification experiments are made on the water tanks process using an optimal input signal and a white Gaussian noise of equal power divided equally between the two input channels of the process. The resulting twenty estimates of the true parameters using IDF and white Gaussian noise respectively, system identification set, and application ellipsoid are shown in Figure 15. As in Figure 9, the limiting direction of the application ellipsoid from the initial estimate coincides and has the same length as in the true application ellipsoid.

The application performances are evaluated using the models obtained in the MPC controlling the water tanks process. The trajectories of the process, along with the references, are shown i Figure 1. The models from IDF outperform the models found using white Gaussian noise, in terms of reference tracking. The difference in performance is more evident in tank 2 than in tank 1.

To quantify the application performance, the application cost is evaluated using the output signals from the process. The distribution of the cost is shown in Figure 16. White Gaussian noise gives, on average, a larger application cost than IDF. The averaged application cost obtained using white Gaussian noise is four times larger than the averaged application cost using IDF. However, none of the costs evaluated are lower than the acceptable performance degradation $1/\gamma$. The discrepancy is due to the model structure not fully capturing the true system, the second order approximation of the application cost not matching the true application cost, the use of an initial estimate in the design and the presence of noise.

Additional experiments

For additional comparison, Experiment 1 is repeated but with two different settings. In both settings eight parameters are estimated instead of only two. That is, $\theta = [a_1,\ a_2,\ a_3,\ a_4,\ \gamma_1,\ \gamma_2,\ k_1,\ k_2]^{\mathrm{T}}$. The initial estimate is once again taken from the process specification and is $\theta = [0.03,\ 0.03,\ 0.03,\ 0.03,\ 0.7,\ 0.7,\ 1.6,\ 1.6]^{\mathrm{T}}$. In the first setting (Experiment 2), the same MPC as before is used and twenty experiments are made. In the second setting (Experiment 3), the process noise is assumed to be zero ($K = 0$), the MPC has no integral action and only ten experiments are made.

In Experiment 2, nineteen of the estimates using IDF and five of the estimates using white Gaussian noise of equal power divided equally between the two input channels of the process lie inside the true application ellipsoid. In Experiment 3, where only ten experiments are made, four of the estimates using IDF and none of the estimates using white Gaussian noise lie inside the true application ellipsoid.

The resulting trajectories of the process for the two settings are shown in Figure 17 and Figure 18, respectively. The difference in performance between using IDF and white Gaussian noise is more evident in Experiment 3 than in any of the other experiments. The difference in performance is mainly due to the lack of integral action in the MPC. A controller with no integral action needs a more accurate estimate of a system's steady state than a controller with integral action. The IDF is able to design the input signal in such a way, that a better estimate of the steady state is obtained than when using white Gaussian noise of equal power.

Summary

The input design based on the initial estimate do differ from the design based on the true parameter values. In Experiment 1, where two parameters are estimated, the input power is 0.009 V^2 in the former case and 0.004 V^2 in the latter. However, the optimal input spectra have the same low-pass characteristics and approximately the same level of magnitude, see Figure 14.

The benefit of using IDF over white Gaussian noise is larger when MPC without integral action is used than MPC with integral action. An advanced controller can compensate more for discrepancies between models and systems and achieve a better performance than a less advanced controller.

In the experiments made, the MPC based on models obtained from white Gaussian noise had a more varying input signal than MPC based on models obtained from IDF. Meaning, the model error varied more in the former case than in the latter. As a result, a less cohesive behavior of the MPC was obtained when models based on white noise were used instead of models based on IDF.

In general, IDF gives better application performance than the approach of using white Gaussian noise. The averaged application cost using white Gaussian noise is four, two and six times larger than the averaged cost using IDF in Experiment 1, 2 and 3, respectively. However, the application requirement evaluated on the real process is not fulfilled in any of the experiments. In fact, due to the discrepancy between the system used in the design and the real process and the presence of noise, the requirement is likely to never be fulfilled. When applying IDF in real-life, the application set is more of a design tool to achieve the best possible application performance than a classification tool to deem models acceptable or not.

Also note that even though IDF does not always outperform white Gaussian noise by much, see for example Experiment 2. IDF does provide an approximate lower bound on the power of the input signal to use, given the allowed length of the experiment. Without IDF, the input power has to be determined in some other way. In addition, it could happen that the optimal input spectrum (although not flat) is approximately the same as the spectrum of the white Gaussian noise obtained by the random number generator used (for example `randn` in MATLAB) – since ideal white Gaussian noise can not be realized and a band-limited white Gaussian noise is provided instead. In such cases, IDF and white Gaussian noise give similar performance.

Challenges with Industrial Application

Several challenges arise when adapting IDF to the realities of industrial processes. Three central issues for the application-oriented input design framework are identified and discussed.

First, to define an application cost for an industrial process is difficult. The cost must reflect the actual cost related to production. The resulting control objective for MPC in a process application then entails pushing one or several signals toward their bounds while maintaining other signals within their specifications. A translation of the MPC objective to a performance degradation measure is by no means straightforward, see for example [20] and the references therein.

Second, the identification experiment is ideally performed during regular production. Therefore, the identification experiment is not allowed to interfere with normal operation of the plant. However, it is not enough to consider only the normal operation of the process to be identified. An excited input or the resulting output of the identified process can influence other processes further down or up in the production chain. Consequently, a much wider scope of the application concept than just the plant at hand must be captured by the application-oriented input design.

The final issue also relates to conducting the identification experiment during normal operation of the process. To maintain production, closed-loops are not to be broken. Instead, the application-oriented input design has to be incorporated into the loop. The goal of the controller becomes twofold: to ensure acceptable control performance and to generate informative data for identification. Three strategies for closed-loop application-oriented input design are experiment design in closed-loop, stealth identification and MPC-X presented in [14], [21] and [22], respectively. In experiment design in closed-loop, the framework presented here is used to design the reference signal to the closed-loop system instead of the input signal to the open-loop system. In stealth identification, the application-oriented identification experiment is constructed in such a way that it is not noticed by the controller. In other words, it is performed in an open-loop fashion. In MPC-X, the constraint on the system identification and application set is added to the MPC formulation. Consequently, the MPC provides an input signal designed both for controlling and identifying the plant. For a discussion and analysis of the challenges of MPC-X, see [23] where it is applied to an industrial distillation column.

The three issues addressed above are complex open questions in the framework of application-oriented input design. They require more elaborated tools to be developed and more industrial validation cases to be performed than what are available in the literature. However, initial attempts have been made to apply the framework on industrial processes.

The Grander Scope of Experiment Design

In this article the starting point is that it is known that the true system belongs to the used model set, that is $\mathcal{S} = \mathcal{M}(\theta^0)$. This means that in the experiment design problems (15) and (16) only the noise induced error is taken into account. Anyone who has used system identification in practice knows that one of the major issues is to determine a suitable model structure; unleashing too many degrees of freedom results in a noise sensitive estimate while using too few may give rise to unacceptable systematic errors. Application-oriented input design alleviates this problem. To understand the mechanisms the following simplified setting is studied.

Assume that the application cost $V_{\text{app}}(\theta)$ is quadratic in the error $\theta - \theta^0$ and that the parametrization has been chosen such that its Hessian is diagonal so that

$$V_{\text{app}}(\theta) = \sum_{k=1}^{n} \sigma_k (\theta_k^0 - \theta_k)^2 \tag{21}$$

for some $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n \geq 0$.

Let the system be of FIR type

$$y(t) = \varphi^T(t)\theta^0 + e(t), \quad \varphi^T(t) = \begin{bmatrix} u(t-1) & \ldots & u(t-n) \end{bmatrix},$$

where $\{e(t)\}$ is zero mean Gaussian white noise with variance λ. The averaged Fisher information matrix (S12) for this model is given by

$$\bar{\mathbf{I}}_{\text{F}} = \lim_{N\to\infty} \frac{1}{N\lambda} \sum_{t=1}^{N} \mathrm{E}\left\{\varphi(t)\varphi^T(t)\right\} \approx \frac{1}{N\lambda} \sum_{t=1}^{N} \varphi(t)\varphi^T(t) \tag{22}$$

Now consider the approximate experiment design problem (16). For well posed problems, there is a cost associated with making the averaged Fisher information matrix large. Thus ideally at the optimum the inequality constraint (16b) becomes an equality. In such a case

$$\bar{\mathbf{I}}_{\mathrm{F}} = \frac{\alpha}{\lambda}\mathrm{Diag}(\sigma_1, \dots, \sigma_n) \tag{23}$$

where $\alpha = \chi^2_\alpha(n)\gamma\lambda/N$. Note, however, that the equality can only occur if $\sigma_1 = \dots = \sigma_n$. Consider the identification criterion (S6) when (23) holds. Using (22) and (23) gives

$$\begin{aligned} V_N(\theta, Z^N) &= \frac{1}{2N}\sum_{t=1}^{N}(y(t) - \varphi^T(t)\theta)^2 \\ &\approx \frac{\alpha}{2}V_{\mathrm{app}}(\theta) + \frac{1}{2N}\sum_{t=1}^{N} e^2(t) - \sum_{k=1}^{n}\varepsilon_k(\theta_k^0 - \theta_k) \end{aligned} \tag{24}$$

where ε_k is the kth element of $\frac{1}{N}\sum_{t=1}^{N} e(t)\varphi(t)$. There is thus a strong connection between the identification criterion and the application cost: If it was not for the last term, the identified model would minimize the application cost. In fact, this nuisance term disappears as the sample size N grows since, due to (22) and (23), $\{\varepsilon_k\}$ are (approximately) independent Gaussian random variables with zero mean and variance $\lambda\alpha\sigma_k/N$.

The strong link between the application cost and the identification criterion that is obtained in application-oriented input design is the reason for why the model structure selection problem is alleviated. Suppose that only the first $\bar{n}$ parameters influence the application cost, that is that $\sigma_{\bar{n}+1} = \dots = \sigma_n = 0$. Then, in the idealized case, the corresponding elements of the averaged Fisher information matrix are zero, that is the data do not contain any information about these parameters. The interpretation of this is that application-oriented input design tries to avoid exciting system properties that are of no consequence for the application, that is it tries to make the system behave in as simple way as possible while still exhibiting the properties of importance for the application. A direct consequence of this is that it is possible to use models of restricted complexity; in the idealized case the last $n - \bar{n}$ parameters can be omitted. On the contrary, for arbitrary excitation it is very difficult to control which system properties that a model of restricted complexity picks up. Notice that there is no penalty for using a more complex model structure, as the excess parameters do not influence the application cost.

In relation to this discussion, there is one more important issue to be highlighted. Notice that the ideal averaged Fisher information matrix (23) is proportional to α, which in turn is proportional to $\chi^2_\alpha(n)/N$. Since the magnitude of the averaged Fisher information matrix is proportional to the input power, this means that the input power is proportional to this ratio. In other words the required input energy is proportional to $\chi^2_\alpha(n)$. Now this constant is of the order n, which in this article denotes the number of parameters in a full order model. This suggests that large input energy is required in application-oriented input design for complex systems, implying either very large amplitudes in the excitation or very long experiments. However, this is counter to practical experience. Engineering abound with examples where complex phenomena have been adequately modeled based on relatively simple experiments. For example, simple bump test experiments are standard procedure in the petrochemical industry for obtaining models used in MPC. To resolve this apparent paradox a few steps of analysis are needed. Minimizing the right-most expression in (24) with respect to θ_k gives (recall that $V_{\text{app}}(\theta)$ is quadratic in θ)

$$\theta_k^0 - \theta_k = \frac{1}{\alpha \sigma_k}\, \varepsilon_k$$

The achieved application cost is obtained by inserting this expression into (21), giving

$$V_{\mathrm{app}}(\theta) = \sum_{k=1}^{n} \frac{1}{\alpha^2 \sigma_k} \varepsilon_k^2 \tag{25}$$

Each term in (25) contributes with the same amount $\lambda/(N\alpha) = 1/(\chi^2_\alpha(n)\gamma)$ to the average application cost $\mathrm{E}\{V_{\mathrm{app}}(\theta)\}$, giving an average application cost of $n/(\chi^2_\alpha(n)\gamma) \approx 1/\gamma$ since $\chi^2_\alpha(n) \sim n$. Suppose now that, as also discussed above, only the first $\bar{n}$ parameters influence V_{app}. This means that there are only $\bar{n}$ terms in (25) implying that an average application cost of the order $1/\gamma$ can be obtained by replacing the factor $\chi^2_\alpha(n)$ by $\chi^2_\alpha(\bar{n})$, thus reducing the required energy of the experiment. In practice, there may be many parameters that only have minor impact on the application cost, that is the corresponding terms in (21) are small. By setting these parameters to zero in the application, the experimental effort can be further reduced. In summary, the required experimental effort is more related to the required quality of the model than the complexity of the system that is being modeled. An in-depth discussion of this is provided in [24] and [25], [26].

Often equality cannot be achieved in (16b) due to the restrictions that exist in how the averaged Fisher information matrix can be shaped. However, much of the principles discussed above carry over to this situation as well. See [27] and [24] for details.

A key issue in experiment design is that the optimal design depends on the unknown true parameter vector. In the examples in Sections "Simulation of Model Based Control Design" and "Water Tanks Experiments" an initial model is used. A natural extension is to use adaptive input design whereby the application-oriented input design problem is solved on-line as the model is updated. It has been shown that in the long run, that is as the sample size grows, this type of adaptive procedure achieves the same accuracy as if the true parameters were used in the design [28], [29], [30]. It is also worth to note that this type of procedure can provide consistent estimates of certain system properties of interest even though models of restricted complexity are used. One such example is the estimation of non-minimum phase (NMP) zeros. In [31] it is shown that for a system of arbitrary order having a single NMP zero, an FIR model with only two parameters suffices to consistently estimate the NMP zero if adaptive optimal experiment design is used. A remarkable result which is due to the robustness properties of optimal experiment design discussed above.

Summary

A method for designing experiments to be used in model-based control design was presented. The method is called application-oriented input design. A framework called IDF, which is based on application-oriented input design, was also presented. IDF is used to estimate models with the intended application in mind. The framework was illustrated in simulation and experimental studies. In the simulation study, the MATLAB-based identification toolbox MOOSE2 was used. MOOSE2 provides a simple way of formalizing and solving application-oriented input design problems. The examples discussed in this article show that much can be gained in terms of experimental cost by taking the intended application of the model into account when designing experiments.

Three challenges with industrial applications of IDF were highlighted. They are open questions in the front line of application-oriented input design research.

As a final note, the grander scope of experiment design was discussed and a motivation for some of the assumptions made in IDF, such as the true system belongs to the used model set and the use of an initial estimate instead of the true parameters, was provided.

