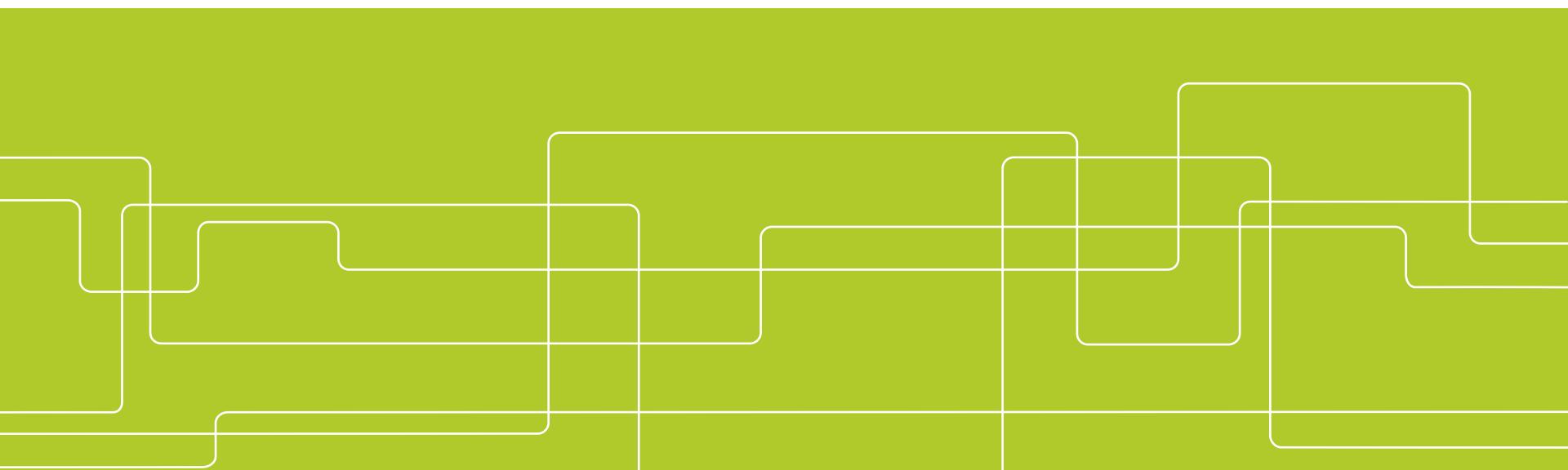




Application Set Approximation in Optimal Input Design for Model Predictive Control

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Introduction

Framework for experiment design in system identification for control.

- Objective:
 - Find optimal input signal to be used in system identification experiment.
- Such that:
 - The control application specification is guaranteed when using the estimated model in the control design.



Notation

The model structure is parametrized by θ .

- True system is given by θ_0 .
- Estimated model is given by $\hat{\theta}$.

Application set

- Application cost: $V_{app}(\theta)$ such that

$$V_{app}(\theta_0) = 0, \quad V'_{app}(\theta_0) = 0, \quad V''_{app}(\theta_0) \geq 0.$$

- Application specification:

$$V_{app}(\theta) \leq \frac{1}{2\gamma}, \quad \gamma > 0.$$

Application set (cont.)

- Acceptable parameter set:

$$\Theta(\gamma) = \left\{ \theta \mid V_{app}(\theta) \leq \frac{1}{2\gamma} \right\}.$$

- Ellipsoidal approximation:

$$\Theta(\gamma) \approx \mathcal{E}_{app}(\gamma) = \left\{ \theta \mid (\theta - \theta_0)^T V''_{app}(\theta_0) (\theta - \theta_0) \leq \frac{1}{\gamma} \right\}.$$

System identification set

Asymptotic property:

$$\hat{\theta} \in \mathcal{E}_{SI}(\eta) = \{\theta \mid (\theta - \theta_0)^T I_F (\theta - \theta_0) \leq \eta\}.$$

(Key result from prediction error/maximum likelihood system identification.)

Optimal input design

- Estimated parameters:

$$\hat{\theta} \in \mathcal{E}_{SI}(\eta) = \{\theta \mid (\theta - \theta_0)^T I_F (\theta - \theta_0) \leq \eta\}.$$

- Acceptable parameters in application:

$$\hat{\theta} \in \Theta(\gamma) \approx \mathcal{E}_{app}(\gamma) = \left\{ \theta \mid (\theta - \theta_0)^T V_{app}''(\theta_0) (\theta - \theta_0) \leq \frac{1}{\gamma} \right\}.$$

- Experiment cost:

$$f_{cost}(\Phi_u, \Phi_y).$$



Optimal input design (cont.)

minimize $f_{cost}(\Phi_u, \Phi_y),$
 Φ_u, Φ_y

subject to $\mathcal{E}_{SI}(\eta) \subseteq \Theta(\gamma),$

$0 \leq \Phi_u(\omega), \forall \omega,$

$0 \leq \Phi_y(\omega), \forall \omega.$

Optimal input design (cont.)

