An ADMM Algorithm for Solving ℓ_1 Regularized MPC

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We present an Alternating Direction Method of Multipliers (ADMM) algorithm for solving optimization problems with an ℓ_1 regularized least-squares cost function subject to recursive equality constraints. The optimization problem has applications in control, such as ℓ_1 regularized MPC, [1].

l_1 Regularized MPC

Control objective: Drive the output y to zero, using few changes of the input u.

MPC idea: A cost function is minimized with respect to $u(t+i)$ for some time horizon $i = 0, \ldots, H_u - 1$. Only $u(t)$ is applied to the system. The cost function is minimized again in an iterative manner for each time step t.

Cost function:

$$V(t) = \sum_{i=1}^{H_p} \|y(t+i-1)\|_2^2 + \lambda \sum_{i=1}^{H_u} \|\Delta u(t+i-1)\|_1,$$

where $\Delta u(t+i) = u(t+i) - u(t+i-1)$. The ℓ_1-norm of Δu promotes sparse Δu.

Optimization problem solved in each time step t:

$$\begin{array}{ll} \text{minimize} & V(t), \\ \text{subject to} & x(t+i) = Ax(t+i-1) + Bu(t+i-1), \\ & y(t+i-1) = Cx(t+i-1),\ i = 1, \ldots, H_p. \end{array} \tag{1}$$

ADMM

Optimization problem: ADMM is an algorithm for solving

$$\begin{array}{ll} \text{minimize} & f(x), \\ \text{subject to} & x \in \mathscr{C}, \end{array} \Leftrightarrow \begin{array}{ll} \text{minimize} & f(x) + I_{\mathscr{C}}(x_c), \\ \text{subject to} & x = x_c, \end{array} \tag{2}$$

where $f(x)$ is a convex function, $\mathscr{C}$ is a convex set and $I_{\mathscr{C}}(x_c)$ is the indicator function of $\mathscr{C}$.

ADMM at iteration k:

$$\textbf{Step 1} \quad x^{k+1} := \arg\min_{x} \{f(x) + (\rho/2)\|x - x_c^k + x_d^k\|_2^2\}.$$

$$\textbf{Step 2} \quad x_c^{k+1} := \arg\min_{x_c} \{I_{\mathscr{C}}(x_c) + (\rho/2)\|x^{k+1} - x_c + x_d^k\|_2^2\}.$$

$$\textbf{Step 3} \quad x_d^{k+1} := x_d^k + (x^{k+1} - x_c^{k+1}).$$

Here x_d is the dual variable of (2) scaled by $1/\rho$, $\rho > 0$.

Stopping criteria: ADMM is iterated until stopping criteria based on the norms of the primal and dual residuals of (2) are fulfilled. The residuals are

$$e_p^k = (x^k - x_c^k), \quad e_d^k = -\rho(x_c^k - x_c^{k-1}).$$

l_1 Regularized MPC and ADMM: The optimization problem (1) can be formulated as (2), and solved using ADMM. **Step 1** and **Step 3** are straightforward to perform. **Step 2** is a projection, which we solve using a Riccati recursion.

Control of Tank Process

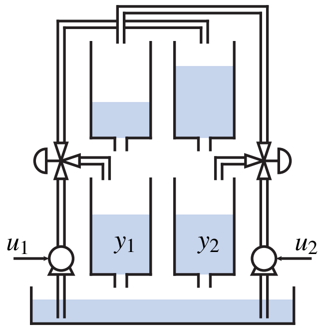

Figure 1: Tank process.

A tank process is controlled using l_1 regularized MPC. The MPC controls the pump voltages, u_1 and u_2. The water levels of the two lower tanks, y_1 and y_2, are the outputs.

The control objective is to drive y_1 and y_2 to the equilibrium points, using few changes of u_1 and u_2.

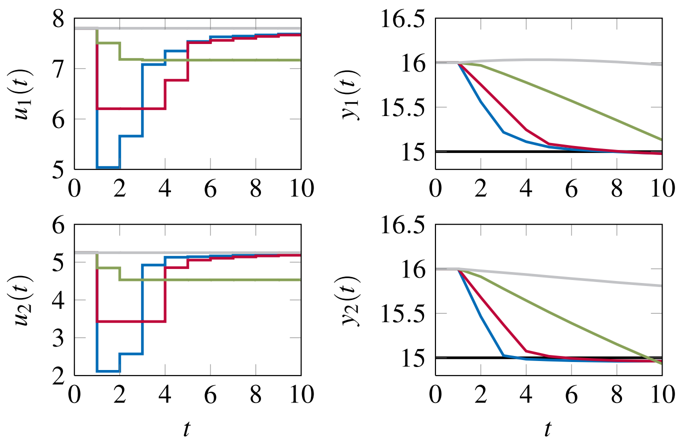

Figure 2: Inputs (u_1 and u_2) and outputs (y_1 and y_2). The sequences (—), (—), (—) and (—) correspond to λ equal to 0.05, 0.1, 2 and 5 respectively. The sequence (—) shows the equilibrium point of the water levels.

Conclusion

- l_1 Regularized MPC.
- ADMM is easy to implement.
- ADMM converges fast to moderate accuracy.
- ADMM enables parallel execution.

Further Reading

[1] M. Gallieri, J. M. Maciejowski "l_{asso} MPC: Smart Regulation of Over-Actuated Systems", to appear in ACC 2012.

[2] M. Annergren, A. Hansson, B. Wahlberg "An ADMM Algorithm for Solving l_1 Regularized MPC", submitted.

[3] S. Boyd, N. Parikh, E. Chu, B. Peleato, J. Eckstein "Distributed Optimization and Statistical Learning via the Alternating Direction Method of Multipliers", Foundations and Trends in Machine Learning 2012.

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