Acknowledgment

The research leading up to this work was partly funded by the Swedish Research Council under contract 621-2009-4017, the European Union's Seventh Framework Programme (FP7/2007-2013) under grant agreement n° 257059 (Autoprofit), and the European Research Council under the advanced grant LEARN, contract 267381. The author's would like to thank the Department of Automatic Control, LTH, Lund University, Sweden; where all the experiments took place.

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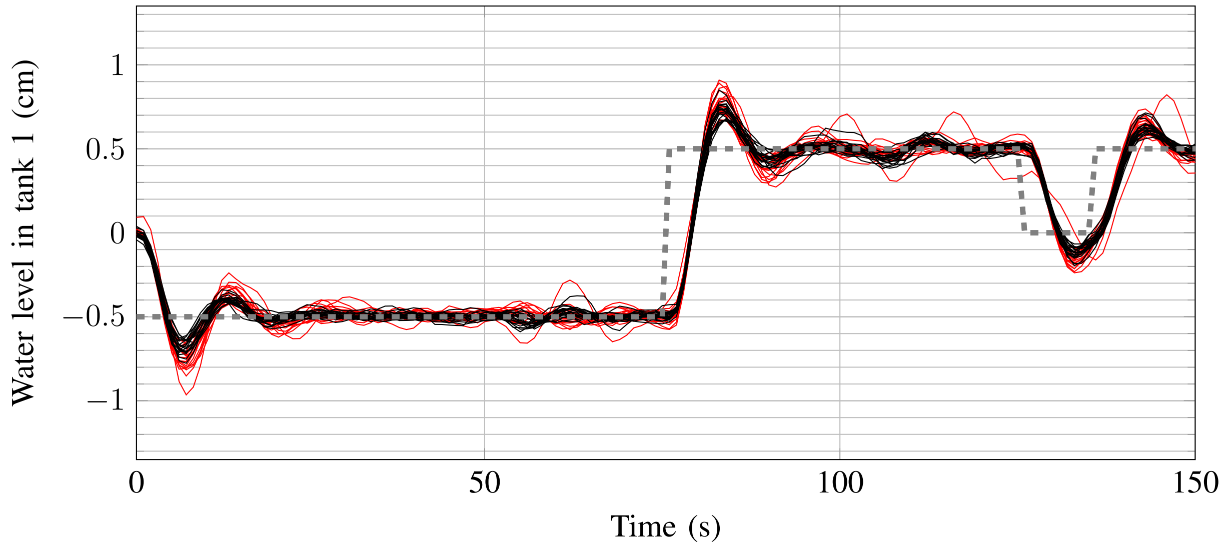
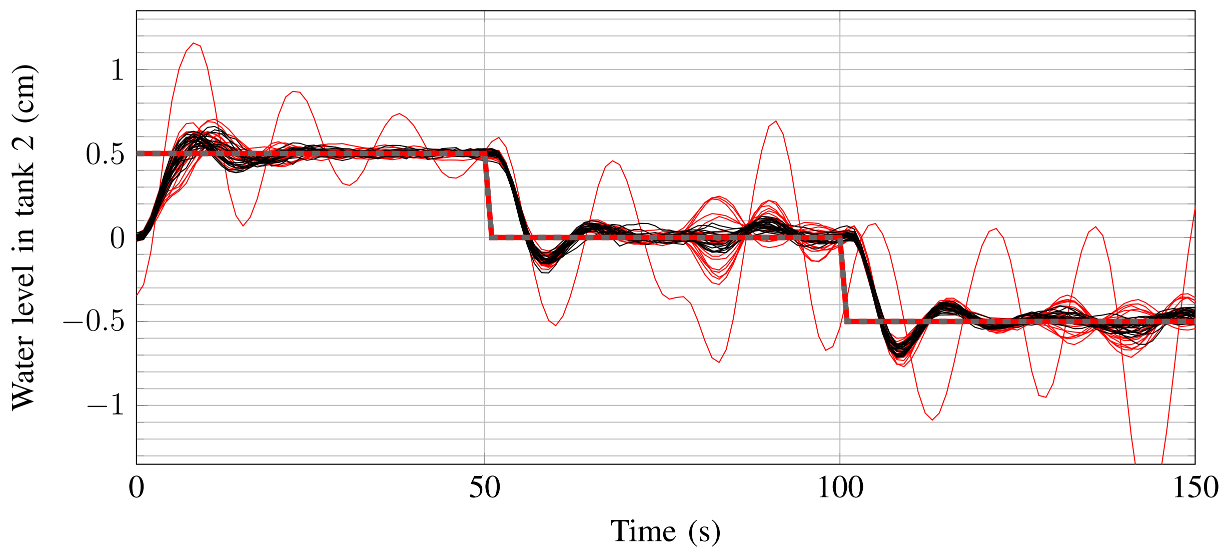
Figures



(a)



(b)

Figure 1. Comparison of application performances in Experiment 1 in "Water Tanks Experiment". The trajectories of the process controlled by MPC with models based on estimates from twenty identification experiments are shown. The trajectories obtained with models from IDF are denoted (——) and the ones obtained from using white Gaussian noise are denoted (——). The desired reference trajectories (- - -) are also plotted. The water levels are shown as deviations from the working points.

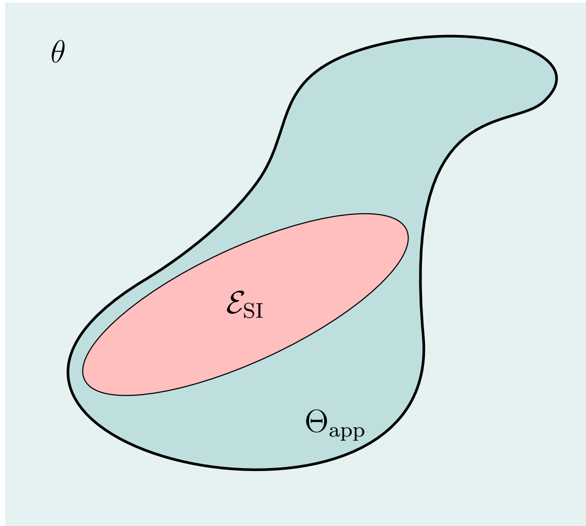
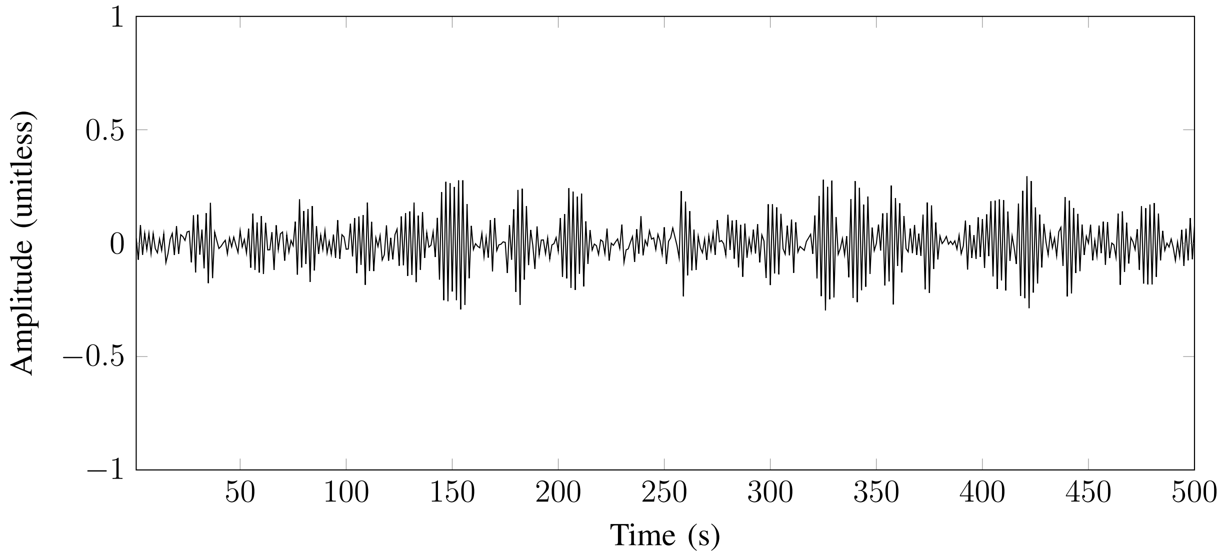
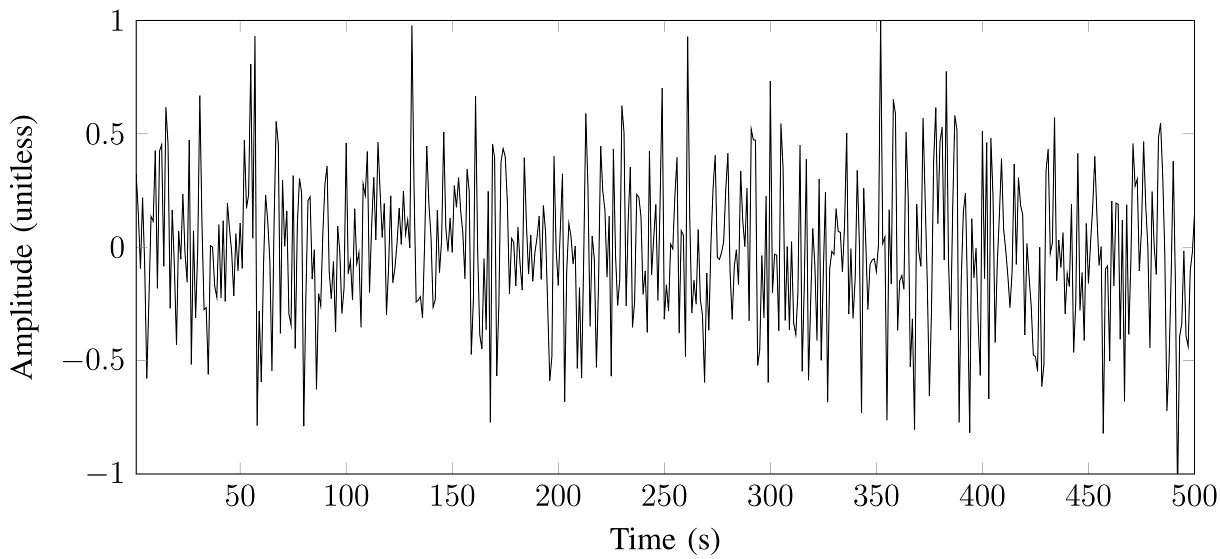


Figure 2. Application-oriented input design concept. In the set of possible parameters θ, the parameters that give an acceptable application performance constitute the set Θ_{app}. The prediction error method gives estimates inside $\mathcal{E}_{\mathrm{SI}}$ with high probability. The purpose of application-oriented design is to make sure that $\mathcal{E}_{\mathrm{SI}}$ lies completely inside Θ_{app}.



(a)



(b)

Figure 3. Comparison of input signals. Resulting input signals for the identification experiment in "Example 2: Acceptable application performance" are displayed. Plot (a) shows the optimal input signal found using IDF. Plot (b) shows a white Gaussian input signal. The variance of the white signal is chosen such that the same accuracy of the estimates is guaranteed with the same probability as for the optimal input signal. The power of the white signal is approximately seven times larger than for the optimal input signal.

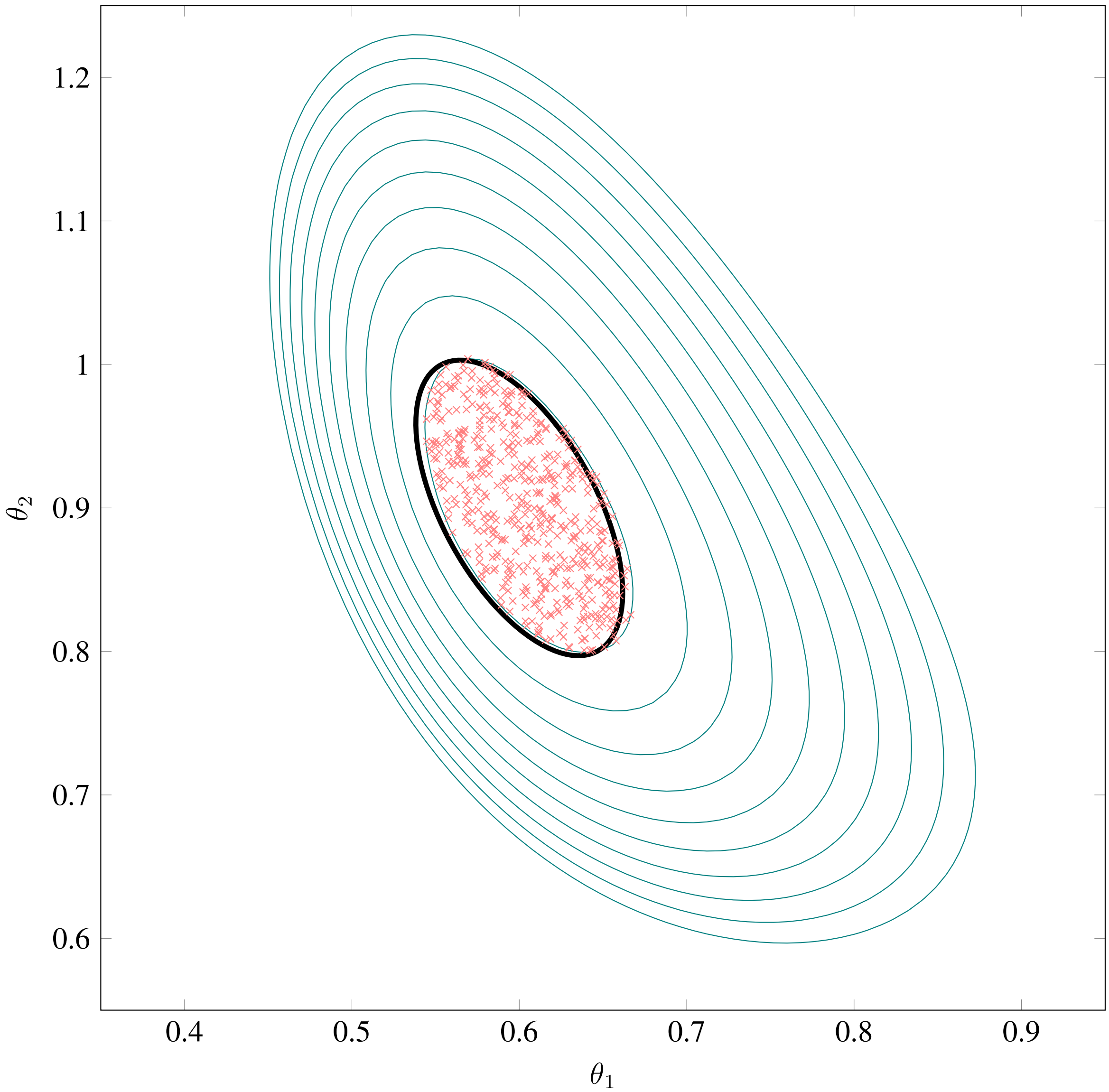


Figure 4. Application set with unconstrained input. Level curves (——) for the application cost of the system in "Simulation of Model Based Control Design" are shown. The innermost level curve corresponds to the required accuracy $1/\gamma$. The ellipsoidal approximation (▬) of the same application cost is also shown along with 539 scenarios (×). Both the ellipsoidal approximation and the scenario approach capture the true application set with high accuracy. Note that, the application set and its approximations plotted here are based on θ^0.