Approximative problem formulation

$$\underset{\Phi_u, \Phi_y}{\text{minimize}} \quad f_{cost}(\Phi_u, \Phi_y),$$

$$\text{subject to} \quad I_F \geq \eta \gamma V''_{app}(\theta_0),$$

$$0 \leq \Phi_u(\omega), \forall \omega,$$

$$0 \leq \Phi_y(\omega), \forall \omega.$$

convex problem



Hessian, V''_{app}

- No analytical expression when using MPC.
- Discretized approximation not applicable due to too many variables.
 - Requires one simulation run of closed loop system for each point.

Can we get analytical expression in one go?

Considered application cost

- Application cost used for MPC:

$$V_{app}(\theta) = \frac{1}{M} \sum_{t=1}^M \|y(t, \theta_0) - y(t, \theta)\|_2^2$$

- Simulation based approximation:

$$\hat{V}_{app}(\theta) = \frac{1}{M} \sum_{t=1}^M \|y(t, \hat{\theta}, \hat{\theta}) - y(t, \theta, \hat{\theta})\|_2^2$$

Key idea

- Use Taylor expansion of $y(t, \theta, \hat{\theta})$:

$$y(\theta) = y(\hat{\theta}) + \sum_{i=1}^n \frac{\partial y(\hat{\theta})}{\partial \theta_i} \delta \theta_i + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \frac{\partial^2 y(\hat{\theta})}{\partial \theta_i \partial \theta_j} \delta \theta_i \delta \theta_j$$

- For $\theta = \hat{\theta} + \delta \theta$, the active constraints in the MPC are the same as for $\theta = \hat{\theta}$.

Key idea (cont.)

- Run MPC for each time instance t with $\theta = \hat{\theta}$.
- Get optimal $u(t, \hat{\theta})$.
- Treat active constraints as equality constraints.
- Calculate explicit expressions of derivatives.
(Perturbation analysis)

Numerical example

- System:

$$\begin{aligned}x(t+1) &= \theta_2 x(t) + u(t) \\y(t) &= \theta_1 x(t) + e(t)\end{aligned}$$

- System settings:

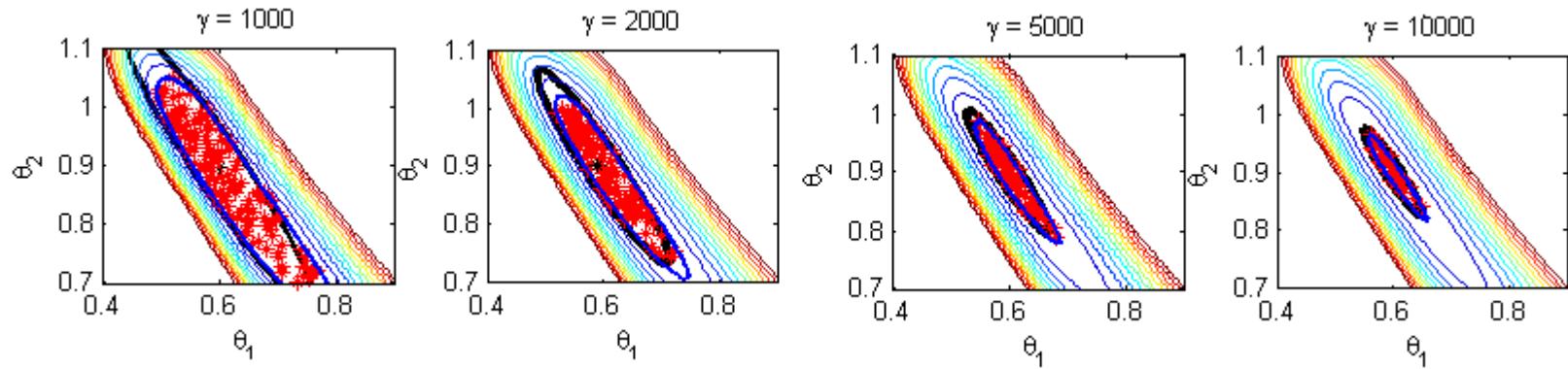
$$\theta_0 = [0.6 \ 0.9], \quad \lambda_e^2 = 0.01, \quad N = 10$$

- MPC settings:

$$\begin{aligned}N_{MPC} &= 5, \\u_{max} &= -u_{min} = 1, \\y_{max} &= -y_{min} = 2\end{aligned}$$

Numerical example

- Same result as with numerical Hessian (DERIVEST).
- Only one simulation run instead of $O(6n^2)$.
- 12 seconds instead of 94 seconds.



Conclusions

- Time efficient calculation of application set approximation.
- MPC on nonlinear plants.
- More complicated noise structures.
- Higher order derivatives.