KTH Electrical Engineering

On Optimal Input Signal Design in System Identification for Control

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Motivation

This poster considers a recently proposed framework for experiment design in system identification for control. We study models obtained by means of a prediction error system identification method. The degradation in control performance due to uncertainty in the model estimate is specified by an application cost function. The objective is to find a minimum variance input signal, to be used in system identification experiment, such that the control application specification is guaranteed with a given probability when using the estimated model in the control design.

Application Set

Application Cost: $V_{app}(\theta)$ such that

$$V_{app}(\theta_0) = 0,\ V'_{app}(\theta_0) = 0 \text{ and } V''_{app}(\theta_0) \succeq 0,$$

where θ_0 is the true parameter. The cost emphasizes an important performance quality of the system, *e.g.*, static gain.

Application Specification: $V_{app}(\theta) \leq 1/(2\gamma),\ \gamma > 0.$

Approximation: $V_{app}(\theta) \approx 1/2(\theta - \theta_0)^{\mathrm{T}} V''_{app}(\theta_0)(\theta - \theta_0).$

Parameter Region:

$$\mathcal{E}_{V_{app}} = \{\theta \mid (\theta - \theta_0)^{\mathrm{T}} V''_{app}(\theta_0)(\theta - \theta_0) \leq \frac{1}{\gamma}\}.$$

Example 1: First order FIR system

$$y(t) = b_1^0 u(t-1) + b_2^0 u(t-2) + d(t).$$

A feedback P-regulator is used to reduce the influence of $d(t)$ in $y(t)$. By using the controller gain

$$K = \frac{\beta^2}{\beta b_1 - b_2}, \quad 0 < \beta \leq 1,$$

we obtain nominal closed loop poles in $-\beta$ and $-\beta b_2/(\beta b_1 - b_2)$. Let

$$V_{app}(\theta) = [F(\beta b_1 - b_2) - F(\beta b_1^0 - b_2^0)]^2,$$

where $F(x)$ is the static gain. Then

$$V''_{app}(\theta_0) = 2[F'(\beta b_1^0 - b_2^0)]^2 \begin{bmatrix} \beta^2 & -\beta \\ -\beta & 1 \end{bmatrix},$$

with eigenvalues $\beta^2 + 1$ and 0, and eigenvectors $[-\beta,\ 1]^T$ and $[1,\ \beta]^T$.

System Identification Set

Identification Cost: $V_{SI}(\theta)$ is the asymptotic LS criterion with

$$V_{SI}(\theta_0) = \lambda_e,\ V'_{SI}(\theta_0) = 0 \text{ and } V''_{SI}(\theta_0) = 2\mathbf{R},$$

where λ_e is the variance of measurement noise, $\mathbf{R} = \mathrm{E}\{\varphi(t)\varphi^{\mathrm{T}}(t)\}$ and $\varphi(t)$ is the regression vector.

Parameter Region: $(\hat{\theta}_N - \theta_0)^T \mathbf{R}(\hat{\theta}_N - \theta_0)$ is χ^2-distributed, where $\hat{\theta}_N$ is the LS estimate. We can find an η such that the estimated parameter will lie inside an ellipsoid

$$\mathcal{E}_{V_{SI}} = \{\theta \mid (\theta - \theta_0)^{\mathrm{T}} \mathbf{R}(\theta - \theta_0) \leq \eta\}$$

with a given probability.

Example 2: First order FIR system yields

$$\mathbf{R} = \begin{bmatrix} r_0 & r_1 \\ r_1 & r_0 \end{bmatrix},\ r_\tau = \mathrm{E}\{u(t)u(t-\tau)\}.$$

Optimal Input Signal Design

Convex Optimization Problem (SDP):

$$\underset{r_\tau}{\text{minimize}}\ r_0,\ \text{subject to } \mathbf{R} \succeq \kappa\gamma V''_{app}(\theta_0).$$

Geometric Interpretation: $\mathbf{R} \succeq \kappa\gamma V''_{app}(\theta_0)$ is equivalent to $\mathcal{E}_{V_{SI}} \subseteq \mathcal{E}_{V_{app}}$.

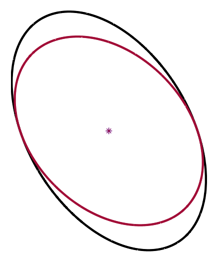

Figure 1: $\mathcal{E}_{V_{SI}}$ (red) and $\mathcal{E}_{V_{app}}$ (black).

Example 1 and 2: We can analytically solve the optimization problem. The optimal input signal can be realized by an AR-process: $u(t) = -au(t-1) + e_u(t)$ with $a = \beta$. Thus,

- AR-pole equal to nominal closed loop pole,
- increased gain of controller yields increased bandwith of excitation signal.

Conclusion

The optimal input signal design developed in [2] relates the experiment design to the intended application of the model. This framework has been illustrated on a basic control problem. More examples are considered in [1].

References

[1] B. Wahlberg, H. Hjalmarsson, M. Annergren, "On Optimal Input Design in System Identification for Control", 49th IEEE CDC, 2010, to appear.

[2] H. Hjalmarsson, "System Identification of Complex and Structured Systems", European Journal of Control, 2009.

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