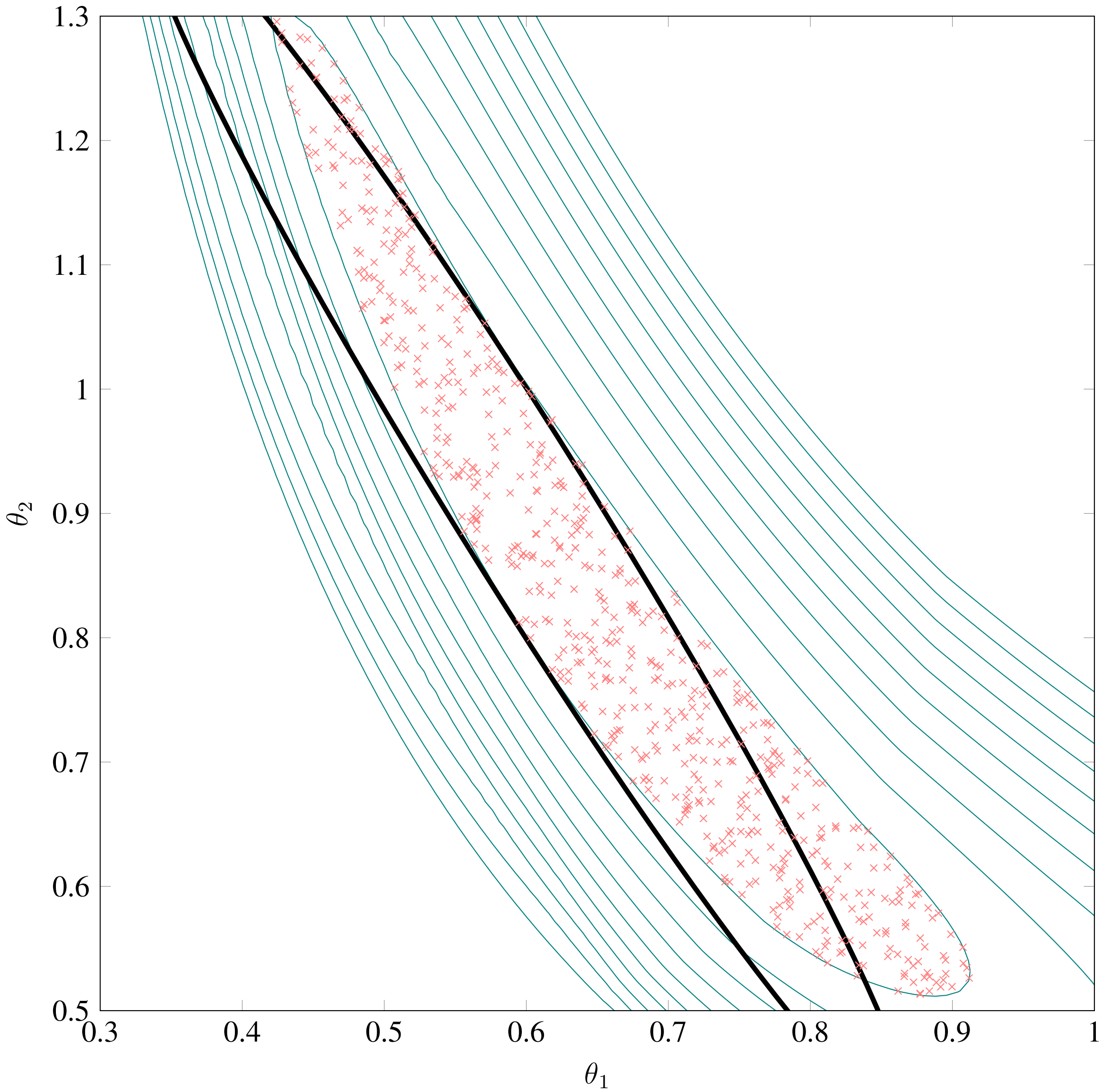


Figure 5. Application set with constrained input. Level curves (——) for the application cost of the system in "Simulation of Model Based Control Design" are shown. The innermost level curve corresponds to the required accuracy $1/\gamma$. The ellipsoidal approximation (——) of the same application cost is also shown along with 612 scenarios (×). The scenario approach is better at capturing the true application set than the ellipsoidal approximation. Note that, the application set and its approximations plotted here are based on θ^0.

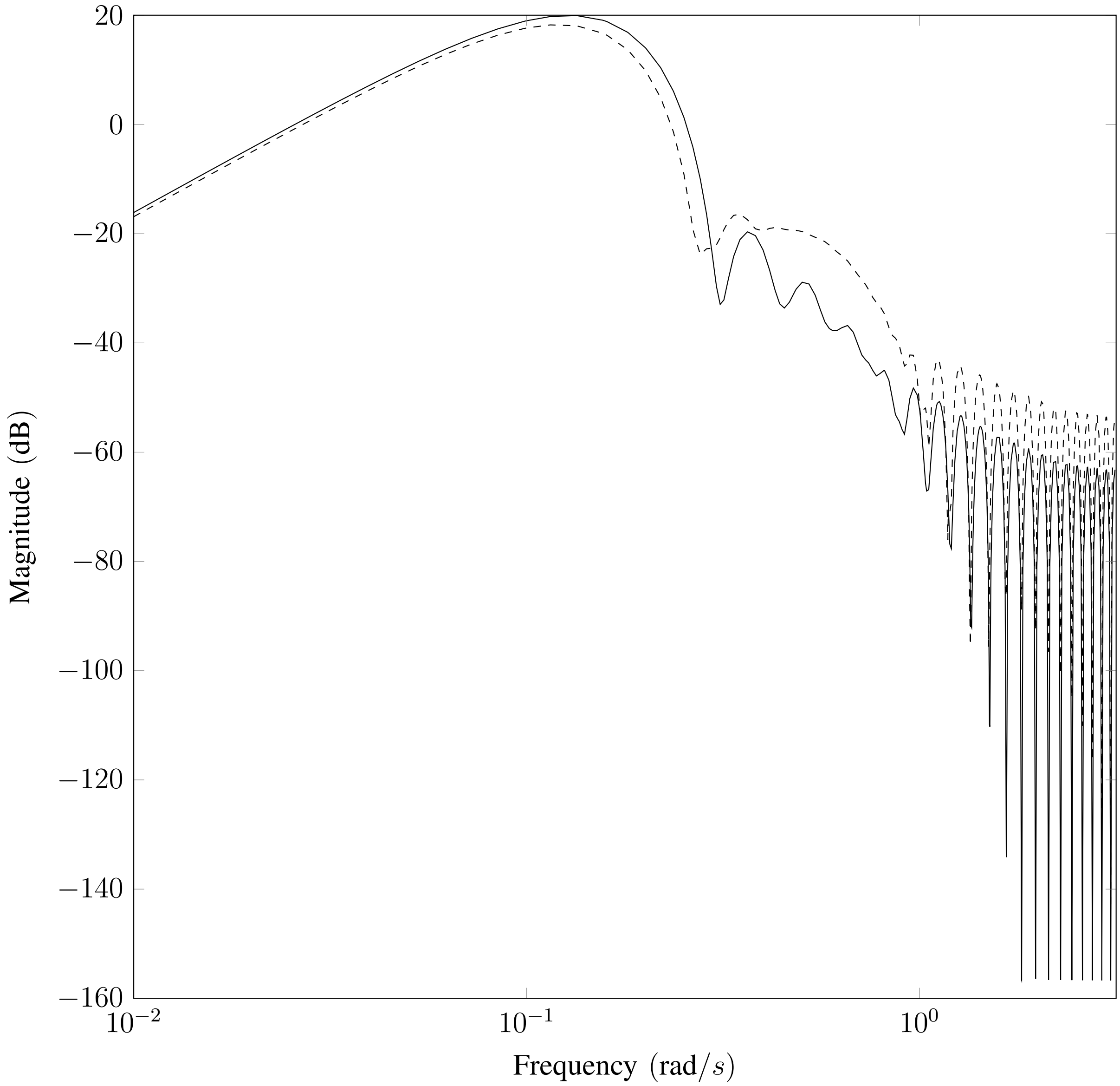


Figure 6. Input spectra with unconstrained input. Resulting input spectra for the unconstrained system in "Simulation of Model Based Control Design" using the ellipsoidal relaxation are shown. The input spectrum from the initial estimate (——) resembles the true input spectrum (- - -).

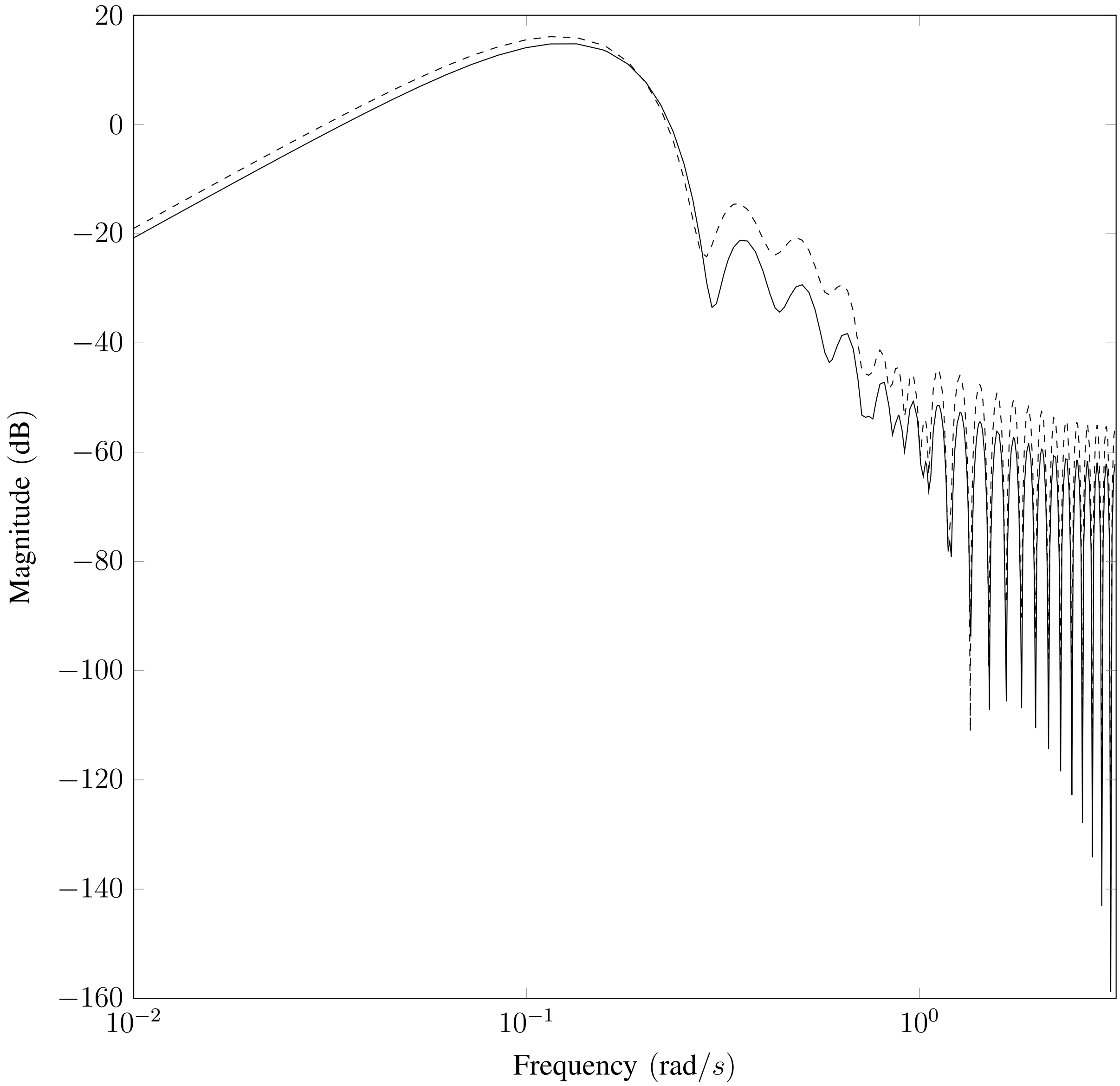


Figure 7. Input spectra with constrained input. Resulting input spectra for the constrained system in "Simulation of Model Based Control Design" using the ellipsoidal relaxation are shown. The input spectrum from the initial estimate (——) resembles the true input spectrum (- - -).

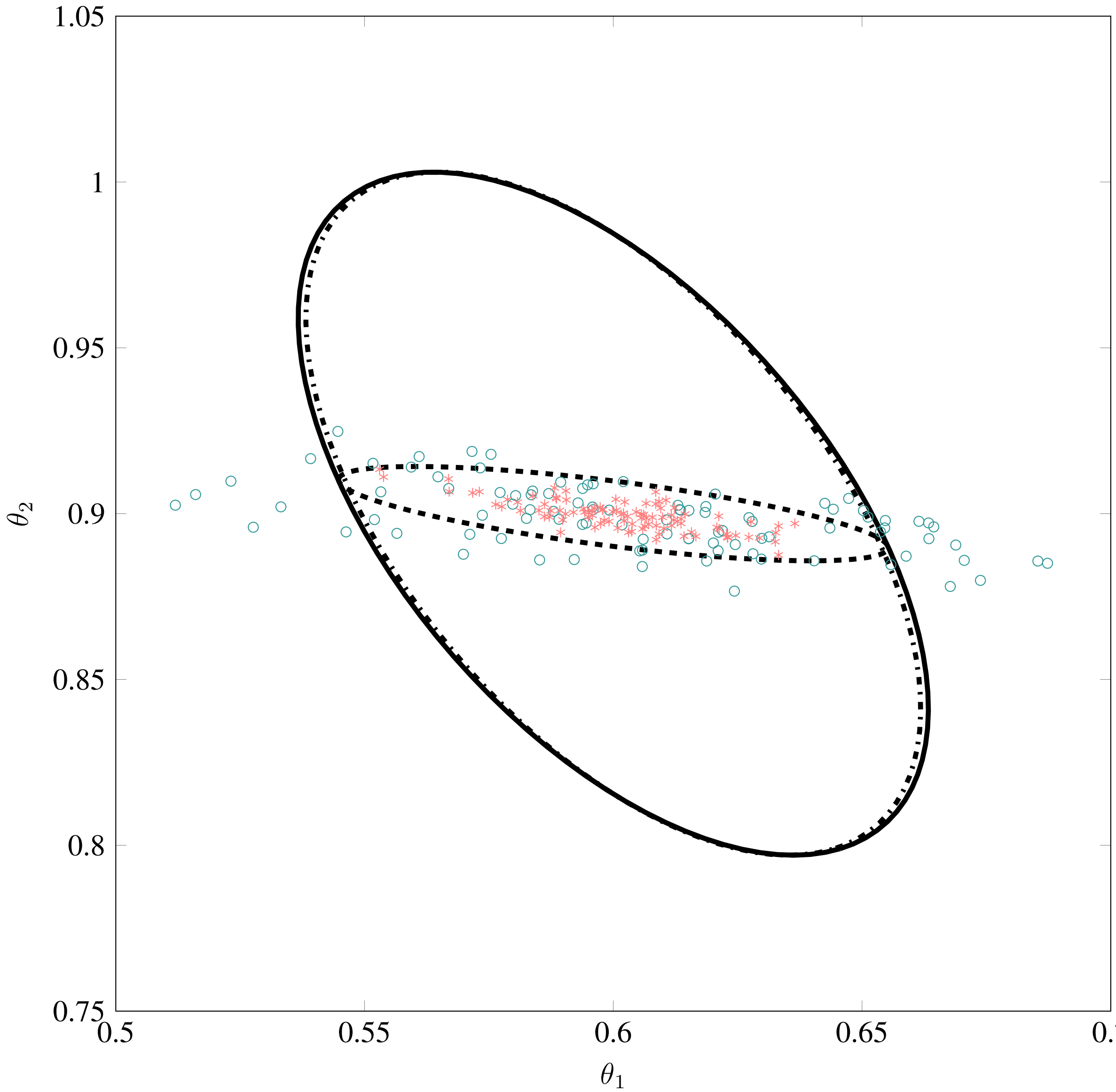


Figure 8. Application-oriented input design with unconstrained input. Resulting parameter sets for the unconstrained system in "Simulation of Model Based Control Design" using the ellipsoidal relaxation are shown. The application ellipsoid from the initial estimate (——) is similar in shape and size to the true application ellipse (·-·-). The system identification set (- - -) is almost contained within the true application ellipse. The estimates from one hundred identification experiments are also plotted, estimates using IDF are denoted (*) and estimates using white Gaussian noise are denoted (○).

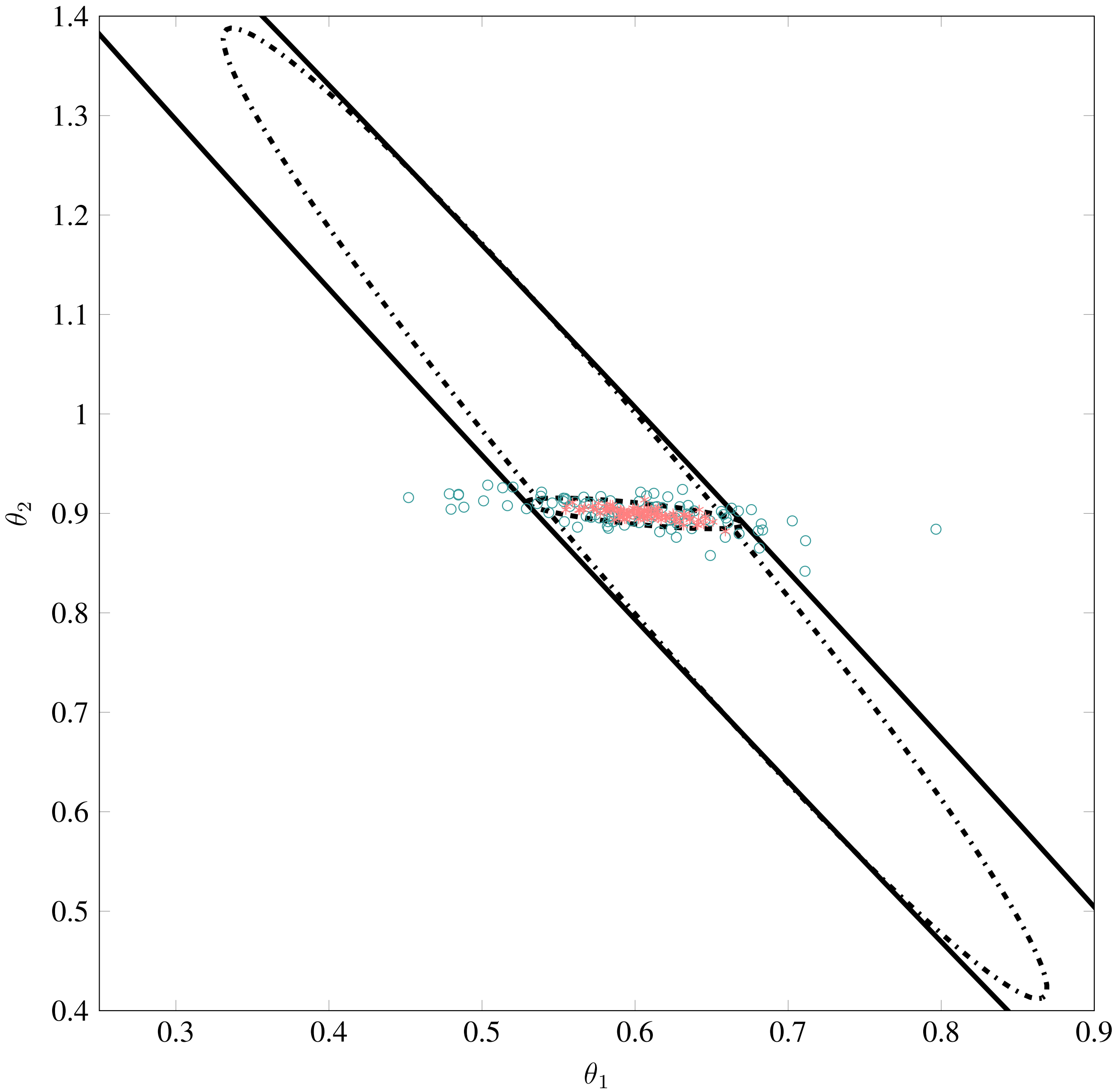
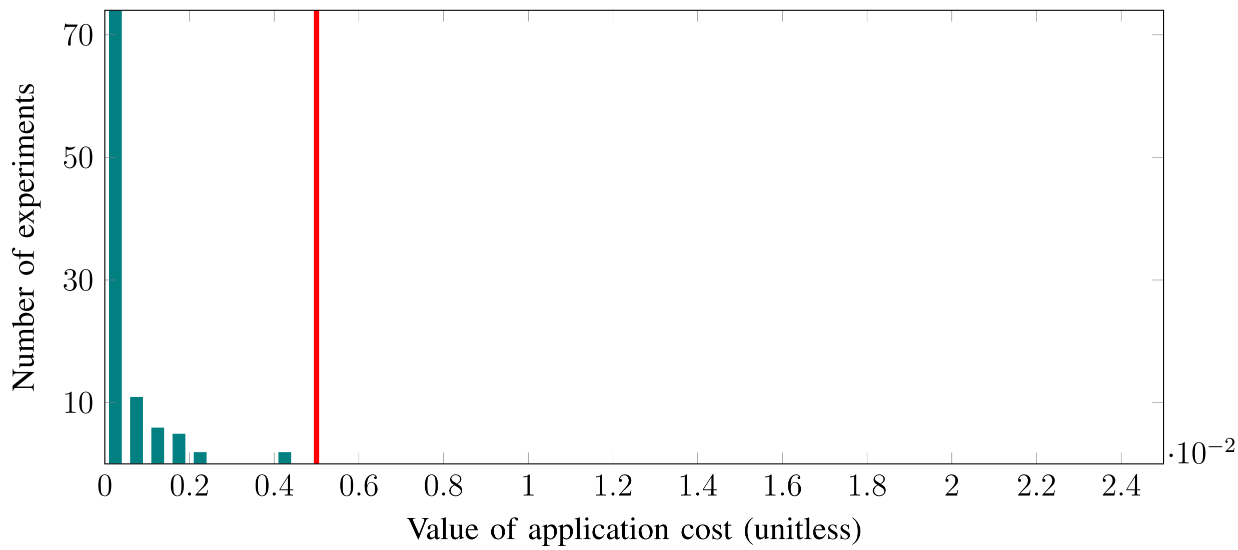
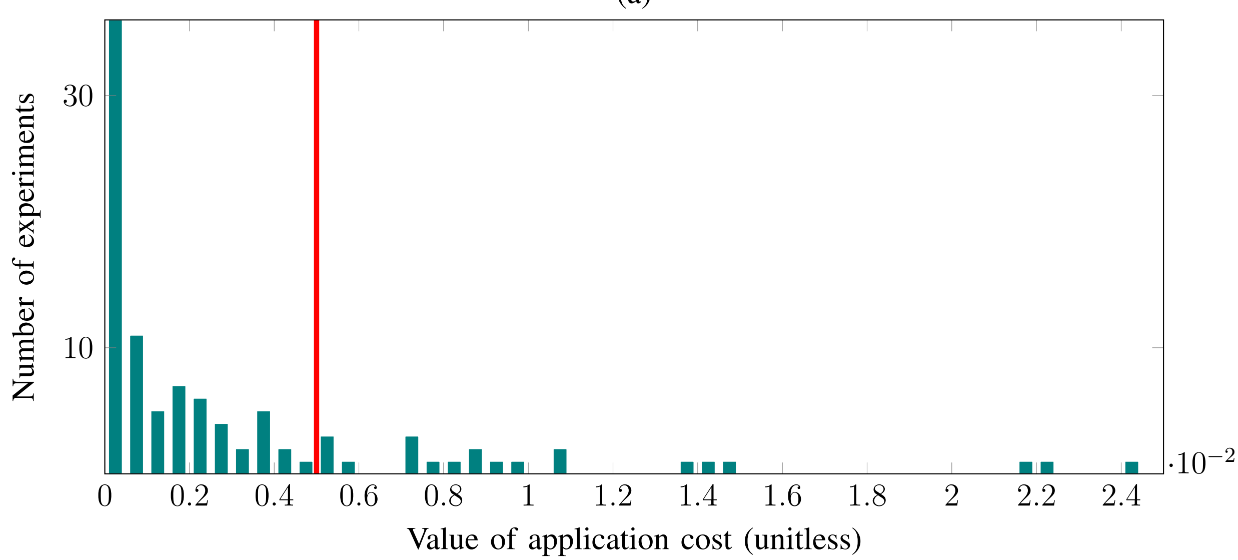


Figure 9. Application-oriented input design with constrained input. Resulting parameter sets for the constrained system in "Simulation of Model Based Control Design" using the ellipsoidal relaxation are shown. The application ellipsoid from the initial estimate (—) is similar in shape but not in size to the true application ellipse (·-·-). However, the shorter semi-axis has the correct length. The system identification set (- - -) is almost contained within the true application ellipse. The estimates from one hundred identification experiments are also plotted, estimates using IDF are denoted (∗) and estimates using white Gaussian noise are denoted (○).

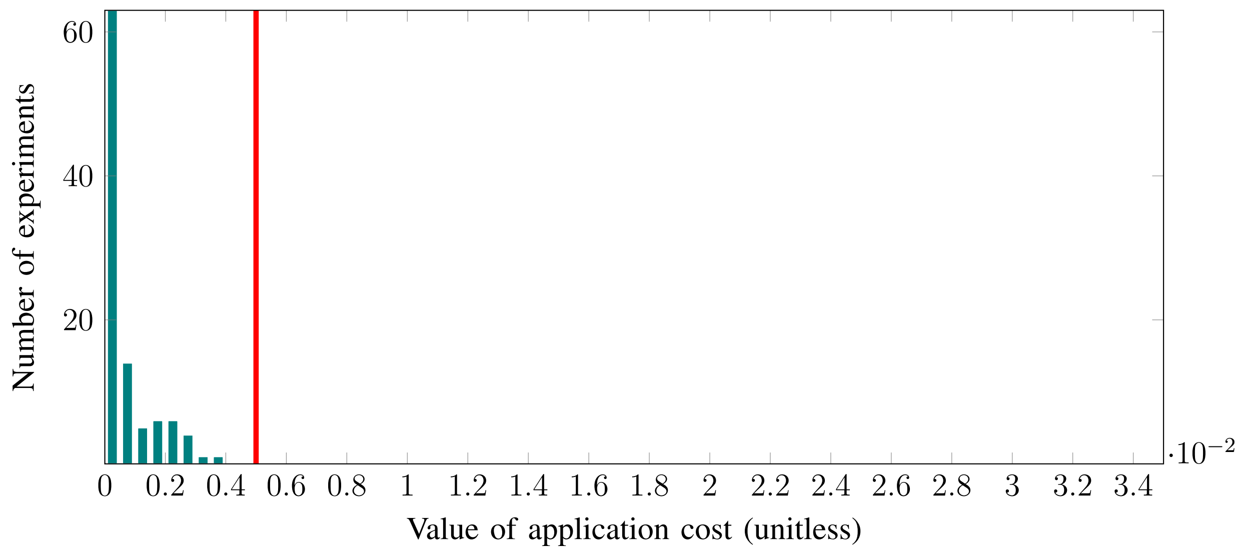


(a)

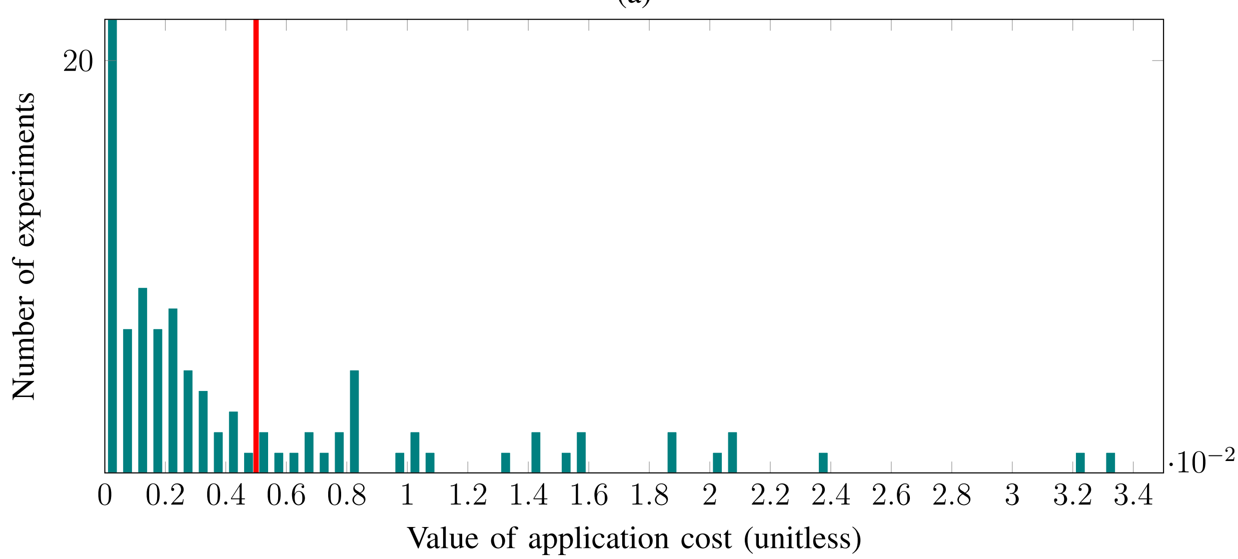


(b)

Figure 10. Application costs. The distribution of the application costs for the unconstrained system in "Simulation of Model Based Control Design" are shown. The costs from IDF are shown in (a) and the costs obtained from white Gaussian noise with power equal to that of the optimal input are shown in (b). The acceptable level of degradation (—) is also shown, its value is $5 \cdot 10^{-3}$. The averaged cost from IDF and white noise are $5 \cdot 10^{-4}$ and $33 \cdot 10^{-4}$, respectively. Each bin is $5 \cdot 10^{-4}$ wide.



(a)



(b)

Figure 11. Application costs. The distribution of the application costs for the constrained system in "Simulation of Model Based Control Design" are shown. The costs from IDF are shown in (a) and the costs obtained from white Gaussian noise with power equal to that of the optimal input are shown in (b). The acceptable level of degradation (—) is also shown, its value is $5 \cdot 10^{-3}$. The averaged cost from IDF and white noise are $7 \cdot 10^{-4}$ and $52 \cdot 10^{-4}$, respectively. Each bin is $5 \cdot 10^{-4}$ wide.

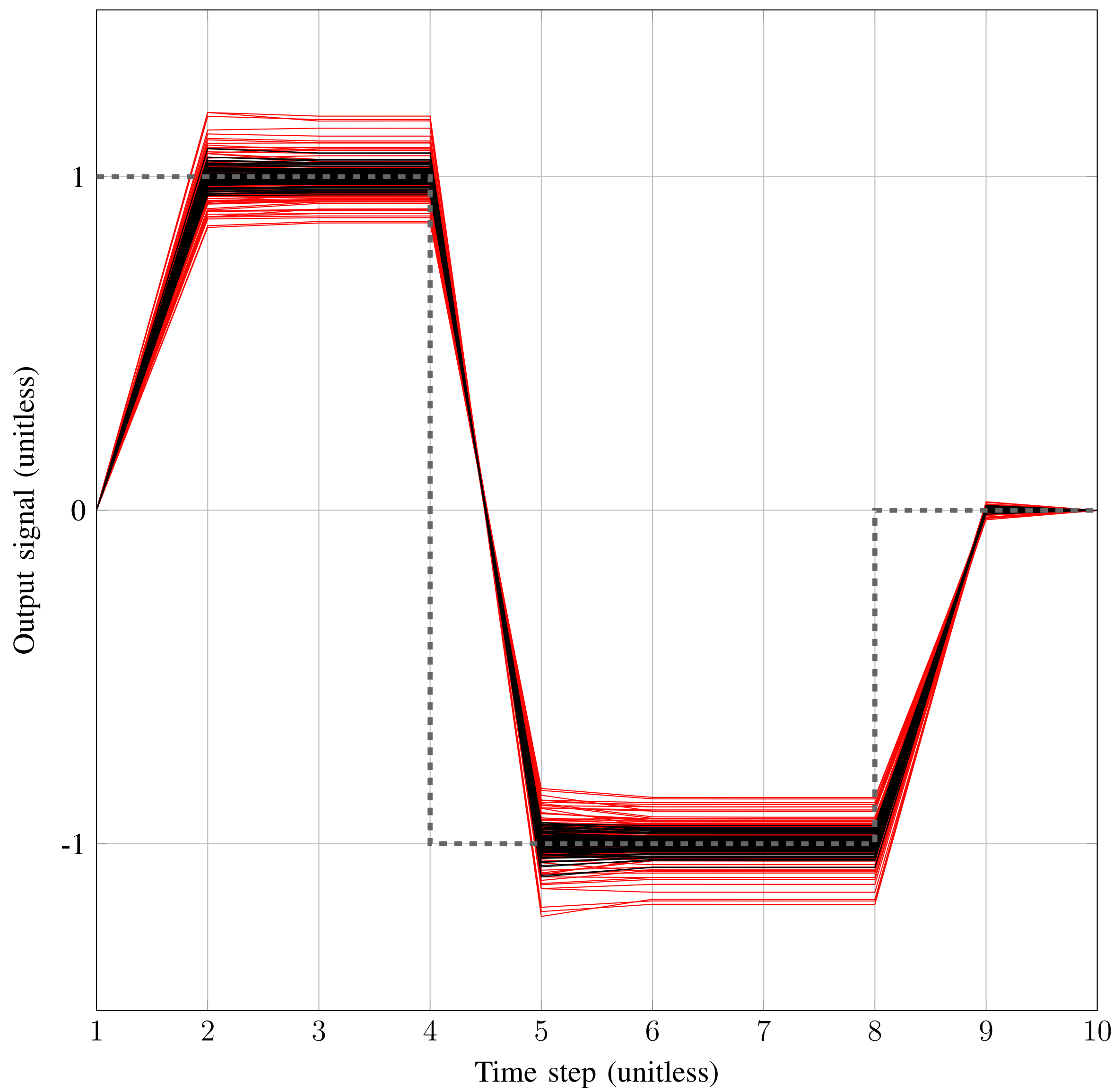


Figure 12. Comparison of application performances for the constrained system in “Simulation of Model Based Control Design”. The trajectories of the process controlled by MPC with models based on estimates from one hundred identification experiments are shown. The models found with IDF were used for the trajectories denoted (——). The models found with white Gaussian noise were used for the trajectories denoted (——). The desired reference trajectories (- - -) are also plotted.

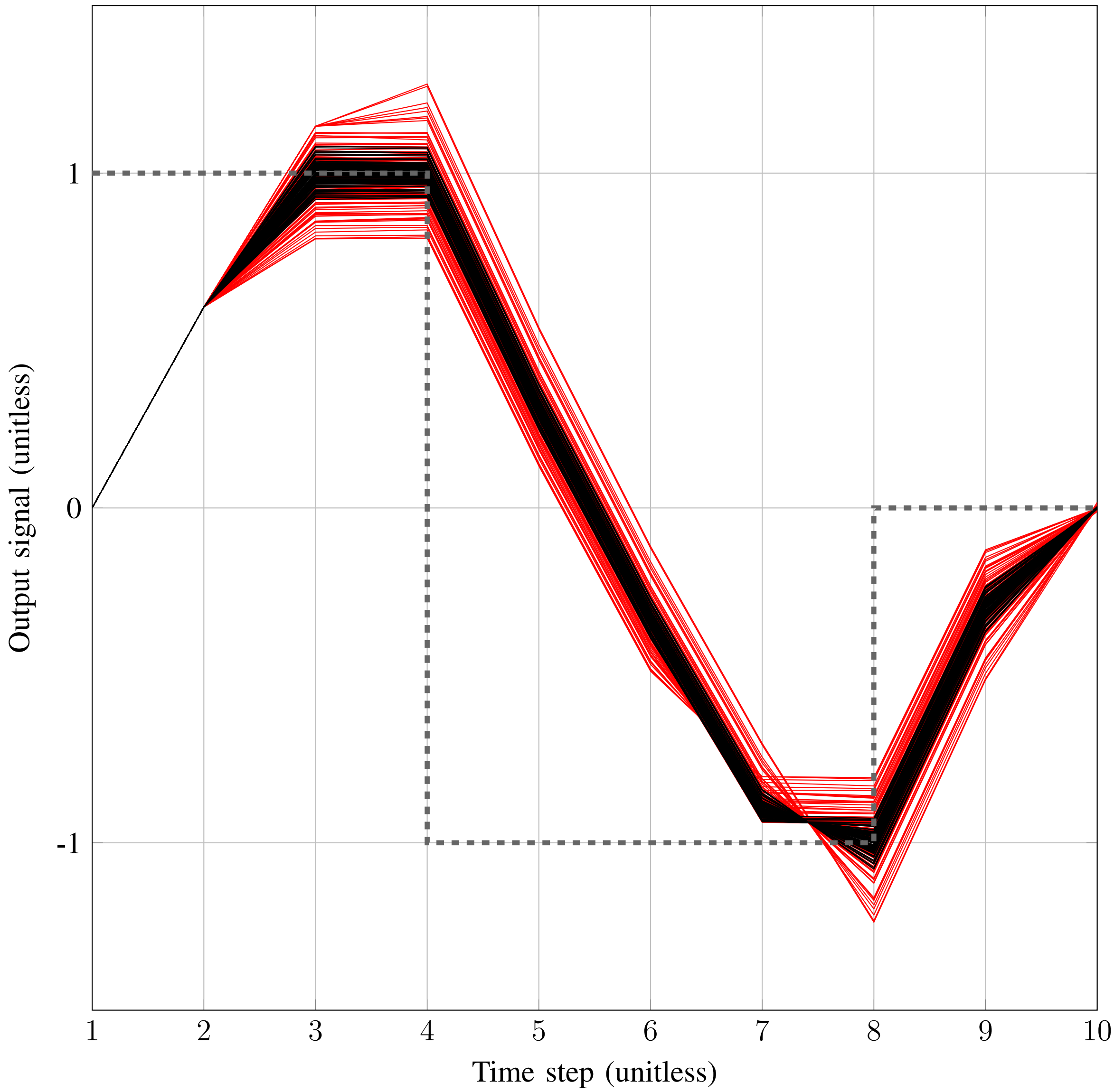


Figure 13. Comparison of application performances for the unconstrained system in “Simulation of Model Based Control Design”. The trajectories of the process controlled by MPC with models based on estimates from one hundred identification experiments are shown. The models found with IDF were used for the trajectories denoted (——). The models found with white Gaussian noise were used for the trajectories denoted (——). The desired reference trajectories (- - -) are also plotted.

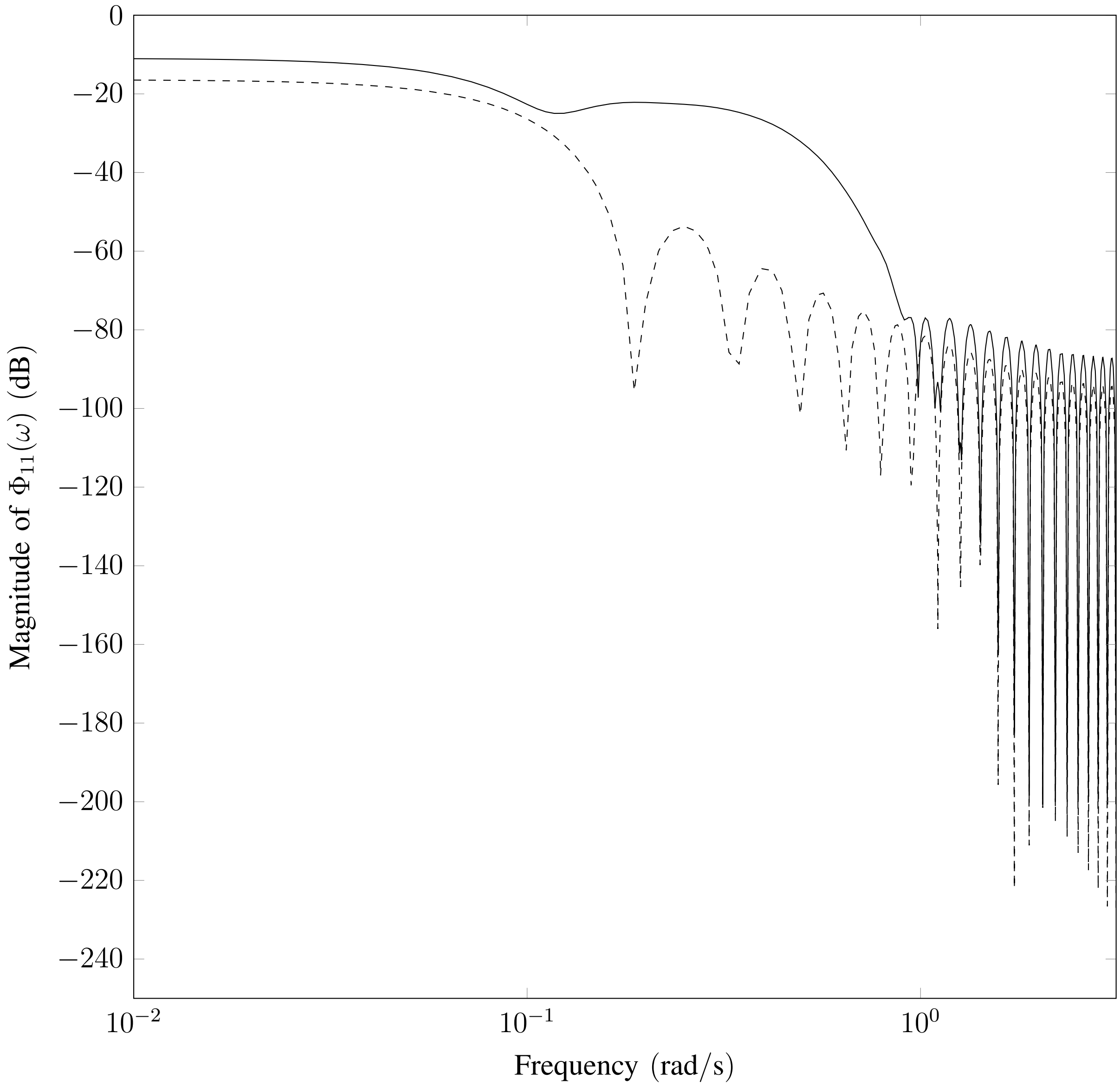


Figure 14. Optimal input spectra for water tanks process. The optimal spectra in Experiment 1 in “Water Tanks Experiment” are shown. The input spectra obtained using application-oriented input design based on θ^0 and initial estimates of the parameters are denoted (——) and (- - -), respectively. The cross spectrum between u_i and u_j is denoted $\Phi_{ij}(\omega)$. The spectra related to u_2 are all zero, that is $\Phi_{12}(\omega) = \Phi_{21}(\omega) = \Phi_{22}(\omega) = 0$ for all ω.

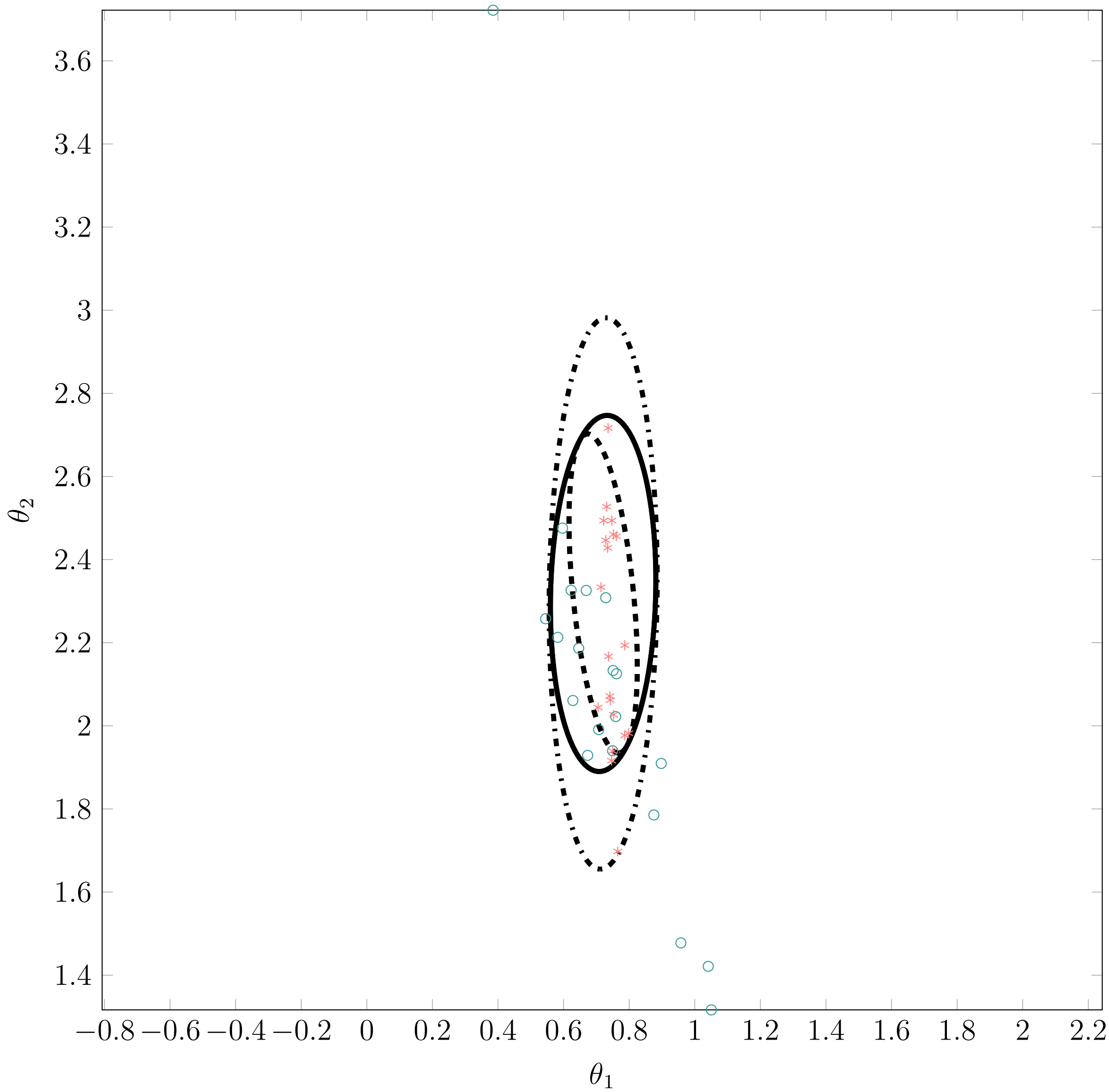


Figure 15. Application-oriented input design for water tanks process. Resulting parameter sets in Experiment 1 in "Water Tanks Experiment" using the ellipsoidal relaxation are shown. The application ellipsoid from the initial estimate (—) is similar in shape but not in size to the true application ellipse (·-·-). However, as in Figure 9, the shorter semi-axis has the correct length. The system identification set using IDF (- - -) is contained within the true application ellipse. The estimates from twenty identification experiments are also plotted, estimates using IDF are denoted (*) and estimates using white Gaussian noise are denoted (○).

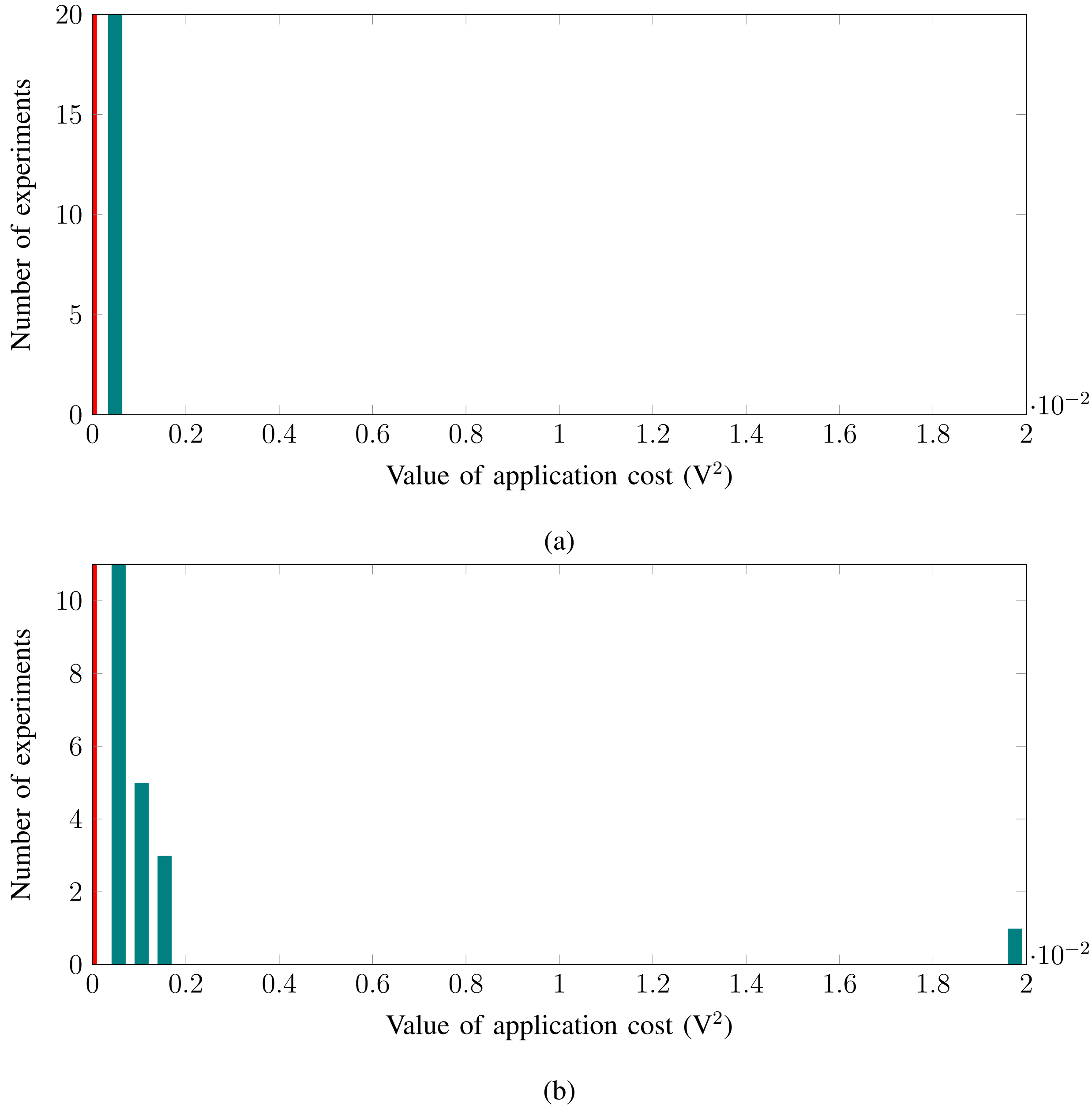
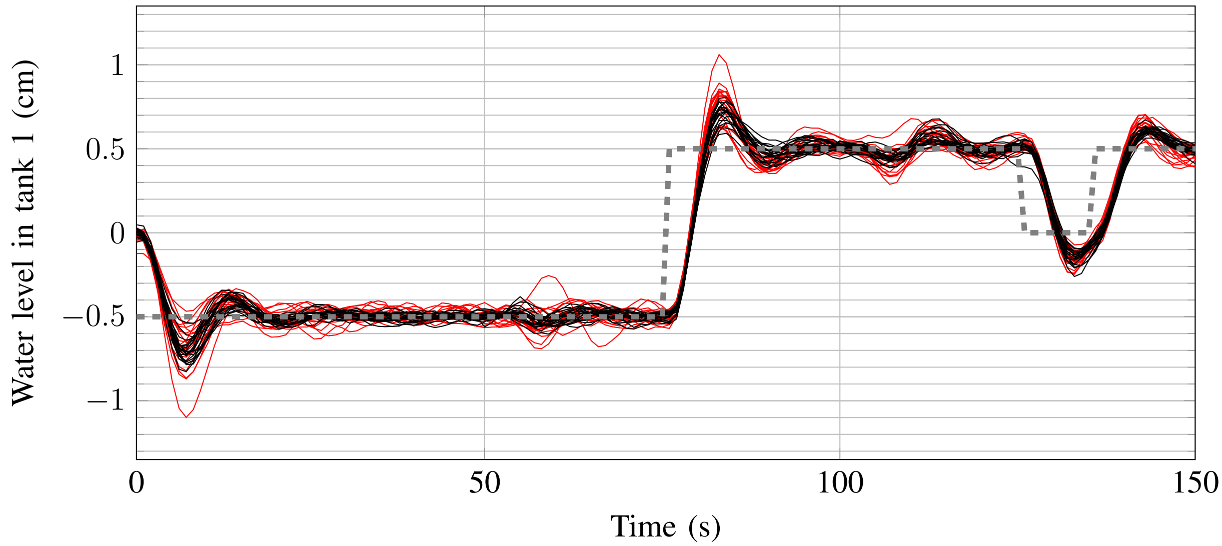
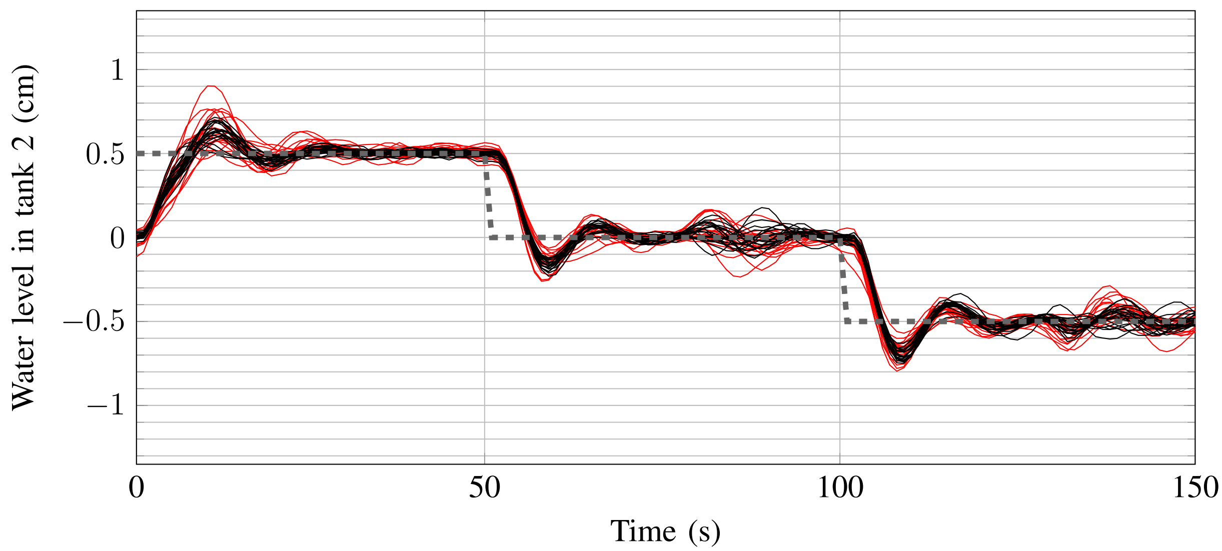


Figure 16. Application costs for water tanks process. The distribution of the application costs in Experiment 1 in "Water Tanks Experiment" are shown. The costs from IDF are shown in (a) and the costs obtained from white Gaussian noise with power equal to that of the optimal input are shown in (b). The acceptable level of degradation (—) is also shown (to the far left), its value is $5 \cdot 10^{-5}$ V^2. The averaged cost from IDF and white noise are $4 \cdot 10^{-4}$ V^2 and $17 \cdot 10^{-4}$ V^2, respectively. Each bin is $5 \cdot 10^{-4}$ V^2 wide.

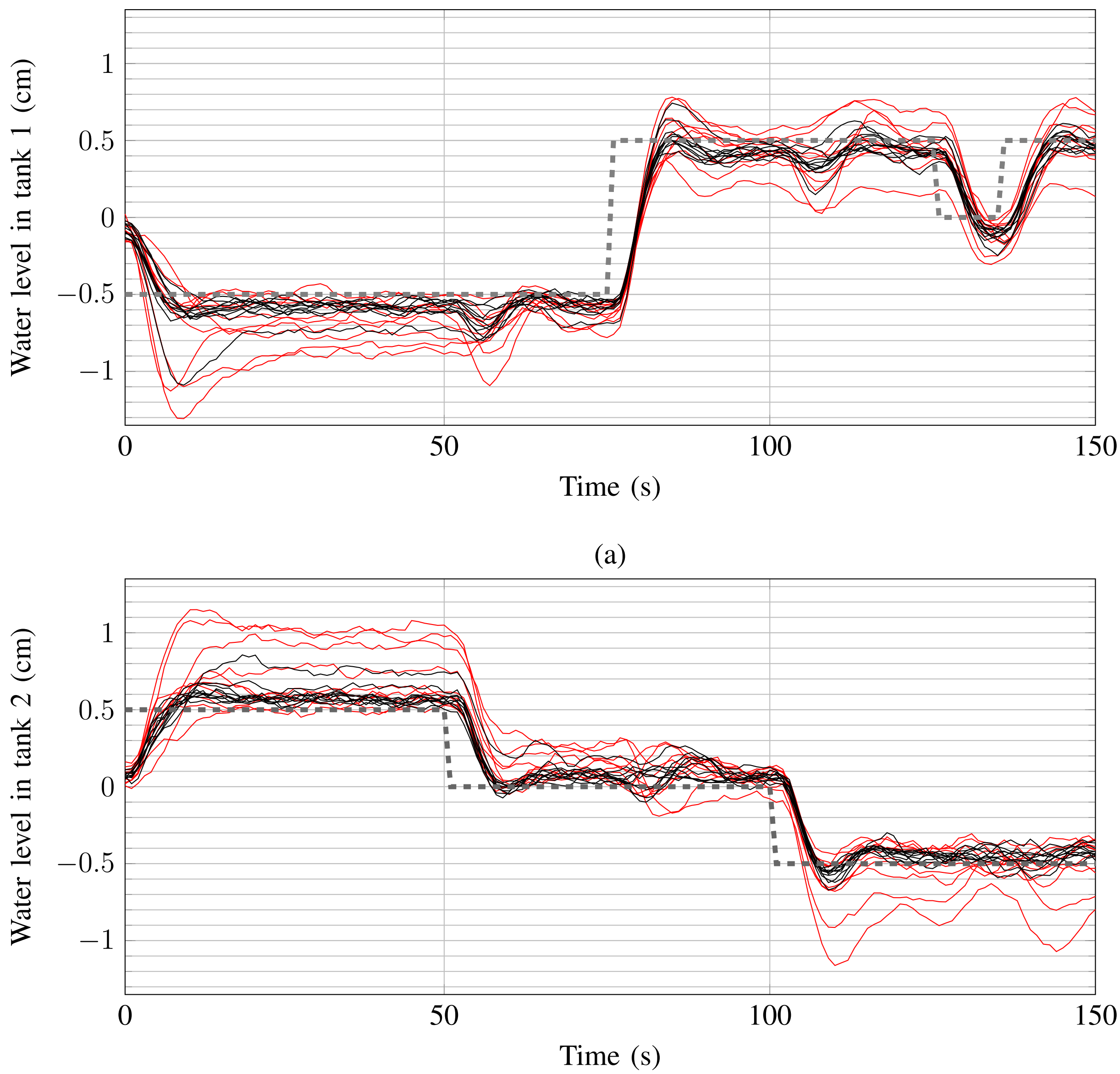


(a)



(b)

Figure 17. Comparison of application performances of Experiment 2 in “Water Tanks Experiment’. The trajectories of the process controlled by MPC with models based on estimates from twenty identification experiments are shown. The trajectories obtained with models from IDF are denoted (——) and the ones obtained from using white Gaussian noise are denoted (——). The desired reference trajectories (- - -) are also plotted. The water levels are shown as deviations from the working points.



(a)

(b)

Figure 18. Comparison of application performances of Experiment 3 in "Water Tanks Experiment". The trajectories of the process controlled by MPC with models based on estimates from ten identification experiments are shown. The trajectories obtained with models from IDF are denoted (——) and the ones obtained from using white Gaussian noise are denoted (——). The desired reference trajectories (- - -) are also plotted. The water levels are shown as deviations from the working points.

Tables

TABLE I

GLOSSARY OF SYMBOLS.

Symbol	Description
t	time in seconds
ω	angular frequency in radians
$e^{i\omega}$	complex exponential where i is the imaginary unit
x^0	true values of x
$\hat{x}_N$	estimated values of x based on N observations
x^*	optimal values of x
x^N	sequence $\{x(t)\}_{t=1}^{N}$
$f'(x_0)$	gradient of f with respect to x, evaluated at x_0
$f''(x_0)$	Hessian of f with respect to x, evaluated at x_0
$\mathrm{E}\{x(t)\}$	expected values of $x(t)$
$\|x(t)\|$	Euclidean norm of $x(t)$
$\mathrm{vec}\, X$	row vector that contains the rows of the matrix X stacked adjacent to each other
$X \succeq Y$	$[X - Y]$ is a positive semidefinite matrix
$X \succ Y$	$[X - Y]$ is a positive definite matrix
$\otimes$	Kronecker product
Φ	spectral density
Φ^+	positive part of spectral density
$\bar{\mathbf{I}}_{\mathrm{F}}$	averaged Fisher information matrix
I_d	identity matrix in $\mathbb{R}^{d\times d}$
$\mathrm{Diag}(x_1, \ldots, x_n)$	diagonal matrix in $\mathbb{R}^{n\times n}$ with the ith diagonal element equal to x_i

TABLE II

GLOSSARY OF ACRONYMS.

Acronym	Description
FIR	finite impulse response
IDF	input design algorithm
LMI	linear matrix inequality
MPC	model predictive control
NMP	non-minimum phase
PEM	prediction error method

TABLE III

INPUT DESIGN FRAMEWORK (IDF).

Step	Action
Step 0	**Initial model**
	Find an initial estimate of the model parameters. The estimate can, for example, be obtained through a short system identification experiment using white Gaussian noise as input signal or through some physical knowledge of the system.
Step 1	**Application cost**
	Evaluate the application cost in accordance with the scenario approach or the ellipsoidal approximation. The evaluation is done in simulations with the initial model acting as the true system.
Step 2	**Optimal spectrum**
	Find the optimal spectrum of the input signal to be used in the system identification experiment. The spectrum is obtained by solving optimization problem (15) when using the scenario approach or (16) when using the ellipsoidal approximation.
Step 3	**Optimal input signal**
	Use the optimal spectrum to generate an input signal with the desired statistical properties. Different methods of input signal generations are mentioned in "Input generation".
Step 4	**Estimate**
	Find the optimal estimate of the model parameters. The estimate is obtained through a system identification experiment using the optimal input signal. The identification method is either the prediction error method or the maximum likelihood method.

TABLE IV

PHYSICAL PARAMETERS OF THE WATER TANKS PROCESS.

Parameter	Description
a_i	cross sectional area of outlet of tank i
A_i	cross sectional area of tank i
γ_j	fraction of flow from pump i pumped to lower tank
k_j	voltage to volumetric flow rate proportionality constant of pump i
l_i	water level to voltage proportionality constant of sensor i

TABLE V

SETTINGS OF "EXAMPLE 2: ACCEPTABLE APPLICATION PERFORMANCE".

The optimal input spectrum is found by solving optimization problem (16) with:

Setting	Description
$\theta^0 = [-0.95\ 1]^{\mathrm{T}}$	true parameters
$\lambda^0 = 1$	true variance of noise $\{e_0(t)\}$
$N = 500$	number of measurements used in the experiment
$1/\gamma = 0.01$	maximal allowed application performance degradation
$\alpha = 0.95$	probability of achieving allowed degradation $1/\gamma$
$\Phi_u^+(\omega) = \frac{1}{2}r_0 + \sum_{k=1}^{9} r_k e^{-i\omega k}$	positive real part of an FIR-shaped input spectrum with ten nonzero autocorrelation coefficients
$\Theta_{\text{app}}=\left\{\theta \middle\| \frac{1}{f^2+1}(\theta-\theta^0)^{\mathrm{T}}\left[\begin{smallmatrix} 1 & -f \\ -f & f^2 \end{smallmatrix}\right](\theta-\theta^0)\leq\frac{1}{\gamma}\right\}$	ellipsoidal application set with infinitely long semi-axis in the direction of $[f\ 1]^{\mathrm{T}}$
$f_{cost} = r_0$	experiment cost equal to the power of the input signal

TABLE VI

VALUES APPROXIMATING THE TRUE WATER TANKS PROCESS.

Parameter	Value when K is estimated	Value when K is assumed zero
a_1	0.06 cm^2	0.06 cm^2
a_2	0.06 cm^2	0.06 cm^2
a_3	0.27 cm^2	0.32 cm^2
a_4	0.14 cm^2	0.14 cm^2
A_i	4.9 cm^2 for $i = 1, \dots, 4$	4.9 cm^2 for $i = 1, \dots, 4$
γ_1	0.72	0.73
γ_2	0.70	0.73
k_1	2.32 cm^3/(sV)	2.36 cm^3/(sV)
k_2	1.86 cm^3/(sV)	1.80 cm^3/(sV)
l_i	0.5 V/cm for $i = 1, 2$	0.5 V/cm for $i = 1, 2$
K	$\begin{bmatrix} 0.34 & 0.01 \\ 0.08 & 0.052 \\ 0.20 & -0.01 \\ 0.06 & 0.07 \end{bmatrix}$ cm/V	0 cm/V
Λ_0	$\begin{bmatrix} 2.5 & 0.3 \\ 0.3 & 13.7 \end{bmatrix} \times 10^{-3}$ V^2	$\begin{bmatrix} 5.1 & 1 \\ 1 & 14.2 \end{bmatrix} \times 10^{-3}$ V^2

Sidebars

Sidebar 1: Related Work

The first results on optimal input design for identification experiments trace back to the statistical literature in the beginning of the 20th century. In the statistics framework, the input signal used in the identification experiment is chosen such that the error of the estimated parameters is minimized subject to any constraints at hand. The error is typically chosen as a scalar function of the covariance matrix of the estimated parameters. For an overview of optimal experiment design in a statistical context, see [32]. In the 1970's, the optimal input design framework in statistics is applied to parameter estimation in system theory. See the survey [33] and book [34] for overviews of the research made.

The system identification community recognized the need of taking the purpose of a model into account when evaluating its accuracy early on. As argued in [35], a survey in system identification from 1971: "If the ultimate purpose is to design a control system then it seems logical that the accuracy of an identification should be judged on the basis of the *performance* of the control system designed from the results of the identification". However, it is not until 1986 in [36] that the model purpose is explicitly incorporated in the input design. The authors introduce the concept of performance degradation due to errors in the transfer function estimates. In the new formulation, the input signal is chosen such that the performance degradation is minimized instead of, as before, the error of the estimated parameters.

During the late 1980's and 1990's, a lot of effort is made to bridge the gap between identification and the growing field of robust control, see [37], [38] and [24].

A notion of plant friendly system identification is formalized during the 1990's in the chemical process control community. Plant friendly system identification is related to application-oriented input design. The objective is to find an optimal trade-off between making an identification experiment as informative as possible while intervening with the normal operation of the plant as little as possible. See [39] for an introduction to the subject and [40] for an example of plant friendly input design.

In 2004 the concept of least-costly identification experiment for control is introduced in [41]. It is further formalized in [17]. The authors consider a dual formulation of the optimization problem presented in [36]. That is, instead of minimizing the performance degradation, an acceptable performance degradation enters the input design problem as a constraint; instead of having constraints on experiment cost, such as experiment length and input energy, experiment cost enters the input design problem in the objective function [42]. The introduced framework is developed further in for example [8]. The author adds the notion of cost of complexity to the input design problem. Cost of complexity is a measure of the experimental cost as a function of system complexity, noise properties, and information to be drawn from the data, see for example [43]. The application-oriented input design considered in this paper is based on the framework of [17] and [8].

There are several current trends related to application-oriented input design. One such trend is to develop methods for simultaneous identification and control of the plant. In [44], a dual control strategy is used. Closed-loop identification experiments and controller designs are performed in a sequential manner. The idea is to gradually improve the performance of the system by improving the model. In [22], the MPC formulation includes the identification procedure. Meaning, the input signal provided by the MPC is designed to give satisfactory control performance while ensuring informative data for identification. In [21], the identification procedure is present but hidden from the control strategy. The framework allows the identification data to be collected in open-loop while a controller is acting on the plant. For other examples of simultaneous identification and control using MPC, see [45], [46], [47], and [48].

Methods of simultaneous identification and control are intertwined with results on signal generation and time domain formulations of the input design problem. In signal generation, an input signal is realized in accordance to an input spectrum while fulfilling signal constraints in the time domain. In [49] and [50], a receding horizon procedure inspired by model predictive control is used. In time domain formulations, signal constraints can be added explicitly in the input design problem instead of implicitly in the signal generation, see [51], [52] and [53].

For an overview of optimal experiment design, see the surveys [54] and [8] and thesis [14]. For an introduction to concepts of least-costly experiment design, see [55].

Sidebar 2: Water Tanks Process

The water tanks process is a laboratory process frequently used in graduate control courses [56]. It consists of four water tanks, two water pumps and a computer. The identification experiment and control design are performed using MATLAB. The system runs at a sampling frequency of 1 Hz, which allows for on-line optimization in model predictive control.

A layout of the water tanks process used in the experimental setup is shown in Figure S1. The main components are two lower tanks, two upper tanks and two pumps. Pump 1 delivers water into tank 1 and tank 3, while pump 2 delivers water to tank 2 and tank 4. The fraction of the water flow that is delivered to the upper and lower tanks, respectively, can be changed by adjusting two valves.

Pressure sensors are located at the bottom of each tank. The signals from the sensors in the two lower tanks are the output signals of the process. They provide information about the water levels. The input signals are the voltages applied to the two pumps.

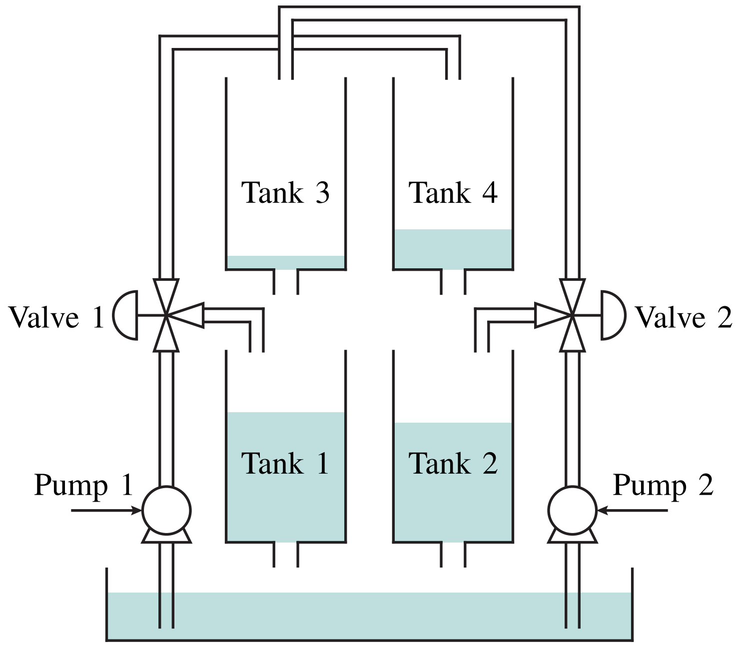


Figure S1. Water tank process. Water is pumped from the basin into the four tanks. The flow from pump 1 fills tanks 1 and 3 while the flow from pump 2 fills tanks 2 and 4. The flow is divided between the tanks according to the settings of the two valves.

Sidebar 3: What is model predictive control?

Model predictive control (MPC) is a flexible and generally applicable control method that is rapidly becoming more and more used in industry. Because of the increased speed of processors and the advent of explicit MPC, predictive control is now finding its way into faster and faster processes.

The advantages of MPC are the simple treatment of multivariate processes and the ability to handle constraints on state variables and signals. The constraints can come from the physics of the plant, such as input or output saturations, or they can be design constraints, for example state levels that result in a deteriorated product. MPC can also handle traditionally difficult constraints such as production costs or environmental aspects.

The core of any MPC implementation is a model of the process that is to be controlled. Typically it is a linear, discrete time model on the form

$$x(t+1) = Ax(t) + Bu(t) + v(t),$$

$$y(t) = Cx(t) + w(t).$$

Here $x(t)$ is the state vector, $u(t)$ is the input, $y(t)$ is the output, and $v(t)$ and $w(t)$ are noise signals. It is possible to use other types of models, for example transfer functions [57]. Versions of MPC with nonlinear models also exist [57].

The model is used to predict future outputs. The control signals are computed based on these predictions. As an example, the input can be calculated using the cost function

$$J(t) = \sum_{i=0}^{N_y} \|\hat{y}(t+i|t) - r(t+i)\|_Q^2 + \sum_{i=0}^{N_u} \|\hat{u}(t+i+2|t) - \hat{u}(t+i+1|t)\|_R^2, \tag{S2}$$

and solving the optimization problem

$$\underset{U(t)}{\text{minimize}} \; J(t), \tag{S3a}$$

$$\text{subject to } \hat{y} \in \mathcal{Y}, \tag{S3b}$$

$$\hat{u} \in \mathcal{U}. \tag{S3c}$$

Here $\hat{y}(t+i|t)$ and $\hat{u}(t+i+1|t)$ are i-step ahead predictions of the output and input signals of the system and$U(t) = \{\hat{u}(t), \dots, \hat{u}(t+N_u)\}$. The values of N_y and N_u define the prediction and control horizon, respectively. That is, they determine how many time steps into the future the MPC takes into account when constructing the input signal. The reference signal is denoted r. The symbols Q and R denote tunable weighting matrices. The norm $\|x\|_A$ is equal to $\sqrt{x^T Ax}$. The regions $\mathcal{Y}$ and $\mathcal{U}$ are the constraint sets for outputs and inputs, respectively.

The solution to optimization problem (S3) gives a sequence of optimal inputs over the control horizon, $U(t) = \{\hat{u}(t), \dots, \hat{u}(t+N_u)\}$. However, only the first input of the sequence, $\hat{u}(t)$, is applied to the system. The optimization then starts over in the next time instant. The way that the prediction and control horizons are moving with time has lead to the alternative name *receding horizon control*. The receding horizon is illustrated in Figure S2.

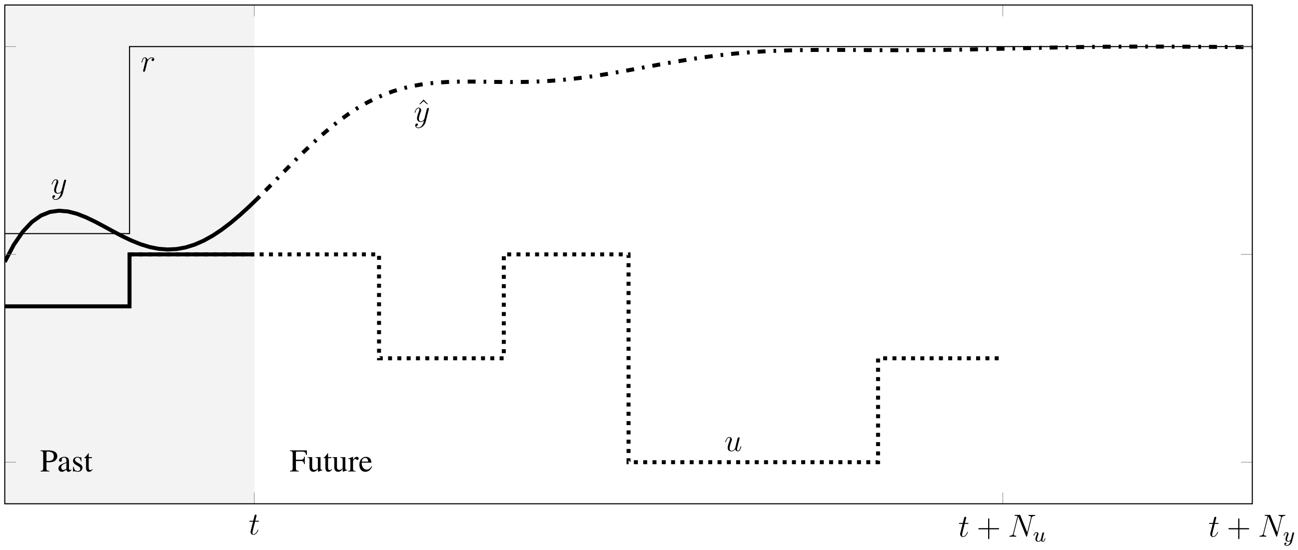


Figure S2. Receding horizon idea. MPC is used to make the system follow the reference trajectory r (——). At time t, the plant output y is predicted (·-·-) over the prediction horizon N_y. The predictions depend on the control inputs u (·····) over the control horizon N_u. The optimal input sequence is calculated and the first input is applied to the plant. The procedure is repeated at time $t + 1$.

Sidebar 4: Prediction Error Method

The prediction error method (PEM) is a method of identifying unknown system parameters θ in the model (S4). PEM can be used for transfer function or state space representations of the system. For details, see [58]. The parameter estimates come from experimental observations of the output and input signal sequences, denoted $Z^N = \{y(t), u(t)\}_{t=1}^N$. The resulting estimates are denoted $\hat{\theta}_N$, where N stands for the number of observations used in the identification experiment.

A causal, multivariate, linear, time invariant system can be described using the parametrized model

$$\mathcal{M}(\theta): \quad y(t) = G(q^{-1}, \theta)u(t) + v(t), \tag{S4a}$$

$$v(t) = H(q^{-1}, \theta)e(t). \tag{S4b}$$

Here $u(t)$ is the input, $y(t)$ is the output and $e(t)$ is zero mean white Gaussian noise with variance Λ. The transfer functions G and H are parameterized by $\theta \in \mathbb{R}^n$ and q^{-1} is the backward-shift operator. It is assumed that the true system, denoted $\mathcal{S}$, can be captured by the model (S4). That is, there is a parameter vector θ^0 such that $\mathcal{S} = \mathcal{M}(\theta^0)$.

Based on the model structure (S4), the one-step-ahead predicted output of the system is

$$\hat{y}(t|t-1; \theta) = H^{-1}(q, \theta)G(q, \theta)u(t) + \left[I - H^{-1}(q, \theta)\right] y(t).$$

The prediction error is defined as the difference between the output of the true system and the output predicted by the model. Consequently, the prediction error becomes

$$\epsilon(t,\theta) = y(t) - \hat{y}(t|\theta) = H^{-1}(q,\theta)\left[y(t) - G(q,\theta)u(t)\right].$$

The parameter estimates are found by minimizing a criterion function of the prediction error with respect to θ. The criterion function to be minimized is denoted $V_N(\theta, Z^N)$. The estimates are defined as

$$\hat{\theta}_N = \arg\min_{\theta} V_N(\theta, Z^N). \tag{S5}$$

A common choice for the criterion function is the quadratic criterion

$$V_N(\theta, Z^N) = \frac{1}{2N}\sum_{t=1}^{N} \epsilon(t,\theta)^T \Lambda^{-1} \epsilon(t,\theta). \tag{S6}$$

The quadratic criterion is used in all examples in this article.

The estimated parameters converge, under mild conditions, to the true values almost surely as the number of observations tends to infinity. Furthermore, the sequence of random variables

$$N(\hat{\theta}_N - \theta^0)^{\mathrm{T}} \bar{\mathbf{I}}_{\mathrm{F}} (\hat{\theta}_N - \theta^0),$$

where $\bar{\mathbf{I}}_{\mathrm{F}}$ is the averaged Fisher information matrix, converges in distribution to the χ^2 distribution with n degrees of freedom. Hence, for a sufficiently large N, the estimates $\hat{\theta}_N$ are contained inside

$$\mathcal{E}_{\mathrm{SI}} = \left\{ \theta \;\middle|\; (\theta - \theta^0)^{\mathrm{T}} \bar{\mathbf{I}}_{\mathrm{F}} (\theta - \theta^0) \leq \frac{\chi^2_\alpha(n)}{N} \right\}, \tag{S7}$$

with probability α. Here $\chi^2_\alpha(n)$ is the α-percentile of the χ^2 distribution with n degrees of freedom.

The averaged Fisher information matrix is an affine function of the input spectrum Φ_u. Thus, the estimates can be directly affected by designing the spectrum of the input signal.

Sidebar 5: Fisher Information

A causal, multivariate, linear, time invariant system can be described using the parametrized model

$$\mathcal{M}(\theta): \quad y(t) = G(q^{-1}, \theta)u(t) + v(t), \tag{S8a}$$

$$v(t) = H(q^{-1}, \theta)e(t). \tag{S8b}$$

Here $u(t)$ is the input, $y(t)$ is the output and $e(t)$ is zero mean white Gaussian noise with variance Λ. The transfer functions G and H are parameterized by $\theta \in \mathbb{R}^n$ and q^{-1} is the backward-shift operator. It is assumed that the true system, denoted $\mathcal{S}$, can be captured by the model (S8). That is, there is a parameter vector θ^0 such that $\mathcal{S} = \mathcal{M}(\theta^0)$.

An estimate of θ can be evaluated in terms of the mean square error. For any unbiased estimate, the Cramér-Rao inequality gives a lower bound on the mean square error of the estimate. Formally, when $\hat{\theta}_N$ is an unbiased estimate of θ, based on the random variable $y^N = \{y(t)\}_{t=1}^N$ with conditional probability density $f_y(\theta; y^N)$, then

$$E_{y^N|\theta^0}\left\{\left(\hat{\theta}_N - \theta^0\right)\left(\hat{\theta}_N - \theta^0\right)^T\right\} \geq \mathbf{I}_{\mathrm{F}}^{-1}, \tag{S9}$$

where $\mathbf{I}_{\mathrm{F}}$ is the Fisher information matrix defined as

$$\mathbf{I}_{\mathrm{F}} \triangleq \mathrm{E}\left\{\left(\frac{\mathrm{d}}{\mathrm{d}\theta}\log f_y(\theta; y^N)\right)\left(\frac{\mathrm{d}}{\mathrm{d}\theta}\log f_y(\theta; y^N)\right)^T\right\}\Bigg|_{\theta=\theta^0}. \tag{S10}$$

Based on the model structure (S8), the predicted output of the system is

$$\hat{y}(t|\theta) = H^{-1}(q, \theta)G(q, \theta)u(t) + \left[I - H^{-1}(q, \theta)\right]y(t).$$

It can be shown that, under the assumption of Gaussian noise,

$$\mathbf{I}_{\mathrm{F}} = \sum_{t=1}^{N} \mathrm{E}\left\{ \left(\frac{\mathrm{d}}{\mathrm{d}\theta} \hat{y}(t,\theta) \right) \Lambda^{-1} \left(\frac{\mathrm{d}}{\mathrm{d}\theta} \hat{y}(t,\theta) \right)^{\mathrm{T}} \right\} \Bigg|_{\theta=\theta^0}, \tag{S11}$$

see [58]. However, as N becomes large it is more convenient to work with the averaged Fisher information matrix instead,

$$\bar{\mathbf{I}}_{\mathrm{F}} = \frac{1}{N} \sum_{t=1}^{N} \mathrm{E}\left\{ \left(\frac{\mathrm{d}}{\mathrm{d}\theta} \hat{y}(t,\theta) \right) \Lambda^{-1} \left(\frac{\mathrm{d}}{\mathrm{d}\theta} \hat{y}(t,\theta) \right)^{\mathrm{T}} \right\} \Bigg|_{\theta=\theta^0}. \tag{S12}$$

The averaged information matrix can be expressed in the frequency domain as

$$\begin{aligned} \bar{\mathbf{I}}_{\mathrm{F}} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \Gamma_u(e^{i\omega}, \theta^0)(\Lambda^{-1} \otimes \Phi_u(e^{i\omega}))\Gamma_u^{\mathrm{T}}(e^{-i\omega}, \theta^0) d\omega + \\ \frac{1}{2\pi} \int_{-\pi}^{\pi} \Gamma_e(e^{i\omega}, \theta^0)(\Lambda^{-1} \otimes \Lambda(e^{i\omega}))\Gamma_e^{\mathrm{T}}(e^{-i\omega}, \theta^0) d\omega, \end{aligned} \tag{S13a}$$

when $N \to \infty$ and where

$$\Gamma_u = \begin{bmatrix} \mathrm{vec}\, F_u^1 \\ \vdots \\ \mathrm{vec}\, F_u^n \end{bmatrix}, \ \Gamma_e = \begin{bmatrix} \mathrm{vec}\, F_e^1 \\ \vdots \\ \mathrm{vec}\, F_e^n \end{bmatrix}, \tag{S13b}$$

$$F_u^i = H^{-1} \frac{dG(\theta)}{d\theta_i}, \ F_e^i = H^{-1} \frac{dH(\theta)}{d\theta_i}, \ \text{for all } i = 1 \ldots n. \tag{S13c}$$

Here θ_i denotes the i:th component of the vector θ. Furthermore, $\mathrm{vec}\, X$ denotes a row vector that contains the rows of the matrix X stacked adjacent to each other, and $\otimes$ is the Kronecker product. For details on the Fisher information matrix in an identification setting, see for example [58].

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1999 - 2001. He is a Fellow of the IEEE for his contributions to system identification using orthonormal basis functions. He is a co-founder of Centre of Autonomous Systems and the Linnaeus Center ACCESS on networked systems at KTH. He is the KTH director and PI for the Wallenberg Autonomous Systems Program (WASP), that was granted over 200 million USD over ten years for research into autonomous systems and software development. His research interests include system identification, modeling and control of industrial processes, and statistical signal processing with applications in smart transportation systems.